Mathematics

Math 6

2023-2024

Aligned with Ohio's Learning Standards
for Mathematics (2017)

Department of Academic Services
Office of Teaching and Learning
Curriculum Division

COLUMBUS CITY SCHOOLS

Curriculum Map

Year-at-a-Glance

The Year-at-a-Glance provides a high-level overview of the course by grading period, including:

- Units;
- Standards/Learning Targets; and
- Timeframes.

Scope and Sequence

The Scope and Sequence provides a detailed overview of each grading period, including:

- Units;
- Standards/Learning Targets;
- Timeframes;
- Big Ideas and Essential Questions; and
- Strategies and Activities.

Curriculum and Instruction Guide

The Curriculum and Instruction Guide provides direction for standards-based instruction, including:

- Unpacked Standards / Clear Learning Targets;
- Content Elaborations;
- Sample Assessments;
- Instructional Strategies;
- Instructional Resources; and
- ODE Model Curriculum with Instructional Supports.

COLUMBUS CITY SCHOOLS

Math 6

Year-at-a-Glance

Grading Period 1

Reporting Category: Ratios and Proportions — 7 weeks

1. Ratios and Rates
    a. Understanding Ratios & Rates
    b. Equivalent Ratios
    c. Solving Ratio & Rate Problems
    d. Converting Customary Measurement Units
2. Fractions, Decimals, and Percents
    a. Understand Percents
    b. Relate Fractions, Decimals, and Percents
    c. Percent of a Number

Grading Period 2

Reporting Category: The Number System — 7 weeks

1. Compute with Multi-Digit Numbers and Fractions
    a. Division with Multi-Digit Whole Numbers
    b. Operations with Multi-Digit Decimals
    c. Division with Fractions
2. Integers, Rational Numbers, and the Coordinate Plane
    a. Integers on the Number Line
    b. Opposites & Absolute Value
    c. Comparing and Ordering Integers
    d. Understand and Compare Rational Numbers
    e. The Coordinate Plane
    f. Graphing Reflections
    g. Absolute Value and Distance

Grading Period 3

Reporting Category: Expressions and Equations — 8 weeks

1. Numerical and Algebraic Expressions
    a. Powers and Exponents
    b. Numeric and Algebraic Expressions
    c. Factors and Multiples
    d. Distributive Property
    e. Equivalent Expressions
2. Equations and Inequalities
    a. Substitution To Solve
    b. One-Step Equations
    c. Inequalities
3. Relationships Between Two Variables
    a. Independent and Dependent Variables
    b. Writing Equations from Tables
    c. Graphing Relationships
    d. Represent Real-World Relationships

Grading Period 4

Reporting Category: Geometry and Statistics — 11 weeks

1. Statistical Measures and Displays
    a. Statistical Questions
    b. Dot Plots and Histograms
    c. Measures of Center & Box Plots
    d. Graphical Displays
2. Area
    a. Area of Parallelograms, Triangles, Trapezoids, Regular Polygons
    b. Polygons on the Coordinate Plane
3. Volume and Surface Area
    a. Volume of Rectangular Prisms
    b. Surface Area of Regular Prisms, Triangular Prisms, Pyramids

Standards for Mathematical Practice

The Standards for Mathematical Practice (SMP) describe skills that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The design of each item on Ohio's state tests encourages students to use one or more Standards for Mathematical Practice.

Modeling and Reasoning are included in the eight Standards for Mathematical Practice within Ohio's Learning Standards. Each grade's blueprint identifies modeling and reasoning as an independent reporting category that will account for a minimum of 20 percent of the overall points on that grade's test.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Practice - Grade 6

Modeling and Reasoning on Ohio's State Tests in Mathematics

COLUMBUS CITY SCHOOLS

Math 6

Scope and Sequence

Students should be assessed using teacher-based resources and the ALEKS program. Students are automatically enrolled in the ALEKS course Middle School Course 1/Lvl 6. Teachers can move students into RTI 6 (Promote at 65%) if students show a need for remediation - usually if a student scores less than 15% on the Initial Knowledge Check. If students have a need for deeper remediation, consider moving them to MS RTI Tier 3 (promote at 85%) Refer to our guide at https://tinyurl.com/CCS-ALEKS-GUIDE

Textbook information
McGraw-Hill - Reveal Math Course 1

Grading Period 1

Reporting Category: Ratios and Proportions — **7 weeks**

Module 1: Ratios and Rates 🔗 — **5 weeks**

Lesson	Standards/Learning Targets	Essential Questions	Strategies/Activities
1.1 Understand Ratios	6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. Supporting Standard(s): 6.NS.4	How can I describe how two quantities are related? What is a ratio and how do we use them to compare quantities? How can I express a ratio?	• Use the language of ratios: "for each," "for every." • Distinguish between part-to-part, part-to-whole, and whole-to-part comparisons. • Introduce ratios and rates with real-world experiences such as taste. • Use visual models to represent ratios • Write ratios symbolically to describe relationships between two quantities.

