Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **Essential Understandings** ● Comparisons can be additive or multiplicative depending on th mathematical situation. ● In multiplicative comparisons, the relationship between quantities is described in terms of how many times larger one i than the other	**Common Misconceptions** **Key words are misleading.** Some key words typically mean addition or subtraction. But not always. Consider: There were 4 jackets left on the playground on Monday and 5 jackets left on the playground on Tuesday. How many jackets were left on the playground? "Left" in this problem does not mean subtract. **Many problems have no key words.** For example, How many legs do 7 elephants have?, does not have a key word. However, students should be able to solve the problem by thinking and drawing a picture or building a model. **It sends a bad message.** The most important strategy when solving a problem is to make sense of the problem and to think. Key words encourage students to ignore meaning and look for a formula. Mathematics is about meaning (Van de Walle, 2012).	**Academic Vocabulary/ Language** ▪ multiplication ▪ equation ▪ multiplicative ▪ additive ▪ comparison **Tier 2** ▪ interpret ▪ represent ▪ comparison

Learning Targets	I can apply my understanding of multiplication to explain the relationship between quantities in terms of how many times larger than. I can write verbal statements about multiplicative comparisons as equations. I can explain the difference between multiplicative (as many times as) and additive (more) comparisons when evaluating the mathematical situation.

Assessment of Learning ● Students can represent multiplicative comparisons. ● Students can explain how multiplication can compare quantities (Sara is 5 times as old as Marsha). ● Students can write equations and represent multiplicative comparisons. **Sample Questions** 1. Margo has 5 pieces of gum. Laura has 6 times as many pieces of gum as Margo. How many pieces of gum does Laura have? Explain how you would solve this problem. 2. John says that he is thinking of a number that is 7 times bigger than 3. Write an equation to express the relationship. 3. Write an expression that shows how much bigger 24 is than 8. Explain your thinking. 4. Marisa is 8 years old. Her mom is 5 times older than she is and her grandmother is 8 times older than Marisa. What multiplication sentence can be written to represent the relationship between Marisa's age and her mom's age? Between Marisa's age and her grandmother's age? How old are Marisa's mother and grandmother?
Ohio Department of Education Model Curriculum Instructional Strategies and Resources Students need to solve word problems involving multiplicative comparison (product unknown, partition unknown) using multiplication or division as shown in Table 2 of the Standards. They should use drawings or equations with a symbol for the unknown number to represent the problem. Students need to be able to distinguish whether a word problem involves multiplicative comparison or additive comparison (solved when adding and subtracting in Grades 1 and 2). Equations must be connected explicitly to problems and representations. Students should be able to explain where the numbers and operations in an equation originate.
Connections Across Standards Know relative sizes of measurement units within one system of units (4.MD.1). Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money (4.MD.2)

3.OA.3 & 8 (Prior Grade Standard)	**5.OA.2 (Future Grade Standard)**
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.OA.2

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. See Table 2 of the Standards. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Essential Understanding

- Comparisons can be additive or multiplicative depending on the mathematical situation.
- In multiplicative comparisons, the relationship between quantities is described in terms of how many times larger one is than the other.

Common Misconceptions

Key words are misleading. Some key words typically mean addition or subtraction. But not always. Consider: There were 4 jackets left on the playground on Monday and 5 jackets left on the playground on Tuesday. How many jackets were left on the playground? "Left" in this problem does not mean subtract.

Many problems have no key words. For example, How many legs do 7 elephants have?, does not have a key word. However, most students should be able to solve the problem by thinking and drawing a picture or building a model.

It sends a bad message. The most important strategy when solving a problem is to make sense of the problem and to think. Key words encourage students to ignore meaning and look for a formula. Mathematics is about meaning (Van de Walle, 2012).

Academic Vocabulary/ Language

- multiplication
- equation
- additive
- multiplicative
- symbol for unknown

Tier 2

- solve
- comparison
- distinguish

Learning Targets	I can create drawings that represent multiplicative comparisons to solve real-world problems that require me to multiply and divide whole numbers. I can apply my understanding of multiplication and division to solve word problems using equations and a symbol for an unknown.

	I can explain the difference between a multiplicative comparison and an additive comparison when evaluating the mathematical situation.

Assessment of Learning

- Students can explain the difference between multiplicative (as many times as) and additive (more) comparisons.
- Students can represent and solve multiplicative comparison word problems by multiplying or dividing.
- Students can represent and solve multiplicative comparison word problems by using a model or pictorial representation.
- Students can solve multiplication and division word problems by using equations and a symbol for an unknown.

Sample Questions

1. Draw a picture showing how to share 17 cookies among 5 friends.
2. If Mary is 11 and her sister is 22, explain how her sister is 11 years older OR 2 times older.
3. Mrs. March Is buying pencils for her classroom. She bought 6 packs of green pencils and 4 packs of pink pencils. There are 10 green pencils in each pack and 12 pink pencils in each pack. What is the total number of pencils Mrs. Marsh bought for her classroom?
4. Kior builds a structure that is 20 inches high. Kior's structure is 4 times taller than her brother's structure. How tall is her brother's structure? Explain using a model, a picture, and write an equation to support your answer.
5. Mr. Bell has 17 marbles in his classroom. Mrs. Lester has twice as many marbles as Mr. Bell. Mr. Bell borrowed all of Mrs. Lester's marbles so his students could play a game. Each student needs 4 marbles to play the game. How many students will be able to play the game? Explain your thinking.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students need to solve word problems involving multiplicative comparison (product unknown, partition unknown) using multiplication or division as shown in Table 2 of the Standards. They should use drawings or equations with a symbol for the unknown number to represent the problem. Students need to be able to distinguish whether a word problem involves multiplicative comparison or additive comparison (solved when adding and subtracting in Grades 1 and 2). Equations must be connected explicitly to problems and representations. Students should be able to explain where the numbers and operations in an equation originate.

TABLE 2. COMMON MULTIPLICATION AND DIVISION SITUATIONS[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 X 6 = ?	3 X ? = 18, AND 18 ÷ 3 = ?	? X 6 = 18, AND 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? **Measurement example.** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? **Measurement example.** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? **Measurement example.** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? **Area example.** What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? **Area example.** A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? **Area example.** A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? **Measurement example.** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? **Measurement example.** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? **Measurement example.** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

Connections Across Standards

Know relative sizes of measurement units within one system of units (4.MD.1).

Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money (4.MD.2).

3.OA.3 & 8 (Prior Grade Standard)	5.OA.2 (Future Grade Standard)
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

Ohio's Learning Standards-Clear Learning Targets

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4.OA.3

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

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Common Misconceptions

Students have difficulty estimating a two-step problem. Students do not always solve all of the steps needed for a multistep problem. Students may not be able to identify which part of the equation is unknown in order to represent it as a variable. Students may not know how to interpret a remainder.

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Academic Vocabulary/ Language

- operations
- equations
- mental computation
- estimation
- rounding
- remainder
- unknown quantity
- multistep

Tier 2

- reasonableness
- represent

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Essential Understandings

- Use the four operations with whole numbers to solve problems.
- Estimation strategies, including rounding, can be used to determine the reasonableness of answers.
- An unknown can be in any position of a multiplicative comparison problem.

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</table>

Learning Targets	I can solve real-world multi-step word problems using the four operations, including problems with remainders that must be interpreted. I can solve multi-step word problems using the four operations by using equations where a symbol is used for the unknown. I can evaluate the reasonableness of the answer by applying mental math, estimation, and rounding strategies.

Assessment of Learning

- Students can represent a multi-step word problem with models, pictures, and equations with a letter standing for the unknown quantity.
- Students can interpret the remainder when needed.
- Students can determine if the solution to a multi-step word problem is reasonable using estimation strategies.

Sample Questions

1. There are 37 members on three teams. How many vans will be necessary to carry them if each van carries 11 people? Justify your thinking.
2. Explain how Jack could estimate how much he needs in order to buy 32 pieces of candy at 19 cents each.
3. There are 483 students in Sue's school. 63 third grade students left the school on a field trip. There are about 20 students in each class. How many classrooms are being used today? Explain your answer.
4. Zoe is getting married. She has 178 guests attending. The party location can set up tables with 10 at each table or tables with 8 at each table. How many tables will Zoe need under each situation? Explain your thinking.
5. Is the product of 27×36 over or under 900? Explain how you know.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Present multi-step word problems with whole numbers and whole-number answers using the four operations. Students should know which operations are needed to solve the problem. Drawing pictures or using models will help students understand what the problem is asking. They should check the reasonableness of their answer using mental computation and estimation strategies.

Students should have the opportunity to share and justify their interpretations. Teacher modeling is important for developing reasonableness with two-step problems. Questions such as (i.e., What is the question asking me to do? Does it make sense to add first? How will this action affect the result of my first step?) should be asked to help students gain a deeper understanding of the problem and to assess the reasonableness of answers.

Connections Across Standards

Know relative sizes of measurement units within one system of units (4.MD.1).
Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money (4.MD.2).

3.OA.3 & 8 (Prior Grade Standard)	**5.OA.1 (Future Grade Standard)**
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

<table>
<tr><td colspan="2">4.OA.4
Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</td><td rowspan="2">Common Misconceptions
When listing multiples of numbers, students may not list the number itself. Emphasize that the smallest multiple is the number itself.
Some students may think that larger numbers have more factors. Having students share all factor pairs and how they found them will clear up this misconception.</td><td rowspan="2">Academic Vocabulary/ Language
▪ factor
▪ product
▪ multiples
▪ odd/even numbers
▪ prime
▪ composite

Tier 2
▪ recognize
▪ determine
▪ explain
▪ show
▪ find</td></tr>
<tr><td colspan="2">Essential Understandings
● A number can be multiplicatively decomposed into factor pairs and expressed as a product of these factor pairs.
● A prime number has only two factors: one and itself (only one factor pair).
● A composite number has more than two factors (more than one factor pair).
● Any whole number is a multiple of each of its factors.</td></tr>
<tr><td>Learning Targets</td><td colspan="3">I can decompose a whole number between 1 and 100 into its factors.
I can explain the relationship between factors to show how a whole number is a multiple of each of its factors.
I can evaluate if a number is prime or composite between 1 - 100 by applying my understanding of factors.</td></tr>
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Assessment of Learning

- Students can define and find factor pairs of any number between 1 and 100.
- Students can find multiples of a number (multiples up to 100).
- Students can determine and explain if a number is prime or composite between 1 - 100.
- Students will use models to explain and justify if a given whole number in the range 1-100 is prime or composite.

Sample Questions

1. Jasmine says that all odd numbers are prime numbers. Devon says that Jasmine is wrong because 9 is odd but 9 is also composite. Who is correct? Justify your reasoning?
2. Explain how to find all the single digit factors of 24.
3. Giovanni listed the factors for 12 as 1, 2, 3, 4, 6, 12. Is he correct? How did you know?
4. Lisa and Jaden sold baked goods at their school's fair. Lisa made cookies, and she put them into packages that held 4 cookies each. Jaden made cookies also, but he put them in packages that held 6 cookies each. Lisa said, "I sold over 30 cookies." Jaden said, "I sold over 30 cookies also." When they compared the amount they sold, they found that they each sold a different number of cookies. How many cookies could Lisa and Jaden each have sold? Identify possible amounts that Lisa and Jaden could have sold. Show your thinking.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students need to develop strategies for determining if a number is prime or composite, in other words, if a number has a whole number factor that is not one or itself. Starting with a number chart of 1 to 20, use multiples of prime numbers to eliminate later numbers in the chart. For example, 2 is prime but 4, 6, 8, 10, 12,… are composite. Encourage the development of rules that can be used to aid in the determination of composite numbers. For example, other than 2, if a number ends in an even number (0, 2, 4, 6 and 8), it is a composite number.