1.2 Tables of Equivalent Ratios	6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
	a. Make tables of equivalent ratios relating quantities with whole number measurements; find missing values in the tables; and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
	
	Supporting Standard(s):
	6.RP.1, 6.RP.2, 6.RP.3b
		How do we make sense of whether two or more ratios are equivalent?
		How can you find two ratios that describe the same relationship?
		How can I use models (tape diagrams, double number lines, ratio tables, coordinate plane, etc.) to display an understanding of ratios and proportional relationships?
			• Recognize ratios as multiplicative relationships.
			• Use models to solve problems involving ratios and unit rates such as ratio tables, tape diagrams, and double number lines.
			• Identify or create equivalent ratios.
			• Make tables of equivalent ratios relating quantities with whole number measurements to do the following:
			  ○ Find missing values;
			  ○ Compare ratios
			• Building-Up and Breaking-Down are strategies where students take a ratio and either build it up

				using addition or take it down using subtraction to get a new equivalent ratio. Although it is a good strategy to introduce ratios, it is not truly proportional reasoning because it primarily uses additive reasoning which does not take into account the constant ratio between the two quantities. However, it can be an important benchmark in understanding. To move students from additive reasoning to more multiplicative reasoning, ask students for more efficient ways to move across the ratio table stressing multiples.
	1.3 Graphs of Equivalent Ratios	6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements; find missing values in the tables; and plot the pairs of values on the coordinate plane. Use tables to compare ratios. Supporting Standard(s): 6.RP.1	How can I use graphs when working with ratios?	• Make tables of equivalent ratios relating quantities with whole number measurements to do the following: ○ Plot pairs of values in the first quadrant of the coordinate plane; ○ Compare ratios; and ○ Develop the concept of proportion without solving proportions explicitly.
	1.4 Compare Ratio Relationships		How do ratios describe relationships between two quantities?	• Make tables of equivalent ratios relating quantities with whole number measurements to do the following: ○ Compare ratios.
	1.5 Solve Ratio Problems	6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. 6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	How can rates, ratios, and proportional reasoning help us better understand the use of ratios and rates in the world around us? How would you use ratio and rate reasoning in real world situations?	• Make tables of equivalent ratios relating quantities with whole number measurements to do the following: ○ Develop the concept of proportion without solving proportions explicitly.
	1.6 Convert Customary Measurement Units	6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or	How can I use graphs to convert from one unit to another in either the US Customary or metric system?	• Apply ratio reasoning to convert measurement units within the same system. • Solve real-life problems involving measurement units that need to be converted.

		equations. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. 6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	Why is it important to know how to measure in everyday life & how to convert (change) units of length, capacity, and weight?	• Students should solve real-life problems involving measurement units that need to be converted. Representing these measurement conversions with tools such as ratio tables, t-charts, double number line diagrams, or tape diagrams/bar models will help students internalize the size relationships between same system measurements.
	1.7 Understand Rates and Unit Rates	6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. 6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements; find missing values in the tables; and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. Supporting Standard(s): 6.RP.1	What is a rate and unit rate? How do ratios, rates, and proportions compare to each other?	• As students move from additive reasoning to multiplicative reasoning, draw attention to the common ratio and connect it to the unit rate. This is the foundation for proportional reasoning that will be more solidified in Grade 7 and which extends to slope in Grade 8. • Explain to students that the same or fixed ratio is associated with a quality such as steepness, flavor, or speed remains the same or fixed as the variable changes together.
	1.8 Solve Rate Problems	6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. 6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. b. Solve unit rate problems including those involving unit pricing and constant speed.	How can ratio and rates be applied to real-world scenarios? How can you use rates to describe changes in real-life problems?	• One method for solving problems involving ratios is by finding the unit rate first. Students ask themselves “How many for one?”. They recognize that a relationship exists, and then they calculate the rate so that one of the quantities is one. They can use this strategy to compare two rates, or they can multiply the unit rate by one of the quantities to find a missing value.