Explicit instruction of divisibility rules is not appropriate, however, student discovery of these rules could be an enrichment within this standard for students who show proficiency with factors, multiples, and prime/composite numbers.

Connections Across Standards

Use multiplication and division with whole numbers to solve problems and make multiplicative comparisons (4.OA.1-2).

3.OA.1 (Prior Grade Standard)	**(Future Grade Standard)**
Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.	N/A

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

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4.OA.5

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

For example, given the rule "Add 3" and the starting number 1 generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

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Common Misconceptions

Students may assume all patterns have the same rule due to limited exposure. This standard is the first formal approach to patterns. Students should have ample opportunities working with and creating patterns.

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Academic Vocabulary/ Language

- number pattern
- shape pattern
- rule

Tier 2

- generate
- identify
- apparent
- features
- explicit
- rule
- analyze

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Essential Understanding

- Explore patterns that consist of repeated sequences of shapes.
- Explore patterns that consist of growing sequences of designs.
- Explore patterns that consist of repeatedly adding the same whole number or repeatedly multiplying by the same whole number.
- Identify features of given or generated patterns.
- Make and describe generalizations about patterns.
- Connect rules and terms of patterns to numerical concepts.

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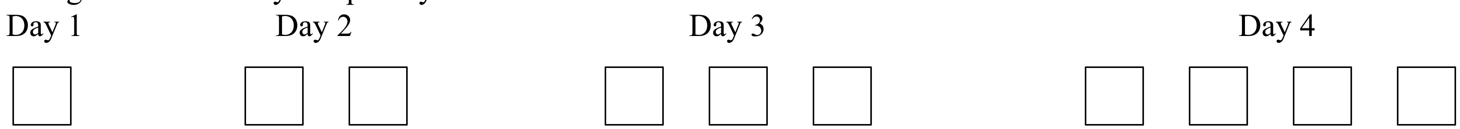
Learning Targets	I can generate a number pattern that follows a given rule. I can generate a shape pattern that follows a given rule. I can evaluate a number pattern and determine additional patterns found within the sequence. I can evaluate a shape pattern and determine additional patterns found within the sequence.

Assessment of Learning

- Students can generate a pattern, using shapes and numbers, that follow a given rule.
- Students can extend a pattern, using shapes and numbers, that follows a given rule.
- Students can construct and describe generalizations about patterns including numbers and shapes.
- Students can make connections between rules and numeric patterns.

Sample Questions

1. If a number pattern is created by the rule “add three”, will there be more odd numbers of even numbers created? Justify your reasoning.
2. Kellie began designing the pattern below with toothpicks. Each day she continues the pattern. How many toothpicks will she need to make her design on the 8th day? Explain your answer.



3. Extend the following pattern and describe the pattern. 6, 12, 24, 48, _____, _______, _______.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

In order for students to be successful later in the formal study of algebra, their algebraic thinking needs to be developed. Understanding patterns is fundamental to algebraic thinking. Students have experience in identifying arithmetic patterns, especially those included in addition and multiplication tables. Examples of repeated patterns can include using shapes and numbers. Contexts familiar to students are helpful in developing students’ algebraic thinking. Students should generate numerical or geometric patterns that follow a given rule. They should look for relationships in the patterns and be able to describe and make generalizations.

Connections Across Standards

Extend the understanding of fraction equivalence (4.NF.1- 2).

3.OA.9 (Prior Grade Standard)	**5.NBT.2 and 5.OA.3 (Future Grade Standard)**
Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.	NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10. OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right by applying concepts of place value, multiplication, or division. **Essential Understandings** ● Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000. ● In the base-ten system, the value of each place is 10 times the value of the place to the immediate right.	**Common Misconceptions** Students may have misconceptions about writing numerals from verbal descriptions. Numbers like one thousand do not cause a problem; however a number like one thousand two causes problems for students. Many students will understand the 1000 and the 2 but then instead of placing the 2 in the ones place, students will write the numbers as they hear them, 10002 (ten thousand two). Students often assume that the first digit of a multi-digit number indicates the "greatness" of a number. The assumption is made that 954 is greater than 1002 because students are focusing on the first digit instead of the number as a whole. Students need to be aware of the greatest place value.		**Academic Vocabulary/Language** ▪ place value ▪ digit **Tier 2** ▪ recognize ▪ represents ▪ apply ▪ concept
Learning Targets	I can apply place value concepts for multi-digit whole numbers. I can evaluate a multi-digit number and determine that the digit to the left is 10 times greater than a given digit. I can apply place value concepts to multiplying or dividing numbers.		

Assessment of Learning

- Students can explain the value of each digit as ten times the value of the digit to its right.
- Students can describe patterns found in place value (i.e. 60,000 is ten times more than 6,000; 6,000 is ten times more than 600)
- Students can demonstrate place value understanding by working flexibly with numbers.
- Students can relate multiplication and division to place value understanding.

Sample Questions

1. Explain why each column in a multi-digit number increases by 10 times.
2. Describe the size difference between 120 and 12.
3. How many different ways can you use base ten blocks to show 293?
4. James wrote the number 13,285. Marcus wrote a five-digit number that has one 3 in it. The 3 in Marcus' number is worth 10 times as much as the 3 in James' number. Write three different numbers that Marcus should have written. Circle one of the numbers you wrote. Use pictures, numbers, or words to show how you know that the number 3 is worth ten times as much as the 3 in James' number.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students also need to create numbers that meet specific criteria. For example, provide students with cards numbered 0 through 9. Ask students to select 4 to 6 cards; then, using all the cards make the largest number possible with the cards, the smallest number possible and the closest number to 5000 that is greater than 5000 or less than 5000.

Students should be able to explain the value of each digit as ten times the value of the digit to its right. Students should be able to represent this with models and explain the relationships. Students have worked with place value and number relationships since first grade. These understandings should be reinforced and applied to the larger numbers examined in fourth grade.

Connections Across Standards

Solve multi-step word problems with whole numbers and assess the reasonableness of answers using mental computation and estimation strategies (4.OA.3).

Solve problems involving metric measurement and conversions from a larger unit to a smaller unit (4.MD.1-2).

3.NBT.3 (Prior Grade Standard)	**5.NBT.1 (Future Grade Standard)**
Multiply one-digit whole numbers by multiples of 10 in the range 10-90, e.g., 9 × 80, 5 × 60 using strategies based on place value and properties of operations.	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.NBT.2 Read and write multi-digit whole numbers using standard form, word form, and expanded form.[G] Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Essential Understandings

- Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.
- Numbers can be expressed in standard form, word form, and expanded form.

Common Misconceptions

Students may have misconceptions about writing numerals from verbal descriptions. Numbers like one thousand do not cause a problem; however a number like one thousand two causes problems for students. Many students will understand the 1000 and the 2 but then instead of placing the 2 in the ones place, students will write the numbers as they hear them, 10002 (ten thousand two). There are multiple strategies that can be used to assist with this concept, including place-value boxes and vertical-addition method. Students often assume that the first digit of a multi-digit number indicates the "greatness" of a number. The assumption is made that 954 is greater than 1002 because students are focusing on the first digit instead of the number as a whole. Students need to be aware of the greatest place value.

Academic Vocabulary/ Language

- place value
- digit
- expanded form
- standard form
- written form
- word form
- greater than (>)
- less than (<)
- equal to (=)
- multi-digit

Tier 2

- compare
- explain

Learning Targets	I can apply and explain place value concepts for multi-digit whole numbers. I can generalize the value of a multi-digit whole number that is expressed in standard form, word form or expanded form. I can apply the concepts of place value when comparing the size of two multi-digit numbers and record the comparison using the inequalities symbols of <, >, =.

Assessment of Learning

- Students can read and write a number in word form and standard form within 1,000,000.
- Students can compose and decompose numbers using expanded form within 1,000,000.
- Students can compare numbers using place value and record results with symbols (>, <, =).

Sample Questions

1. Write an inequality comparing 813 and 831.
2. Using place value, explain why 811 is greater than 799 and write an expression using < or >.
3. Write the number that is represented by the expanded form: 14,000 + 80 + 6.
4. Erin and Josh are comparing numbers. Erin has 98 and Josh has 120. Erin knows that you need to start at the left when comparing numbers. She thinks that her number is bigger because it starts with a 9 and Josh's number starts with a 1. Josh argues that his number is larger because he has more digits in his number. Who is correct and why? Use what you know about place value and the value of digits in explaining your answer.
5. Jose and Reggie each have a 4-digit number that contains the digits 7, 1, 5, and 3. Jose's number is larger. What could Jose and Reggie's numbers be? What are two other numbers they could be? Explain your thinking.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Provide multiple opportunities in the classroom setting and use real-world context for students to read and write multi-digit whole numbers. Students need to have opportunities to compare numbers with the same number of digits, e.g., compare 453, 698 and 215; numbers that have the same number in the leading digit position, e.g., compare 45, 495 and 41,223; and numbers that have different numbers of digits and different leading digits, e.g., compare 312, 95, 5245 and 10,002.

Numbers can be compared through a variety of strategies. Students should reason about comparison using number lines, estimation, and other place value models.

Connections Across Standards

Solve multi-step word problems with whole numbers and assess the reasonableness of answers using mental computation and estimation strategies (4.OA.3).
Solve problems involving metric measurement and conversions from a larger unit to a smaller unit (4.MD.1-2).

(Prior Grade Standard)	**5.NBT.3a (Future Grade Standard)**
N/A	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form [G], *e.g.,* $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (\frac{1}{10}) + 9 \times (\frac{1}{100}) + 2 \times (\frac{1}{1000})$. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.NBT.3

Use place value understanding to round multi-digit whole numbers to any place through 1,000,000.

Essential Understandings

- Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.
- Rounding helps solve problems mentally and assess the reasonableness of an answer.

Common Misconceptions

There are several misconceptions students may have about writing numerals from verbal descriptions. Numbers like one thousand do not cause a problem; however a number like one thousand two causes problems for students. Many students will understand the 1,000 and the 2 but then instead of placing the 2 in the ones place, students will write the numbers as they hear them, 10,002 (ten thousand two). There are multiple strategies that can be used to assist with this concept, including place-value boxes and vertical -addition method. Students often assume that the first digit of a multi-digit number indicates the "greatness" of a number. The assumption is made that 954 is greater than 1,002 because students are focusing on the first digit instead of the number as a whole. Students need to be aware of the greatest place value.

Academic Vocabulary/ Language

- place value
- digit
- multi-digit
- rounding
- whole numbers

Tier 2

- explain

Learning Targets	I can apply the concepts of place value to round multi-digit whole numbers to the nearest 10, 100, 1,000,... through 1,000,000.