Module 2: Fractions, Decimals and Percents 🔗			2 weeks
Lesson	**Standards/Learning Targets**	**Essential Questions**	**Strategies/Activities**
2.1 Understand Percents	6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. c. Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means 30/100 times the quantity; solve problems involving finding the whole, given a part and the percent.	What is a percent? How is a percent like a ratio? How would you describe percent of a quantity as a rate per 100?	• Represent percents using models, such as 100 grids, tape diagrams, and double number lines. • Use ratio reasoning to relate a percent of a quantity as a rate per 100. • Use language to explain percent: ask students what "cent" actually means (100). What is a century? How many cents in a dollar?
2.2 Percents Greater Than 100% and Less Than1%	OMIT: ODE introduces Percents Greater Than 100% and Less Than1% in 7th Grade.		
2.3 Relate Fractions, Decimals, and Percents	6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. c. Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means 30/100 times the quantity; solve problems involving finding the whole, given a part and the percent.	How are whole numbers, fractions, decimals, and percents related to one another? What is the connection between a percent and a fraction? How are percents expressed as fractions and decimals?	• Use 100 grids to compare percents to fractions. • Compare different ratios by changing to percents.
2.4 Find the Percent of a Number	6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. c. Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means 30/100 times the quantity; solve problems involving finding the whole, given a part and the percent. Supporting Standard(s): 6.RP.1	How can you use a model to find the percent of a number? How can we use parts and wholes to find the percent of a number? How can you use fractions, decimals, and percents to solve everyday problems?	• Use benchmark percents (1%, 5%, 10%, 20%, 25%, 50%, and 100%) to compute other percents of a given whole number both mentally and with a model. • Use bar diagrams, equivalent ratios, double number lines, and ratio tables to find the percent of a number.

2.5 Estimate the Percent of a Number	6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. c. Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means 30/100 times the quantity; solve problems involving finding the whole, given a part and the percent.	Why are benchmark percents useful? Why is the estimate lower (or higher) than the actual part?	• Use benchmark percents to estimate a percent problem. ○ Round the whole to a convenient number (to the nearest 100?) and round the percent to a convenient benchmark fraction. Then, multiply the rounded whole by the benchmark fraction. • The use of bar diagrams and equivalent ratios is beneficial when estimating the percent of a number.
2.6 Find the Whole	6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. c. Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means 30/100 times the quantity; solve problems involving finding the whole, given a part and the percent. Supporting Standard(s): 6.NS.4	How can you use bar diagrams to find missing parts and wholes? How can you use the percent to determine the number of sections on a double number line? When writing equivalent ratios, do you need to find the percent, part, or whole?	• Use bar diagrams, equivalent ratios, and double number lines to find the whole given the part and the percent.
Reporting Category: The Number System			**7 weeks**
Module 3: Compute with Multi-Digit Numbers and Fractions 🔗			**3 weeks**
Lesson	**Standards/Learning Targets**	**Essential Questions**	**Strategies/Activities**

3.1 Divide Multi-Digit Whole Numbers	6.NS.2 Fluently divide multi-digit numbers using a standard algorithm.	How do adding, subtracting, multiplying, and dividing integers relate to each other?
		How can I check a division problem?
			• It may be beneficial to expose students to multiple standard algorithms and have them try each. After exposure to several algorithms, students can choose which algorithm makes sense to them. Use class discussion to help students create understanding explaining why their preferred algorithm works.
			  ○ Partial Quotients
			  ○ Area Model
			  ○ Explicit-Trade Method
			  ○ Traditional Division

				• Check answers with multiplication. • Grid paper helps students organize their computational work. It is highly encouraged that all students have access to grid paper.
	3.2 Compute with Multi-Digit Decimals	6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.	Why is the placement of decimal points important when I am computing with decimals? How can I maintain the proper place value when I am multiplying decimals? How does estimating decimals help with adding and subtracting them? How do I divide decimals? When adding and subtracting decimals, what is the first step and the most important step?	• Make sure to intentionally connect decimal notation and computation with fraction notation and computation. • When adding and subtracting decimals continue to emphasize place value. • For students struggling with addition and subtracting with decimals, connect the algorithm with models such as base-ten blocks, decimal squares, or number lines. Models help reinforce why it is important to line up the place value positions before adding and subtracting. • One strategy for solving equations with decimals is to clear the equation of decimals by creating an equivalent equation without decimals. For example, $5 - 0.17 = x$ is equivalent to $500 - 17 = 100x$ by multiplying each side of the equation by 100. • Different Algorithms: ○ Multiplication ■ Area Model ■ Partial Products ■ Lattice Algorithm ■ Traditional Multiplication ○ Division ■ Partial Quotients ■ Explicit-Trade ■ Traditional Division
	3.3 Divide Whole Numbers by Fractions	6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.	How can I divide with fractions?	• The use of models is highly encouraged to give students a visual understanding of fraction division. ○ A variety of models are available to students. It is important for students to be exposed to many different types of models, and they should
	3.4 Divide Fractions by Fractions		How does multiplying and dividing fractions relate to one	