Assessment of Learning ● Students can explain how to round multi-digit whole numbers to any place through 1,000,000. ● Students can explain how rounding helps solve problems mentally. ● Students can assess the reasonableness of an answer. **Sample Questions** 1. Explain how you know that when you add 4,785 and 4,220 it will be less than 10,000. 2. Jerry says that 6,450 rounds to 6,400 and Jill says that it rounds to 6,500. Who is correct? Explain your thinking. 3. If a number was rounded to 500, what would have been the lowest and highest values for the number, if the number was rounded to the tens? Explain your thinking. 4. When might it be valuable to round to the hundreds place? When might it be valuable to round to the tens place? (Students should relate the problem to a real world context). 5. Mary said, "When I round these numbers, 143,679; 105,033 and 121,737, I get the same answer." Heather said, "I disagree. I get all different numbers." Can they both be correct? Explain your reasoning.
Ohio Department of Education Model Curriculum Instructional Strategies and Resources In Grade 4, rounding is not new, and students need to build on the Grade 3 skill of rounding to the nearest 10 or 100 to include larger numbers and place value. What is new for Grade 4 is rounding to digits other than the leading digit, e.g., round 23,960 to the nearest hundred. This requires greater sophistication than rounding to the nearest ten thousand because the digit in the hundreds place represents 900 and when rounded it becomes 1000, not just zero. Students should also begin to develop some rules for rounding, building off the basic strategy of; "Is 48 closer to 40 or 50?" Since 48 is only 2 away from 50 and 8 away from 40, 48 would round to 50. Now students need to generalize the rule for much larger numbers and rounding to values that are not the leading digit. Remind students that rounding is a way to estimate. Using friendly or compatible numbers is another way to estimate. Estimating helps students determine reasonableness of their solutions or calculations. Students should also be able to describe how rounding is different than estimating and when rounding might be helpful and when it might not be. For example 44 rounds to 40 but may be best thought of as 45 for easier estimations.
Connections Across Standards Solve multi-step word problems with whole numbers and assess the reasonableness of answers using mental computation and estimation strategies (4.OA.3). Solve problems involving metric measurement and conversions from a larger unit to a smaller unit (4.MD.1-2).

3.NBT.1(Prior Grade Standard)	**5.NBT.4 (Future Grade Standard)**
Use place value understanding to round whole numbers to the nearest 10 or 100.	Use place value understanding to round decimals to any place, millions through hundredths.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

<table>
<tr><td>4.NBT.4
Fluently [G] add and subtract multi-digit whole numbers using a standard algorithm [G].</td><td rowspan="2">Common Misconceptions
Often students mix up when to 'carry' and when to 'borrow'. Also students often do not understand why they need to regroup and just subtract the smaller digit from the larger one. Emphasize place value and the meaning of each of the digits.</td><td rowspan="2">Academic Vocabulary/ Language
▪ add
▪ subtract
▪ algorithm
▪ multi-digit

Tier 2
▪ fluently
▪ arithmetic</td></tr>
<tr><td>Essential Understandings
● There are different algorithms that can be used to add or subtract.
● Fluency is being efficient, accurate, and flexible with strategies.</td></tr>
<tr><td>Learning Targets</td><td colspan="2">I can apply the flexible and efficient strategies for addition and subtraction to multi-digit whole numbers.
I can analyze and connect strategies for adding and subtracting multi-digit whole numbers using a standard algorithm.</td></tr>
</table>

Assessment of Learning

- Students can determine when an algorithm is efficient and when it is not for adding and subtracting multi-digit whole numbers.
- Students can explain and connect strategies for adding and subtracting multi-digit whole numbers using a standard algorithm.
- Students can apply an efficient standard algorithm accurately and flexibly.

Sample Questions

1. The library loaned out 348 books on Monday, 425 books on Tuesday and 612 books on Wednesday. How many books did the library loan out during those three days in all? Explain your thinking.
2. Mrs. Allen bought a package of 1,000 stickers. She gave away 430 stickers during the first half of the school year. How many stickers does Mrs. Allen have remaining? Justify your answer.
3. What two addends could equal a sum of 146?
4. Write an addition problem in which an addend is 1,435 and the sum will round to 5,000 when rounding to the nearest thousand. Explain your thinking.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

A crucial theme in multi-digit arithmetic is encouraging students to develop strategies that they understand, can explain, and can think about, rather than merely follow a sequence of directions that they don't understand. It is important for students to have seen and used a variety of strategies and materials to broaden and deepen their understanding of place value before they are required to use standard algorithms. The goal is for them to understand all the steps in the algorithm, and they should be able to explain the meaning of each digit. For example, a 1 can represent one, ten, one hundred, and so on. For multi-digit addition and subtraction in Grade 4, the goal is also fluency, which means students must be able to carry out the calculations efficiently and accurately. Start with a student's understanding of a certain strategy, and then make intentional, clear-cut connections for the student to the standard algorithm. This allows the student to gain understanding of the algorithm rather than just memorize certain steps to follow.

Students who understand the algorithm must also know when it is efficiently used. Many computations are completed more efficiently using varied strategies and/or mental mathematics. This is why so much time is spent in earlier grades decomposing numbers and adding multiples of tens, hundreds, and so on.

Connections Across Standards

Multiply or divide to solve word problems involving multiplicative comparisons (4.OA.2).

Solve multi-step word problems with whole numbers and assess the reasonableness of answers using mental computation and estimation strategies (4.OA.3).

Develop strategies to determine the area and perimeter of rectangles in real world situations (4.MD.3).

3.NBT.2 (Prior Grade Standard)	**5.NBT.5 (Future Grade Standard)**
Fluently add and subtract within 1,000 using strategies and algorithms [G] based on place value, properties of operations, and/or the relationship between addition and subtraction	Fluently [G] multiply multi-digit whole numbers using a standard algorithm.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

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4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Essential Understandings

- The product is the result of multiplication.
- Factors are the numbers being multiplied together.
- There is a relationship between multiplication and division.

</td><td>

Common Misconceptions

Often students mix up when to 'carry' and when to 'borrow'. Also students often do not understand why they need to "borrow" and just subtract the smaller digit from the larger one. Emphasize place value and the meaning of each of the digits.

</td><td>

Academic Vocabulary/ Language

- multiply
- equation
- area model
- rectangular arrays
- product
- factor

Tier 2

- illustrate
- explain

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Learning Targets

</td><td colspan="2">

I can multiply a whole number up to four digits by a one-digit number or a two-digit number by a two-digit number by applying strategies based on place value and/or operation properties.
I can construct a model such as rectangular arrays, and/or area models or use an equation to justify a two-digit by two-digit multiplication problem.

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Assessment of Learning

- Students can explain and represent multiplication of a multi-digit factor, up to 4 digits, by a one-digit factor.
- Students can explain and represent multiplication of a two-digit factor by a two-digit factor.
- Students can represent multiplication situations and problems with various representations (arrays, area models, equal jumps on a number line, partial products, and decomposition of numbers).
- Students can connect and write an equation for multiplication situations.

Sample Questions

1. Explain two ways to multiply 23 × 15.
2. Draw an area model that shows the problem 142 × 5.
3. Draw three different arrays that would model the product of 48.
4. Solve the following problem using two different strategies. There are 1,534 students at Green Elementary School. Seventy-eight students shopped at the school store each day. How many students shopped at the school store after 5 days?
5. Explain why solving 354 × 5 is more easily solved by breaking the problem into 300 × 5 + 50 × 5 + 4 × 5.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

As students developed an understanding of multiplying a whole number up to four digits by a one-digit whole number, and multiplying two two-digit numbers through various strategies, they should do the same when finding whole-number quotients and remainders. By relating division to multiplication and repeated subtraction, students can find partial quotients. An explanation of partial quotients in this video can be viewed at http://www.teachertube.com, search for Outline of Partial Quotients. This strategy will help them understand the division algorithm. Students will have a better understanding of multiplication or division when problems are presented in context. Students should be able to illustrate and explain multiplication and division calculations by using equations, rectangular arrays and the properties of operations. These strategies were used in Grade 3 as students developed an understanding of multiplication. To give students an opportunity to communicate their understanding of various strategies, organize them into small groups and ask each group to create a poster to explain a particular strategy and then present it to the class. Students should have an understanding of terms such as sum, difference, fewer, more, less, ones, tens, hundreds, thousands, digit, whole numbers, product, factors and multiples.

Connections Across Standards

Multiply or divide to solve word problems involving multiplicative comparisons (4.OA.2).

Solve multi-step word problems with whole numbers and assess the reasonableness of answers using mental computation and estimation strategies (4.OA.3).

Develop strategies to determine the area and perimeter of rectangles in real world situations (4.MD.3).

3.NBT.3 (Prior Grade Standard)	**5.NBT.5 (Future Grade Standard)**
Multiply one-digit whole numbers by multiples of 10 in the range 10-90, e.g., 9 × 80, 5 × 60 using strategies based on place value and properties of operations.	Fluently [G] multiply multi-digit whole numbers using a standard algorithm.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

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4.NBT.6

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

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Common Misconceptions

When working with division, students often do not think about the importance of place value. They treat each digit in the dividend separately without looking at the value of the entire number. Encourage students to explore different strategies and consider the relationship between multiplication and division. Estimating by using multiplication prior to dividing, helps students see what a reasonable quotient will be.

</td>
<td rowspan="2">

Academic Vocabulary/ Language

- quotient
- remainder
- dividend
- divisor

Tier 2

- illustrate
- explain

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Essential Understandings

- There are two major division situations: fair sharing (group size unknown) and repeated subtraction (number of groups unknown). See Table 2 of the Standards .
- There is a relationship between multiplication and division.
- The dividend divided by the divisor is the quotient.
- A remainder can be stated, can be discarded, or can force the quotient to increase to the next whole number depending on the context.
- Equations, rectangular arrays, and/or area models can be used to illustrate and explain multiplication and division.

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</table>

Learning Target	I can apply my understanding of the relationship between multiplication and division, place value, and properties of operations to divide up to four-digit dividends and one-digit divisors to find the quotient and remainder. I can create models of rectangular arrays and/or area models or use equations to justify the calculations of division problems.

Assessment of Learning

- Students can demonstrate division of a multi-digit number using place value, rectangular arrays, and area models.
- Students will find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors.
- Students will explore division problems that result in remainders.

Sample Questions

1. Explain how knowing $4 \times 23 = 92$ and $4 \times 50 = 200$ would allow you to more easily solve the problem $292 \div 4$.
2. Divide 584 by 4 in two different ways.
3. Draw and explain an area model for $1{,}426 \div 4$.
4. Write a division problem using a 4-digit dividend and a 1-digit divisor that results in an even quotient. Show your work.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

As students developed an understanding of multiplying a whole number up to four digits by a one-digit whole number, and multiplying two two-digit numbers through various strategies, they should do the same when finding whole-number quotients and remainders. By relating division to multiplication and repeated subtraction, students can find partial quotients. An explanation of partial quotients in this video can be viewed at http://www.teachertube.com, search for Outline of Partial Quotients. This strategy will help them understand the division algorithm. Students will have a better understanding of multiplication or division when problems are presented in context. Students should be able to illustrate and explain multiplication and division calculations by using equations, rectangular arrays and the properties of operations. These strategies were used in Grade 3 as students developed an understanding of multiplication. To give students an opportunity to communicate their understanding of various strategies, organize them into small groups and ask each group to create a poster to explain a particular strategy and then present it to the class. Vocabulary is important. Students should have an understanding of terms such as, sum, difference, fewer, more, less, ones, tens, hundreds, thousands, digit, whole numbers, product, factors and multiples.

Connections Across Standards

Multiply or divide to solve word problems involving multiplicative comparisons (4.OA.2).
Solve multi-step word problems with whole numbers and assess the reasonableness of answers using mental computation and estimation strategies (4.OA.3).
Develop strategies to determine the area and perimeter of rectangles in real world situations (4.MD.3).

3.NBT.3 (Prior Grade Standard)	**5.NBT.6 (Future Grade Standard)**
Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.NF.1

Explain why a fraction *a/b* is equivalent to a fraction *(n × a)/(n × b)* by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Essential Understandings

- The denominator describes the number of equal parts the whole is divided into; the more equal fractional parts used to make a whole, the smaller the size of the parts.
- Equivalent fractions use different sized fractional parts to describe the same amount, e.g., $\frac{1}{2} = \frac{2}{4}$.
- Multiplying the numerator and the denominator by the same number will result in an equivalent fraction.
- There is a multiplicative relationship between the number of equal parts in a whole and the size of the parts.

Common Misconceptions

Students think that when generating equivalent fractions they need to multiply or divide either the numerator or denominator, such as, changing $\frac{1}{2}$ to sixths. They would multiply the denominator by 3 to get $\frac{1}{6}$, instead of multiplying the numerator by 3 also. Their focus is only on the multiple of the denominator, not the whole fraction. Students need to first use a visual model then use a numeric form of the fraction one such as $\frac{3}{3}$ so that the numerator and denominator do not contain the original numerator or denominator.

Academic Vocabulary/ Language

- fractions
- equivalent
- fraction model

Tier 2

- explain
- recognize
- generate

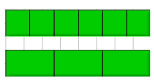
Learning Targets	I can create and explain equivalent fractions using visual models. I can create and explain equivalent fractions. I can create a visual model to explain an equivalent fraction by comparing the number and size of the parts. I can apply the concepts of multiplicative relationship when creating equivalent fractions.

Assessment of Learning

- Students can create and use models to explain why different fractions are equivalent (color tiles, pattern blocks, Cuisenaire Rods, fraction bars, fraction, circles, and number lines).
- Students can explain why fractions are equivalent.
- Students can recognize and create equivalent fractions.

Sample Questions

1. Create 5 fractions that are equivalent to $\frac{3}{5}$ by using models and explain why the different fractions are equivalent.
2. How can $\frac{4}{5}$ and $\frac{8}{10}$ be equivalent if the numerator and denominator in each is different? Explain your thinking.
3. Write the statement $\frac{8}{12}$ is twice as large as $\frac{4}{6}$ on the board. Ask students if they agree or disagree with the statement. Have students use a model to support their thinking.
4. Have students draw a number line (0 to 1). Place the fraction $\frac{4}{8}$ on the number line. Have students place a fraction on the number line that is equivalent to $\frac{4}{8}$ and explain why the fractions are equivalent.
5. Explain how this model shows that $\frac{1}{3} = \frac{2}{6}$.



Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students' initial experience with fractions began in Grade 3. They used models such as number lines to locate unit fractions, and fraction bars or strips, area or length models, and Venn diagrams to recognize and generate equivalent fractions and make comparisons of fractions. Students extend their understanding of unit fractions to compare two fractions with different numerators and different denominators. Students should use models to compare two fractions with different denominators by creating common denominators or numerators. The models should be the same (both fractions shown using fraction bars or both fractions using circular models) so that the models represent the same whole. The models should be represented in drawings. Students should also use benchmark fractions such as $\frac{1}{2}$ to compare two fractions. The result of the comparisons should be recorded using >, < and = symbols. Students should revisit the identity property of multiplication (any number multiplied by one is itself) to understand why you can multiply a fraction by *n/n* to create an equivalent fraction.

Understanding why two or more fractions are equivalent is followed by asking students to generate equivalent fractions. Students should generate and justify why their generations are equivalent. Students should use representations in their justifications. When students discover the numerical process of multiplying (or dividing) the numerator and denominator by the same number, they should understand how that connects to the identity property of multiplication and division. Students who use a procedure must be able to explain why the procedure works. Reducing should not be used as it implies that something is getting smaller, which is not the case with equivalent fractions as they represent the same value.

Connections Across Standards

Gain familiarity with factors and multiples (4.OA.4).

3.NF.3d (Prior Grade Standard)	5.NF.1-2 (Future Grade Standard)
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	1. Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, use visual models and properties of operations to show* $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. *(In general, a /b + c /d = (a /b x d /d) + (c /d x b /b) = (ad + bc)/bd.)* 2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.*For example, recognize an incorrect result* $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$*, by observing that* $\frac{3}{7} < \frac{1}{2}$.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.NF.2

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Essential Understanding

- Visual models, such as rectangular area models, arrays (e.g., egg cartons) and length models (including fraction strips and number lines), can be used to represent and compare fractions.
- To compare fractions using models, each fraction should be represented with the same visual model and the same sized whole.
- Multiplying the numerator and the denominator by the same number will result in an equivalent fraction.

Common Misconceptions

Students think that when generating equivalent fractions they need to multiply or divide either the numerator or denominator, such as, changing $\frac{1}{2}$ to sixths. They would multiply the denominator by 3 to get $\frac{1}{6}$, instead of multiplying the numerator by 3 also. Their focus is only on the multiple of the denominator, not the whole fraction. Students need to first use a visual model then use a numeric form of the fraction one such as $\frac{3}{3}$ so that the numerator and denominator do not contain the original numerator or denominator.

Academic Vocabulary/ Language

- fractions
- equivalent
- numerator
- denominator
- visual fraction model
- >, <, =

Tier 2

- compare
- create
- recognize
- valid
- record

Learning Targets	I can compare two fractions by creating common numerators or common denominators by applying multiplication concepts to both the numerator and the denominator. I can create visual models, such as rectangular area models, arrays, and length models to reason about the comparison of two fractions that refer to the same whole. I can use benchmark fractions to reason about the comparison of two fractions. I can record the comparison of fractions using inequality symbols of <, >, = and defend my answers.

Assessment of Learning

- Students can compare two fractions by creating equivalent fractions with a common denominator.
- Students can record fraction comparisons using >, <, or =.
- Students can explain how fractions are compared when they refer to the same whole.
- Students can create visual models to reason about the comparison of two fractions that refer to the same whole.

Sample Questions

1. Find the larger fraction between $\frac{5}{8}$ and $\frac{3}{7}$ by comparing each to $\frac{1}{2}$. Explain your thinking.
2. Paul's Pizza sells a $\frac{1}{2}$ pizza that feeds 3. Patty's Pizza says that half of their pizza only feeds one person. How is this possible?
3. Two students were asked to write a sentence using the fractions $\frac{2}{3}$ and $\frac{2}{6}$. Dante wrote, "The numerator of $\frac{2}{3}$ and $\frac{2}{6}$ are equal, so the fractions are equal." JJ wrote, "$\frac{2}{6}$ is greater than $\frac{2}{3}$ because 6 is greater than 3." Do you agree with Dante, JJ, or neither? Explain why.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students' initial experience with fractions began in Grade 3. They used models such as number lines to locate unit fractions, and fraction bars or strips, area or length models, and Venn diagrams to recognize and generate equivalent fractions and make comparisons of fractions. Students extend their understanding of unit fractions to compare two fractions with different numerators and different denominators. Students should use models to compare two fractions with different denominators by creating common denominators or numerators. The models should be the same (both fractions shown using fraction bars or both fractions using circular models) so that the models represent the same whole. The models should be represented in drawings. Students should also use benchmark fractions such as $\frac{1}{2}$ to compare two fractions. The result of the comparisons should be recorded using >, < and = symbols.

Understanding why two or more fractions are equivalent is followed by asking students to generate equivalent fractions. Students should generate fractions and justify why they are equivalent. Students should use representations in their justifications. When students discover the numerical process of multiplying (or dividing) the numerator and denominator by the same number, they should understand how that connects to the identity property of multiplication and division. Students who use a procedure must be able to explain why the procedure works. Reducing should not be used as it implies that something is getting smaller, which is not the case with equivalent fractions as they represent the same value.

Connections Across Standards

Gain familiarity with factors and multiples (4.OA.4).

3.NF.3 (Prior Grade Standard)	5.NF.1-2 (Future Grade Standard)
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form* $3 = \frac{3}{1}$ *; recognize that* $\frac{6}{1} = 6$*; locate* $\frac{4}{4}$ *and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	1. Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, use visual models and properties of operations to show* $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$ *. (In general, a /b + c /d = (a /b x d /d) + (c /d x b /b) = (ad + bc)/bd.)* 2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$*, by observing that* $\frac{3}{7} < \frac{1}{2}$ *.*

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

Standard	Common Misconceptions	Academic Vocabulary/ Language
4.NF.3 Understand a fraction *a/b* with *a* > *1* as a sum of fractions $\frac{1}{b}$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model [G]. Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$. c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. **Essential Understanding** • Fractions can be added and subtracted when the wholes are the same size. • Unit fractions can be combined from multiple wholes if all the wholes are the same size. • Fractions with the same denominators can be added and subtracted using visual models, properties of operations, and relationships of addition and subtraction of whole numbers. • Mixed numbers can be written as fractions, e.g., $\frac{14}{3} = 4\frac{2}{3}$, and can be added or subtracted in this form. • Equivalent fractions can be used to add and subtract fractions. (Fractions need not be simplified.)	Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole.	▪ fractions ▪ equivalent ▪ numerator ▪ denominator ▪ decompose ▪ ordering ▪ mixed number **Tier 2** ▪ solve ▪ represent

Learning Targets	I can apply the concepts of adding and subtracting fractions with like denominators when the wholes are the same size when solving real-world problems.. I can decompose a fraction into a sum of fractions with the same denominator in more than one way and create a visual fraction model to justify the answer. I can use an equivalent fraction to represent a mixed fraction to add and subtract fractions to solve word problems.

Assessment of Learning

- Students can compose and decompose a fraction.
- Students can describe that adding and subtracting of fractions must refer to the same size whole and having like denominators.
- Students can represent addition and subtraction of fractions with varied models.
- Students can represent addition and subtraction of mixed numbers with varied models.
- Students can add and subtract fractions and mixed numbers with varied strategies.
- Students can solve word problems with adding and subtracting fractions and mixed numbers.
- Students can solve real-world problems that will result in various equivalent answers, and explain why the solutions are equivalent.

Sample Questions

1. Bob walked $2\frac{3}{8}$ miles and Sue walked $3\frac{1}{8}$ miles. What is the difference in the amount of miles they walked? How far did they walk together? Explain your thinking.
2. Explain why $5\frac{4}{6}$ is the same as $3\frac{2}{6} + 2\frac{2}{6}$.
3. Draw two fraction models to show the difference between $1\frac{2}{8}$ and $3\frac{5}{8}$.
4. Derrick came home and found a fraction of a large pizza on the counter. He eats $\frac{4}{10}$ of the pizza and now there is $\frac{2}{10}$ of the pizza left. What fraction of the pizza was on the counter when he got home?
5. Layla is having a sleepover with 2 friends. They order one party size submarine sandwich and it is cut into 12 equal parts. They eat the entire sandwich, but each person has a different number of parts. What is one way the sandwich was shared? Write an equation to represent your answer equal to $\frac{12}{12}$. Is there another way the friends could have shared the sandwich?
6. If the sum of two mixed numbers is 6, what could the two addends be? Justify your answer.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

In Grade 3, students represented whole numbers as fractions. In Grade 4, they will use this knowledge to add and subtract mixed numbers with like denominators using properties of number and appropriate fraction models. It is important to stress that whichever model is used, it should be the same for the same whole. For example, a circular model and a rectangular model should not be used in the same problem. Understanding of multiplication of whole numbers is extended to multiplying a fraction by a whole number. Allow students to use fraction models and drawing to show their understanding. Present word problems involving multiplication of a fraction by a whole number. Have students solve the problems

using visual models and write equations to represent the problems. Students need instruction on the conceptual understanding of adding and subtracting fractions and what a reasonable answer looks like. Teachers should facilitate class discussions where questions are purposefully posed to help students extend thinking and make connections to other mathematical ideas and relationships.
Connections Across Standards Interpret and represent multiplicative comparisons (4.OA.1). Determine whether a whole number is a multiple on another whole number (4.OA.4).

3.NF.3b (Prior Grade Standard)	**5.NF.1-2 (Future Grade Standard)**
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$*)*. Explain why the fractions are equivalent, e.g., by using a visual fraction model.	1. Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, use visual models and properties of operations to show* $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. *(In general,* $a/b + c/d = (a/b \times d/d) + (c/d \times b/b) = (ad + bc)/bd$.*)* 2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$*, by observing that* $\frac{3}{7} < \frac{1}{2}$.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.NF.4

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction *a/b* as a multiple of *1/b*. For example, use a visual fraction model to represent $\frac{5}{4}$ as the product of $5 \times (\frac{1}{4})$, recording the conclusion by the equation $\frac{5}{4} = 5 \times (\frac{1}{4})$ or $\frac{5}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.

b. Understand a multiple of *a/b* as a multiple of *1/b,* and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (\frac{2}{5})$ as $6 \times (\frac{1}{5})$, recognizing this product as $\frac{6}{5}$. (In general, $n \times (a/b) = (n \times a)/b$.)

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Essential Understanding

- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100 (Fractions need not be simplified).
- Multiplication is repeated addition, i.e., just as $4 \times 3 = 3 + 3 + 3 + 3$, $5 \times \frac{1}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ which equals $\frac{5}{8}$.

Common Misconceptions

Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole.

Academic Vocabulary/ Language

- fractions
- whole number
- multiple
- fraction model

Tier 2

- apply
- extend
- solve
- represent

Learning Targets	I can apply the concept of repeated addition to multiplication when multiplying a whole number by a unit fraction. I can use a visual fraction model to justify multiplying a fraction by a whole number. I can solve word problems involving multiplication of a fraction by a whole number using visual fraction models and equations.

Assessment of Learning

- Students can explain a fraction as a multiple of a unit fraction (e.g. $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$ or $3 \times \frac{1}{5}$).
- Students can solve word problems by multiplying a whole number by a fraction.
- Students can multiply a whole number by a fraction using visual models.

Sample Questions

1. Explain how many fourths are in $\frac{5}{4}$ and write an equation that shows that relationship.
2. What number should go in the blank? $\frac{1}{6} \times$ ____ $= \frac{7}{6}$? Explain your thinking.
3. If each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Show your answer using fractions models or drawings.
4. Explain why $3 \times \frac{2}{6}$ is the same as $6 \times \frac{1}{6}$.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Understanding of multiplication of whole numbers is extended to multiplying a fraction by a whole number. Allow students to use fraction models and drawing to show their understanding. Students need to have conceptual understanding as to why an answer to a problem is reasonable. Students should focus on what an answer will look like prior to actually calculating the answer. Questions like: Does this answer make sense? How do you know the solution is going to be less than a certain number? will help students make sense of problem solving. Present word problems involving multiplication of a fraction by a whole number. Have students solve the problems using visual models and write equations to represent the problems.

Connections Across Standards

Interpret and represent multiplicative comparisons (4.OA.1).
Determine whether a whole number is a multiple on another whole number (4.OA.4).

3.NF.3 (Prior Grade Standard)	**5.NF.4 (Future Grade Standard)**
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$). Explain why the fractions are equivalent, e.g., by using a visual fraction model.[G] c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form* $3 = \frac{3}{1}$*; recognize that* $\frac{6}{1} = 6$*; locate* $\frac{4}{4}$ *and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts, equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(\frac{2}{3}) \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $(\frac{2}{3}) \times (\frac{4}{5}) = \frac{8}{15}$. (In general, $(a/b) \times (c/d) = ac/bd$.) b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express $\frac{3}{10}$ as $\frac{30}{100}$, and add $\frac{3}{10}+\frac{4}{100}=\frac{34}{100}$. In general students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators, but addition and subtraction with unlike denominators is not a requirement at this grade. **Essential Understanding** ● Using equivalent fractions, any fraction with a denominator of ten can be renamed as a fraction with a denominator of 100.	**Common Misconceptions** Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole.	**Academic Vocabulary/ Language** ▪ fractions ▪ whole number ▪ multiple ▪ equivalent fraction ▪ denominator **Tier 2** ▪ express ▪ respective

Learning Targets	I can write fractions with denominators of 10 to equal fractions with denominators of 100. I can add two fractions with the denominators of 10 and 100. I can represent addition of fractions with denominators of 10 and 100. I can create an equivalent fraction for fractions with denominators of 10 and 100. I can apply the reasoning about fractions with denominators of 10 and 100 to add fractions.

Assessment of Learning

- Students can write, represent, and add two fractions with a denominator of 10 and 100.
- Students can write fractions with denominators of 10 to equal fractions with denominators of 100.

Sample Questions

1. Change $\frac{7}{10}$ to an equal fraction with a denominator of 100.
2. How does your knowledge of place value help you prove that $\frac{3}{10} + \frac{5}{100} = \frac{35}{100}$? Explain your thinking.
3. A dime is $\frac{1}{10}$ of a dollar and a penny is $\frac{1}{100}$ of a dollar. What fraction of a dollar is 6 dimes and 3 pennies? Use a model to show your thinking. Write your answer in both fraction and decimal form.
4. Do $\frac{4}{10}$ and $\frac{40}{100}$ have the same location on a number line? Justify why or why not.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students' initial experience with fractions began in Grade 3. They used models such as number lines to locate unit fractions, and fraction bars or strips, area or length models, and Venn diagrams to recognize and generate equivalent fractions and make comparisons of fractions. Students extend their understanding of unit fractions to compare two fractions with different numerators and different denominators. Students should use models to compare two fractions with different denominators by creating common denominators or numerators. The models should be the same (both fractions shown using fraction bars or both fractions using circular models) so that the models represent the same whole. The models should be represented in drawings. Students should also use benchmark fractions such as $\frac{1}{2}$ to compare two fractions. The result of the comparisons should be recorded using >, < and = symbols.

This standard is critical for understanding the relationship between fractions and decimals. Denominators of 10 and 100 have connections to decimals (tenths and hundredths). Understanding the fraction equivalents enables students to work with decimal equivalents. This understanding is then applied to comparing and computing with decimals.

Connections Across Standards

Use units to solve measurement problems (4.MD.1-2).
Generalize place value understanding (4.NBT.2).

3.NF.3 (Prior Grade Standard)	**5.NF.4 (Future Grade Standard)**
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$). Explain why the fractions are equivalent, e.g., by using a visual	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a part of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show* $(\frac{2}{3}) \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $(\frac{2}{3}) \times (\frac{4}{5}) =$

fraction model.[G] c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 =* $\frac{3}{1}$ *; recognize that* $\frac{6}{1}$ *= 6; locate* $\frac{4}{4}$ *and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	$\frac{8}{15}$. *(In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.NF.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.	**Common Misconceptions** Students treat decimals as whole numbers when making comparison of two decimals. They think the longer the number, the greater the value. For example, they think that 0.03 is greater than 0.3.	**Academic Vocabulary/ Language** ▪ fractions ▪ decimal ▪ number line **Tier 2** ▪ notation ▪ rewrite ▪ describe ▪ locate
Essential Understandings ● The place value system of whole numbers can be expanded to represent numbers less than 1. ● A fraction with a denominator of 10 or 100 can be written using decimal notation. ● A number can be written as a fraction, e.g., $\frac{17}{100}$, or as a decimal, e.g., 0.17. ● A decimal point or horizontal bar can be used to show where the unit is located, e.g., $\frac{35}{100}$ = 0.35.		

Learning Targets	I can reason about the relationship between a fraction and a decimal to represent a number with a denominator of 10 or 100 as a decimal. I can locate a decimal on a number line.

Assessment of Learning

- Students can represent and describe a fraction with a denominator of 10 and 100 as a decimal.
- Students can identify the tenths and hundredths place.
- Students can represent a decimal on a number line.
- Students can explain the relationship between a fraction and a decimal.

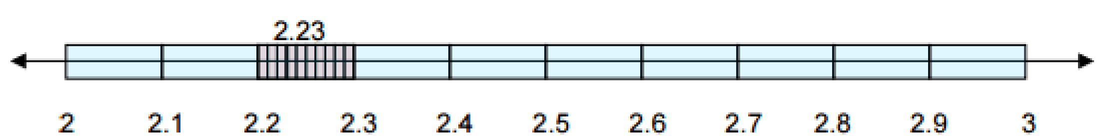
Sample Questions

1. Rewrite 0.52 as a fraction with a denominator of 100.
2. Mr. King asks her students to write 0.60 as a fraction. Ishmael wrote $\frac{6}{10}$. Curtis wrote $\frac{60}{100}$. Explain how both students are correct. Use words and a pictorial model to explain your answer.
3. Have students create a number line. Plot the fractions and decimals on the number line. Explain your reason as to why you put the numbers in the specific place on the number line. ($\frac{4}{10}$ $\frac{20}{100}$ 0.90)

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students' understanding of decimals to hundredths is important in preparation for performing operations with decimals to hundredths in Grade 5. Students need to understand that decimals are an extension of our whole number base ten system. Decimals and fractions both represent parts of a whole. Students need to understand the value of the place and the relationship between places values so that 0.5 is ten times larger than 0.05. When reading decimals the decimal point is read as **and** which separates the whole number from the decimal. Having students read decimals names out loud helps them make connections to fractions. When locating decimals on a number line students should understand that the smaller numbers are farther to the left and the greater number is farther to the right. Often students are able to better understand comparing decimals if the problem is in context to real world situations.

In decimal numbers, the value of each place is 10 times the value of the place to its immediate right. Students need an understanding of decimal notations before they try to do conversions in the metric system. Understanding of the decimal place value system is important prior to generalization of moving the decimal point when performing operations involving decimals.



Students extend fraction equivalence from Grade 3 with denominators of 2, 3, 4, 6 and 8 to fractions with a denominator of 10. Provide fraction models of tenth and hundredths so that students can express a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100.

Connections Across Standards	
Connections Across Standards Use units to solve measurement problems (4.MD.1-2). Generalize place value understanding (4.NBT.2).	
3.NF.2 (Prior Grade Standard) Understand a fraction as a number on the number line; represent fractions on a number line diagram.[G] a. Represent a fraction *1/b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size *1/b* and that the endpoint of the part based at 0 locates the number *1/b* on the number line. b. Represent a fraction *a/b* (which may be greater than 1) on a number line diagram by marking off a lengths *1/b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.	**5.NBT.2 (Future Grade Standard)** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	**Common Misconceptions** Students treat decimals as whole numbers when making comparison of two decimals. They think the longer the number, the greater the value. For example, they think that 0.03 is greater than 0.3.	**Academic Vocabulary/ Language** ▪ fractions ▪ decimal ▪ <, >, = **Tier 2** ▪ compare ▪ justify ▪ conclusion ▪ symbol ▪ recognize ▪ record
Essential Understandings ● Decimals can only be compared when the decimals being compared refer to the same whole. ● Decimals written as tenths or hundredths can be compared using equivalent fractions		

Learning Targets	I can compare two decimals, explain my reasoning, and record the results using <, >, or =. I can explain that comparisons between two decimals are only valid when they refer to the same whole.

Assessment of Learning

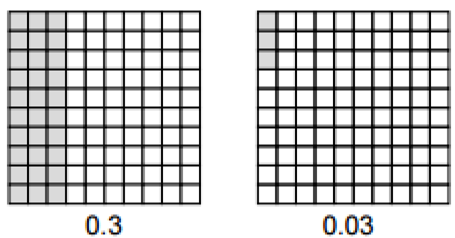
- Students can compare decimals using representations (decimal grids, base ten models, and number lines)
- Students can compare decimals using place value.
- Students can explain how decimals are compared.
- Students can record the results of comparisons using >, <, or = and justify the conclusions.
- Students can recognize that decimals can only be compared when the decimals being compared refer to the same whole.

Sample Questions

1. Khalid made a model to represent .25 and Aubrey made a model to represent .75. Whose model represents the bigger decimal? Explain.
2. Ben has a collection of dimes and pennies and Josiah has 54 pennies. Ben is arguing that he has more money even though he has less coins. What coins could Ben have that would make him correct?
3. What digits could be placed in the blank to make the number sentence true? 0.33 > 0.__9? Justify your answer.
4. Mohamad loves to run. On Monday, he ran 3.10 miles. On Tuesday, he ran 1.95 miles. On Wednesday he ran 2.50 miles. On Friday, he ran more than he did on Tuesday but less than he did on Monday. What could the possible amount of time be that he ran on Friday? Explain your thinking.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

When comparing two decimals, remind students that as in comparing two fractions, the decimals need to refer to the same whole. Allow students to use visual models to compare two decimals. They can shade in a representation of each decimal on a 10 × 10 grid. The 10 × 10 grid is defined as one whole. The decimal must relate to the whole.



Flexibility with converting fractions to decimals and decimals to fractions provides efficiency in solving problems involving all four operations in later grades.

Connections Across Standards

Use units to solve measurement problems (4.MD.1-2).

Generalize place value understanding (4.NBT.2).

3.NF.3d (Prior Grade Standard) Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	**5.NBT.3 (Future Grade Standard)** Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (\frac{1}{10}) + 9 \times (\frac{1}{100}) + 2 \times (\frac{1}{1000})$. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.MD.1 Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table. For example, express the length of a 4-meter rope in centimeters. Because 1 meter is 100 times as long as a 1 centimeter, a two-column table of meters and centimeters includes the number pairs 1 and 100, 2 and 200, 3 and 300,...

Essential Understandings

- Larger units can be expressed in terms of smaller units.
- The number of units used to measure an object will depend on the size of the unit of measure.
- The larger the unit, the smaller the measurement reads; the smaller the unit, the larger the measurement reads.
- Metric units are related by powers of ten.
  - 1 kilometer = 1,000 meters, 1 meter = 100 centimeters, 1 centimeter = 10 millimeters;
  - 1 kilogram = 1,000 grams; and
  - 1 liter = 1,000 milliliters.

Common Misconceptions

Students believe that larger units will give a larger measure. Students should be given multiple opportunities to measure the same object with different measuring units. For example, have the students measure the length of a room with centimeter cubes, with centimeter rulers, and with meter sticks. Students should notice that it takes fewer meter sticks to measure the room than rulers or cubes.

Academic Vocabulary/ Language

Metric System

- kilometer
- meter
- centimeter
- millimeter
- gram
- kilogram
- milliliter
- liter

Tier 2

- relative size
- record

Learning Targets	I can compare the relative sizes of units within the metric system. I can translate the larger units into equivalent smaller units. I can construct a two column table or number pairs to record measurement equivalence.

Assessment of Learning

- Students can explain the relationship between kilometers, meters, centimeters and millimeters.
- Students can explain the relationship between liters and milliliters.
- Students can explain the relationship between kilograms and grams.
- Students can explain the relationship between units to make conversions from larger units to smaller units.
- Students can record measurement equivalence in a two column table.

Sample Questions

1. Explain how to change a kilogram into a gram.
2. What differences do you notice between a kilometer, a meter, and a centimeter?
3. How many meters long is a whale that measures 3,000 centimeters.
4. Yesterday, I ran less than 5 kilometers but more than 1,200 meters. How far could I have run? Explain.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

In order for students to have a better understanding of the relationships between units, they need to use measuring devices in class. The number of units needs to relate to the size of the unit. Allow students to use meter sticks and rulers marked with centimeters to discover the relationship between centimeters and meters.. Have students record the relationships in a two column table or t-charts.

Career Connection

Students will use meter sticks and rulers with centimeters to solve problems with different units. Host a career speaker in the classroom to discuss how measurement and various units are used across their career field (e.g., construction, carpentry, design). Consider inviting a speaker who works on a school-based project, at your school or nearby, to share information about their work on school campuses. Lead a discussion that allows students to reflect on their work with different units and how it applies to the careers shared in the speaker's presentation.

Connections Across Standards

Generalize place value understanding for multi-digit whole numbers (4.NBT.1 - 2).
Use place value operations and properties of operations to perform multi-digit arithmetic (4.NBT.5).
Use the four operations with whole numbers to solve problems (4.OA.2 – 3).
Build fractions from unit fractions (4.NF.3-4).
Understand decimal notation for fractions, and compare decimal fractions (4.NF.5 – 7).

3.MD.2 (Prior Grade Standard)	5.MD.1 (Future Grade Standard)
Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters. Add, subtract, multiply, or divide whole numbers to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems involving notions of "times as much"; see Glossary, Table 2.	Know relative sizes of these U.S. customary measurement units: pounds, ounces, miles, yards, feet, inches, gallons, quarts, pints, cups, fluid ounces, hours, minutes, and seconds. Convert between pounds and ounces; miles and feet; yards, feet, and inches; gallons, quarts, pints, cups, and fluid ounces; hours, minutes, and seconds in solving multi-step, real-world problems.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.MD.2 Solve real-world problems involving money, time, and metric measurement.

a. Using models, add and subtract money and express the answer in decimal notation.
b. Using number line diagrams [G], clocks, or other models, add and subtract intervals of time in hours and minutes.
c. Add, subtract, and multiply whole numbers to solve metric measurement problems involving distances, liquid volumes, and masses of objects.

Essential Understandings

- Solve problems involving measurement.
- Answers to money problems can include the dollar symbol, $, and decimal point placed appropriately in decimal notation.
- Answers to time problems should include a.m. and p.m. as appropriate.

Common Misconceptions

Students believe that larger units will give the larger measure. Students should be given multiple opportunities to measure the same object with different measuring units. For example, have the students measure the length of a room with centimeter cubes, with rulers marked with centimeters, and with meter sticks. Students should notice that it takes fewer meter sticks to measure the room than rulers or tiles.

Academic Vocabulary/ Language

- interval
- decimal notation
- line diagrams
- hours
- minutes
- liquid volume
- mass

Metric System

- kilometer
- meter
- centimeter
- millimeter
- gram
- kilogram
- milliliter
- liter

Tier 2

- relative size
- record

Learning Targets	I can create a model to add and subtract money and express my answer in a decimal notation. I can use number line diagrams, clocks, or other models to add and subtract intervals of time in hours and minutes. I can add, subtract, and multiply whole numbers to solve metric measurement problems involving distances, liquid volumes, and masses of objects.

Assessment of Learning

- Students can represent and solve measurement word problems.
- Students can represent measurement quantities using number lines with measurement scales.
- Students can use models, add and subtract money and express the answer in decimal notation.
- Students can add and subtract intervals of time in hours and minutes using number line diagrams, clocks, or other models.
- Students can convert larger units into equivalent smaller units to solve a problem.

Sample Questions

1. Tara bought a bag of candy at the movie theatre. She spent less than $\frac{75}{100}$ of a dollar but more than $\frac{5}{10}$ of a dollar. How much money could Tara have spent on candy? Explain your thinking.
2. Charlie and 10 friends are planning a party. They purchased 3 quarts of juice. If each glass holds 8oz will everyone get at least one glass of juice? Provide an explanation.
3. Brendan did chores on Saturday. He mowed the lawn for 2 hours and 15 minutes, cleaned his bedroom for 40 minutes and cleaned the garage for 1 hour and 25 minutes. How much time did Brendan do chores on Saturday? Explain your thinking.
4. Isaac and Keith are training to run in a 5-kilometer race next month. Each morning, Isaac runs a route through the neighborhood park while Keith runs on the racetrack at the nearby high school. On Monday, Isaac ran $3\frac{1}{2}$ kilometers before he needed to take a break. Keith ran 7 laps on the track, and then he needed to rest. If each lap Keith ran was 400 meters, who ran a longer distance? Explain how you know which person ran a longer distance.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students are to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit.

Career Connection

Students will use meter sticks and rulers with centimeters to solve problems with different units. Host a career speaker in the classroom to discuss how measurement and various units are used across their career field (e.g., construction, carpentry, design). Consider inviting a speaker who works on a school-based project, at your school or nearby, to share information about their work on school campuses. Lead a discussion that allows students to reflect on their work with different units and how it applies to the careers shared in the speaker's presentation.

Connections Across Standards

Generalize place value understanding for multi-digit whole numbers (4.NBT.1 - 2).
Use place value operations and properties of operations to perform multi-digit arithmetic (4.NBT.5).
Use the four operations with whole numbers to solve problems (4.OA.2 – 3).
Understand decimal notation for fractions, and compare decimal fractions (4.NF.5 – 7).

3.MD.1-2 (Prior Grade Standard)	5.MD.1 (Future Grade Standard)
1. Work with time and money. a. Tell and write time to the nearest minute. Measure time intervals in minutes (within 90 minutes). Solve real-world problems involving addition and subtraction of time intervals (elapsed time) in minutes, e.g., by representing the problem on a number line diagram or clock. b. Solve word problems by adding and subtracting within 1,000, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the $ and ₵ symbol appropriately (not including decimal notation). 2. Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters. Add, subtract, multiply, or divide whole numbers to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems involving notions of "times as much"; Table 2.	Know relative sizes of these U.S. customary measurement units: pounds, ounces, miles, yards, feet, inches, gallons, quarts, pints, cups, fluid ounces, hours, minutes, and seconds. Convert between pounds and ounces; miles and feet; yards, feet, and inches; gallons, quarts, pints, cups, and fluid ounces; hours, minutes, and seconds in solving multi-step, real-world problems.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

<table>
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4.MD.3

Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. *For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter.*

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Common Misconceptions

Students believe that larger units will give the larger measure. Students should be given multiple opportunities to measure the same object with different measuring units. For example, have the students measure the length of a room with centimeter cubes, with rulers marked with centimeters, and with meter sticks. Students should notice that it takes fewer meter sticks to measure the room than rulers or tiles.

</td>
<td rowspan="2">

Academic Vocabulary/ Language

- perimeter
- area
- length
- width
- adjacent
- strategy

Tier 2

- apply
- solve
- explain

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Essential Understandings

- The area of a rectangle can be found by multiplying the lengths of adjacent sides (length and width) of the rectangle.
- Given an area or a perimeter of a rectangle and one side length, the adjacent side length can be determined.

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Learning Targets

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I can use apply addition and multiplication strategies to solve real-world problems involving the perimeter of rectangles.
I can use efficient strategies to solve real-world problems involving the area of rectangles.
I can reason about the relationship between area and perimeter to create efficient strategies to solve real-world multi-step problems when solving for the unknown.

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</table>

Assessment of Learning

- Students can explain how to solve for area and perimeter.
- Students can apply efficient strategies to solve real-world area problems including unknown length or width problems.
- Students can apply efficient strategies to solve real-world perimeters problems including unknown lengths or width problems.

Sample Questions

1. The area of the living room floor is 210 square feet. If it has a width of 14 feet, what is the length?
2. A rectangular figure has a perimeter of 35 cm. What could the lengths of the sides be? Give two possibilities.
3. The areas of two shapes are each 40 square inches, but the perimeters are very different. Sketch the two shapes and calculate the perimeters.
4. A rectangle has a length of 3 cm and a width of 2 cm, with an area of 6 sq.cm. Double the length and width. What is the area of the new rectangle? How does that affect the area? Try doubling the side lengths again and describe the pattern you see.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students used models to find area and perimeter in Grade 3. They need to relate discoveries from the use of models to develop an understanding of the area and perimeter formulas to solve real-world and mathematical problems. Students need to use the relationship between multiplication and division to solve problems involving finding the missing side. Focus for instruction should be understanding the concepts of area and perimeter and not the memorization of the formulas.

Career Connection

Students will use yard and meter sticks and rulers with inches and centimeters to solve problems with different units. Host a career speaker in the classroom to discuss how measurement and various units are used across their career field (e.g., construction, carpentry, design). Consider inviting a speaker who works on a school-based project, at your school or nearby, to share information about their work on school campuses. Lead a discussion that allows students to reflect on their work with different units and how it applies to the careers shared in the speaker's presentation.

Connections Across Standards

Generalize place value understanding for multi-digit whole numbers (4.NBT.1 - 2).
Use place value operations and properties of operations to perform multi-digit arithmetic (4.NBT.5).
Use the four operations with whole numbers to solve problems (4.OA.2 – 3).
Understand decimal notation for fractions, and compare decimal fractions (4.NF.5 – 7).

3.MD.5-8 (Prior Grade Standard)	5.MD.5 (Future Grade Standard)
5. Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. 6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). 7. Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole- number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$ (represent the distributive property with visual models including an area model). d. Recognize area as additive. Find the area of figures composed of rectangles by decomposing into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems. 8. Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the Associative Property of Multiplication. b. Apply the formulas $V = \ell \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.MD.4

Display and interpret data in graphs (picture graphs, bar graphs, and line plots [G]) to solve problems using numbers and operations for this grade.

Essential Understandings

- Data can be organized and represented in a picture graph, a bar graph, or a line plot.
- The key of a picture graph tells how many items each picture or symbol represents.
- The scale of a line plot can be whole numbers, halves, quarters, tenths, or hundredths.
- The scale of a bar graph varies depending on the data set.
- Symbols used in picture graphs and line plots should be consistently spaced and sized for visual accuracy.
- Information presented in a graph can be used to solve problems involving the data in the graph.

Common Misconceptions

Students may not choose the correct interval for when they create a bar graph or may not choose the right value of each symbol in their picture graph. Students need experiences with a variety of data so they can choose the interval that helps to display the data clearly.

Students may choose to display non numerical data in a line plot, for example "Favorite Pizza Toppings".

Academic Vocabulary/ Language

- line plot
- bar graph
- picture graph
- interpret
- data

Tier 2

- solve
- represent

Learning Targets	I can organize and represent data on a picture graph, a bar graph or a line plot. I can interpret the information from picture graphs, bar graphs, and line plots to solve real-world problems using the operations.

Assessment of Learning

- Students can use information in a graph to solve problems.
- Students can organize and represent data in a picture graph, a bar graph, or a line plot.
- Students can display and interpret data using real-world problems

Sample Questions

1. What is the relationship between line plots and number lines?
2. Create a line plot from the measurement of the length of student pencils in the classroom to the nearest centimeter.
3. In Mrs. Rensel’s class, 12 kids have a dog, 8 kids have a cat, 0 kids have a fish and 4 kids have a hamster. Create a bar graph to display this data about their class pets.
4. Create a picture graph from the following data. What questions could be answered by analyzing the data in this picture graph? Write down as many questions as you can think of.

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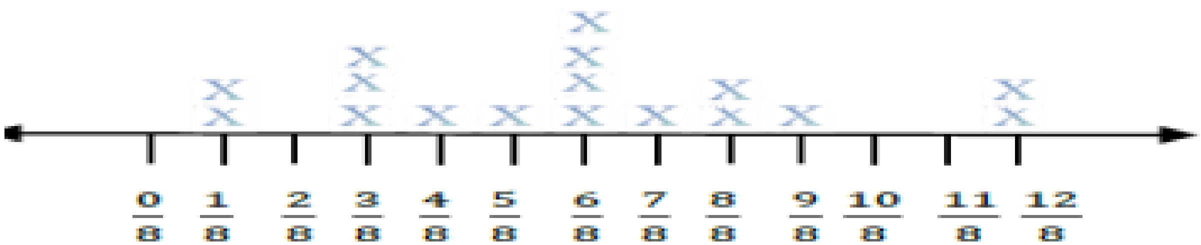
Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Data has been measured and represented on line plots in units of whole numbers, halves or quarters. A line plot is a simple way to organize data. Students have also represented fractions on number lines. Now students are using line plots to display measurement data in fraction units and using the data to solve problems involving addition or subtraction of fractions. Have students create line plots with fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$) and plot data showing multiple data points for each fraction.



Pose questions that students may answer, such as;

- “How many one-eighths are shown on the line plot?” Expect “two one-eighths” as the answer. Then ask, “What is the total of these two one-eighths?” Encourage students to count the fractional numbers as they would with whole-number counting, but using the fraction name.
- “What is the total number of inches for insects measuring $\frac{3}{8}$ inches?”

Students can use skip counting with fraction names to find the total, such as, “three-eighths, six-eighths, nine-eighths. The last fraction names the total. Students should notice that the denominator did not change when they were saying the fraction name. Have them make a statement about the result of adding fractions with the same denominator.

Students need to be shown data in a variety of graphs (bar graphs, picture graphs and line plots) and solve problems involving the data.

<table>
<tr><td colspan="2">Connections Across Standards
Solve grade-level appropriate problems using the four operations (4.OA.1-3).
Extend the understanding of fraction equivalence and ordering (4.NF.1-2).</td></tr>
<tr><td>3.MD.3-4 (Prior Grade Standard)
3. Create scaled bar graphs to represent a data set with several categories. Solve two-step “how many more” and “how many less” problems using information presented in the scaled graphs. For example, create a bar graph in which each square in the bar graph might represent 5 pets, then determine how many more/less in two given categories.
4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by creating a line plot [G], where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.</td><td>5.MD.2 (Future Grade Standard)
Display and interpret data in graphs (picture graphs, bar graphs, and line plots [G]) to solve problems using numbers and operations for this grade, e.g., including U.S. customary units in fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, or decimals.</td></tr>
</table>

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **Essential Understandings** ● Angles are formed when two rays share a common endpoint; the common endpoint of the rays is called a vertex. ● Angles are measured in degrees. ● A protractor is a tool used to measure angles. ● There are 360 degrees in a circle. ● One degree is $\frac{1}{360}$ of a circle.		**Common Misconceptions** Students are confused as to which number to use when determining the measure of an angle using a protractor because most protractors have a double set of numbers. Students should decide first if the angle appears to be an angle that is less than the measure of a right angle (90°) or greater than the measure of a right angle (90°). If the angle appears to be less than 90°, it is an acute angle and its measure ranges from 0° to 89°. If the angle appears to be an angle that is greater than 90°, it is an obtuse angle and its measures range from 91° to 179°. Ask questions about the appearance of the angle to help students in deciding which number to use.	**Academic Vocabulary/ Language** ▪ angle ▪ degree ▪ ray ▪ circle ▪ protractor ▪ endpoint ▪ geometric shape **Tier 2** ▪ recognize ▪ reference
Learning Targets	I can draw, measure, and explain different concepts of angles. I can explain how an angle is made of two rays with common endpoints. I can explain how an angle is measured by its reference to a circle. I can define and explain a "one-degree angle" and how it is used to measure angles.		

Assessment of Learning

- Students can describe an angle.
- Students can explain how to measure an angle.
- Students can describe an angle's relationships to a circle.
- Students can draw two rays with a common endpoint to form an angle.
- Students will explore circles explaining one-degree as $\frac{1}{360}$ of a circle.

Sample Questions

1. Draw and explain the parts of an angle.
2. How many angles can you find in our classroom? How do you know they are angles? Estimate the size of at least three of the angles that you found.
3. Explain how many "one degree angles" it takes to be equivalent to another given angle.
4. Ameila is thinking of an obtuse angle that is less than 115 degrees. What angle could she be thinking of? Explain how you know it is an obtuse angle.
5. Kai and Rebecca were investigating angles and circles, drawing circles and creating angles inside of their circles. Kai drew a small circle and divided it into six equal sections. He measured the angles of each section and found that they were all 60°. Rebecca decided to draw a circle that was larger than Kai's circle. She divided her circle into six equal sections and measured the angles of each section. She expected them to be larger than 60°, but they all measured 60°. Why might Rebecca have thought the sections of his circle would have a larger angle measurement than the sections in Kai's circle?

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Angles are geometric shapes composed of two rays that are infinite in length. Students can understand this concept by using two rulers held together near the ends. The rulers can represent the rays of an angle. As one ruler is rotated, the size of the angle is seen to get larger. Ask questions about the types of angles created. Responses may be in terms of the relationship to right angles. Introduce angles as acute (less than the measure of a right angle) and obtuse (greater than the measure of a right angle). Have students draw representations of each type of angle. They also need to be able to identify angles in two-dimensional figures. Angles are measured in degrees from 0 to 360. One complete rotation is 360 degrees. The measurement of an angle depends upon the fraction of the circle cut off by the rays. The degree measure of an angle can be estimated by comparing it to 90 degrees as more or less than 90 degrees. To find the exact measurement, students need to use a protractor.

Connections Across Standards

Draw and identify lines and angles, and classify shapes by properties of their lines and angles (4.G.1-2).
Use the four operations to solve problems (4.OA.3).
Understand fraction equivalence and ordering (4.NF.1-2).

(Prior Grade Standard)	**(Future Grade Standard)**
N/A	N/A

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

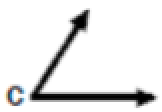
4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.		**Common Misconceptions** Students are confused as to which number to use when determining the measure of an angle using a protractor because most protractors have a double set of numbers. Students should decide first if the angle appears to be an angle that is less than the measure of a right angle (90°) or greater than the measure of a right angle (90°). If the angle appears to be less than 90°, it is an acute angle and its measures ranges from 0° to 89°. If the angle appears to be an angle that is greater than 90°, it is an obtuse angle and its measures range from 91° to 179°. Ask questions about the appearance of the angle to help students in deciding which number to use.	**Academic Vocabulary/ Language** ▪ angle ▪ degree ▪ protractor ▪ ray ▪ endpoint ▪ vertex ▪ acute ▪ obtuse ▪ right angle ▪ straight **Tier 2** ▪ sketch ▪ draw ▪ explain ▪ specified
Essential Understandings ● Geometric measurement: understand concepts of angle and measure angles. ● A straight angle has a measurement of 180 degrees. ● A right angle has a measurement of 90 degrees.			
Learning Targets	I can draw, measure, and explain different concepts of angles. I can use a protractor to measure whole degree angles. I can construct an angle of specified size, using a protractor.		

Assessment of Learning

- Students can measure an angle using a protractor.
- Students can use and explain how to use a protractor to create an angle with a given measurement.
- Students can describe acute, right, straight, and obtuse angles.
- Students can sketch angles when given a measurement.

Sample Questions

1. Measure angle C.



2. What angle could you draw that is greater than 35 degrees but less than 90 degrees? Sketch your angle.
3. Using a ruler, draw any triangle on your paper. Measure the three angles of your triangle using a protractor. Compare your angles with your classmates' angles. Find the sum of the angles of your triangle. What do you notice after you both found the sum of your three angles?
4. Erin measured an angle at 100 degrees, but realized that it was an acute angle, what could Erin have done wrong? Explain your thinking.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students are ready to use a tool to measure angles once they understand the difference between an acute angle and an obtuse angle. Angles are measured in degrees. There is a relationship between the number of degrees in an angle and circle which has a measure of 360 degrees. One complete rotation is 360 degrees. The measurement of an angle depends upon the fraction of the circle cut off by the rays. The degree measure of an angle can be estimated by comparing it to 90 degrees as more or less than 90 degrees. To find the exact measurement, students need to use a protractor. Students are to use a protractor to measure angles in whole-number degrees. They can determine if the measure of the angle is reasonable based on the relationship of the angle to a right angle. They also make sketches of angles of specified measure.

Connections Across Standards

Draw and identify lines and angles, and classify shapes by properties of their lines and angles (4.G.1-2).
Understand fraction equivalence and ordering (4.NF.1-2).
Use the four operations to solve problems (4.OA.3).

(Prior Grade Standard)	**(Future Grade Standard)**
N/A	N/A

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. **Essential Understanding** ● Angles can be decomposed into unit angles. (n degrees is n one degree angles.)	**Common Misconceptions** Students are confused as to which number to use when determining the measure of an angle using a protractor because most protractors have a double set of numbers. Students should decide first if the angle appears to be an angle that is less than the measure of a right angle (90°) or greater than the measure of a right angle (90°). If the angle appears to be less than 90°, it is an acute angle and its measure ranges from 0° to 89°. If the angle appears to be an angle that is greater than 90°, it is an obtuse angle and its measures range from 91° to 179°. Ask questions about the appearance of the angle to help students in deciding which number to use.	**Academic Vocabulary/ Language** ▪ angle ▪ degree ▪ protractor ▪ additive ▪ decompose ▪ equation ▪ symbol ▪ unknown angle measure **Tier 2** ▪ recognize ▪ solve ▪ diagram

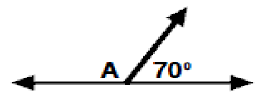
Learning Targets	I can draw, measure, and explain different concepts of angles. I can explain how when angles are joined in non-overlapping parts, the total measure is the sum of the parts. I can apply mathematical concepts of addition and/or subtraction to find the unknown angles on a diagram to solve real-world problems.

Assessment of Learning

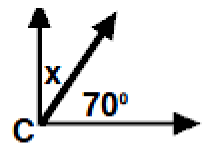
- Students can solve problems involving unknown angles.
- Students can use addition and subtraction to solve for unknown angles in real-world and mathematical problems
- Students will explore and explain decomposing an angle into parts, e.g., 45 degrees + 15 degrees + 30 degrees.

Sample Questions

1. What is the measure of angle A? Justify your answer.



2. Write an equation and solve for *x* if angle C is a right angle.



3. Find three different ways you can decompose 90 degrees and sketch the three examples.
4. If 3 angles are placed together so that they create a straight line (180 degrees) and one of the angles is obtuse, what are possible angle measurements of the other two angles?
5. Sophia drew an acute angle. Ralph drew an angle three times larger than Sophia's angle. What could the measures of the two angles be? Explain your thinking.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students are ready to use a tool to measure angles once they understand the difference between an acute angle and an obtuse angle. Angles are measured in degrees. There is a relationship between the number of degrees in an angle and circle which has a measure of 360 degrees. One complete rotation is 360 degrees. The measurement of an angle depends upon the fraction of the circle cut off by the rays. The degree measure of an angle can be estimated by comparing it to 90 degrees as more or less than 90 degrees. To find the exact measurement, students need to use a protractor. Students are to use a protractor to measure angles in whole-number degrees. They can determine if the measure of the angle is reasonable based on the relationship of the angle to a right angle. They also make sketches of angles of specified measure.

Connections Across Standards

Draw and identify lines and angles, and classify shapes by properties of their lines and angles (4.G.1-2).
Understand fraction equivalence and ordering (4.NF.1-2).
Use the four operations to solve problems (4.OA.3).

(Prior Grade Standard)	**5.MD.5c (Future Grade Standard)**
N/A	Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real- world problems.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

4.G.1

Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

Essential Understandings

- A point is a location in space; it has no length, width, or height.
- A line is a continuous straight path that extends indefinitely in two opposite directions.
- A line segment is a continuous straight path between two points.
- A ray is a continuous straight path that extends indefinitely in one direction from one point.
- Angles are made of two rays with the same endpoint; the endpoint is called the vertex.
- A right angle has a measure of 90˚.
- An acute angle has a measure of less than 90˚.
- An obtuse angle has a measure between 90˚ and 180˚
- A plane is a flat surface that extends infinitely in all directions.

Common Misconceptions

Students believe a wide angle with short sides may seem smaller than a narrow angle with long sides. Students can compare two angles by tracing one and placing it over the other. Students will then realize that the length of the sides does not determine whether one angle is larger or smaller than another angle. The measure of the angle does not change.

Academic Vocabulary/ Language

- right angle
- acute angle
- obtuse angle
- two –dimensional figure
- point
- perpendicular lines
- parallel lines
- line segments
- rays
- lines

Tier 2

- draw
- identify

Learning Targets	I can draw and identify lines and angles and use these to classify shapes. I can draw a point, line, line segment and ray and identify them in two-dimensional figures. I can draw right angles, acute angles, and obtuse angles by comparing then to a right angle that measures 90°. I can identify right, acute, and obtuse angles in two-dimensional figures using the comparison to a 90°. I can draw perpendicular and parallel lines and identify them in two-dimensional figures.

Assessment of Learning

- Students can draw and identify points, lines, line segments, rays, angles, parallel lines, and perpendicular lines in two-dimensional figures.

Sample Questions

These are pretty straight forward skills of having a student properly represent a drawing of each of these and be able to identify each one. Drawings should not be represented the same way each time.

1. Draw a line segment and a ray. Explain how they are different.
2. Why do you think points, lines, line segments, rays, angles, parallel lines, and perpendicular lines are considered to be the building blocks of geometry? Explain your thinking.
3. Find examples of parallel lines and perpendicular lines within the classroom. Where in life might you find parallel lines? Where might you find perpendicular lines? This task can be extended by students finding examples of acute, right and obtuse angles within the classroom.
4. Using one rubber band on a geoboard, create a square using the corner pegs. If you were to only move one of the corners, what other shapes could you make? How many different shapes can you make using one rubber band on the geoboard? Have students copy their geoboard designs to compare results with a classmate.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Angles

Students can and should make geometric distinctions about angles without measuring or mentioning degrees. Angles should be classified in comparison to right angles, such as larger than, smaller than or the same size as a right angle. Students can use the corner of a sheet of paper as a benchmark for a right angle. They can use a right angle to determine relationships of other angles.

Two-dimensional shapes

Two-dimensional shapes are classified based on relationships by the angles and sides. Students can determine if the sides are parallel or perpendicular, and classify accordingly. Characteristics of rectangles (including squares) are used to develop the concept of parallel and perpendicular lines. The characteristics and understanding of parallel and perpendicular lines are used to draw rectangles. Repeated experiences in comparing and contrasting shapes enable students to gain a deeper understanding about shapes and their properties.

Connections Across Standards

Students can use recognition of angle measures to classify two-dimensional figures (4.MD.5 and 4.MD.6).

3.G.1 (Prior Grade Standard)	**5.G.3-4 (Future Grade Standard)**
Draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles).	3. Identify and describe commonalities and differences of triangles based on angle measures (equiangular, right, acute, and obtuse triangles) and side lengths (isosceles, equilateral, and scalene triangles). 4. Identify and describe commonalities and differences of quadrilaterals based on angle measures, side lengths, and the presence or absence of parallel and perpendicular lines, e.g., squares, rectangles, parallelograms, trapezoids [G], and rhombuses.

Ohio's Learning Standards-Clear Learning Targets

Math Grade 4

<table>
<tr><td colspan="2">4.G.2
Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size.</td><td rowspan="2">Common Misconceptions

Students believe a wide angle with short sides may seem smaller than a narrow angle with long sides. Students can compare two angles by tracing one and placing it over the other. Students will then realize that the length of the sides does not determine whether one angle is larger or smaller than another angle. The measure of the angle does not change.</td><td rowspan="2">Academic Vocabulary/ Language
▪ right angle
▪ acute angle
▪ obtuse angle
▪ two –dimensional figures
▪ perpendicular lines
▪ parallel lines

Tier 2
▪ classify
▪ presence
▪ absence
▪ recognize
▪ identify</td></tr>
<tr><td colspan="2">Essential Understandings
● Two lines (or two line segments) in a plane are perpendicular if the angle between them is a right angle.
● Two lines (or two line segments) in a plane are parallel if they do not intersect.
● Two-dimensional figures can be classified (based on the presence or absence of parallel or perpendicular lines or presence or absence of angles of a specified size).</td></tr>
<tr><td>Learning Targets</td><td colspan="3">I can draw and identify lines and angles and use these to classify shapes.
I can categorize 2-D figures into groups based on whether certain sides are parallel or perpendicular.
I can categorize 2-D figures into groups based on whether certain angles are acute, obtuse, or right.</td></tr>
</table>

Assessment of Learning

- Students can describe parallel and perpendicular lines.
- Students can classify two-dimensional shapes using parallel and/or perpendicular line segments.
- Students can classify shapes by their angles.
- Students can use correct language when discussing points, lines, line segments, rays, and angles

Sample Questions

1. Give students an array of shapes and have the students sort them in appropriate groups. Students should be able to articulate in precise mathematical language why the groups are classified the way they are.
2. Have students discuss the following questions: Do two triangles always make a quadrilateral? Can you draw a pentagon with two right angles? Can you draw a hexagon with two right angles? Compare your shapes with a classmate.
3. Is a square a rectangle? Is a rhombus a parallelogram? Is a rhombus always a square? Why or why not?
4. Distribute tangram pieces to students. Pose the following questions? Can you make a square? A rectangle? A parallelogram? A trapezoid? A triangle? Have students copy their designs on paper and provide an explanation.

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Angles

Students can and should make geometric distinctions about angles without measuring or mentioning degrees. Angles should be classified in comparison to right angles, such as larger than, smaller than or the same size as a right angle. Students can use the corner of a sheet of paper as a benchmark for a right angle. They can use a right angle to determine relationships of other angles.

Two-dimensional shapes

Two-dimensional shapes are classified based on relationships by the angles and sides. Students can determine if the sides are parallel or perpendicular, and classify accordingly. Characteristics of rectangles (including squares) are used to develop the concept of parallel and perpendicular lines. The characteristics and understanding of parallel and perpendicular lines are used to draw rectangles. Repeated experiences in comparing and contrasting shapes enable students to gain a deeper understanding about shapes and their properties.

Connections Across Standards

Students can use recognition of angle measures to classify two-dimensional figures (4.MD.5 and 4.MD.6).

3.G.1 (Prior Grade Standard)	**5.G.4 (Future Grade Standard)**
Draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles).	Identify and describe commonalities and differences of quadrilaterals based on angle measures, side lengths, and the presence or absence of parallel and perpendicular lines, e.g., squares, rectangles, parallelograms, trapezoids [G], and rhombuses.