			another?	be allowed to use the tool that best models the problem and makes sense to them. It may be helpful for students to use manipulatives such as snap cubes or fraction bars to begin exploring fraction division. Using grid paper for modeling or folding strips could also be helpful for students to make the mathematics visible to them. ● Methods for Dividing Fractions ○ Dividing across fractions ○ Common Denominator Method ○ Dividing Fractions by Multiplying the Inverse ● Please see the Ohio Model Curriculum for more details on strategies for teaching fraction division.
	3.5 Divide with Whole and Mixed Numbers		How can I change mixed numbers to use them in division problems?	
Module 4: Integers, Rational Numbers, and the Coordinate Plane				**4 weeks**
Lesson	**Standards/Learning Targets**		**Essential Questions**	**Strategies/Activities**
	4.1 Represent Integers	6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. 6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	What are integers? What are negative numbers? How can you figure out if positive or negative numbers are involved when you are figuring out a real world problem?	● This is the first time that students see the number line extending in both directions. Since a number line shows both quantity and direction, there are now two different numbers that have the same quantity such as 4 and −4. Therefore, students shift from using line segments to show value on the number line to using arrows which show both quantity (magnitude) and direction. ● Make sure students are exposed to both horizontal and vertical number lines. Scales are not limited to 1. ● The negative sign can mean several things: ○ A sign attached to a number to form negative numbers; ○ A subtraction sign; or ○ An indication to take the opposite of.
	4.2 Opposites and	6.NS.5 Understand that positive and negative	How can I use a number line to	● Students should learn that the absolute value of a

	Absolute Value	numbers are used together to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. 6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite. 6.NS.7 Understand ordering and absolute value of rational numbers. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.	determine a number's opposite? How can you figure out if positive or negative numbers are involved when you are figuring out a real world problem?	number does not take into account sign or direction; it is only a measure of distance (magnitude) from 0. Discourage students from saying that the "answer is always positive or 0" since that will lead to misconceptions when students encounter problems such as $\|4x-2\| = 18$ in high school.
	4.3 Compare and Order Integers	6.NS.7 Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. d. Distinguish comparisons of absolute value from statements about order.	How can I use a number line to order integers? How does the position of an integer on the number line relate to order?	• Continue to use number lines to give students a visual for ordering integers. ○ Please contact math curriculum leaders if you are interested in an classroom empty number line and middle school number cards. • Expose students to both horizontal and vertical number lines.

		Supporting Standard(s): 6.NS.6, 6.NS,6c		
	4.4 Rational Numbers	6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. 6.NS.7 Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. Supporting Standard(s): 6.NS.7b	What is a rational number? How can I use the number line to order rational numbers?	• Continue using number lines to give students a visual for ordering rational numbers. ○ Please contact math curriculum leaders if you are interested in an classroom empty number line and middle school number cards. • Expose students to both horizontal and vertical number lines.
	4.5 The Coordinate Plane	6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and	What is a coordinate plane? How can I find, identify, or place a point on the coordinate plane? What is a quadrant on the coordinate plane? How do the signs in an ordered pair indicate its location on a coordinate grid? How would understanding a coordinate system benefit you in everyday life?	• Students are seeing all 4 quadrants for the first time in 6th grade. It might be helpful to label the ends of each axis with their respective + or −. • Scales should not be limited to 1.

		other rational numbers on a coordinate plane. 6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.		
	4.6 Graph Reflections of Points	6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., − (−3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. 6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	How are two coordinate pairs with different signs related? How can I reflect an ordered pair?	• Scales should not be limited to 1. • Students should realize that when two ordered pairs differ only by signs, they are reflections across one or both of the axes.
	4.7 Absolute Value and Distance	6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with	What is absolute value? How does absolute value relate to distance on a coordinate grid?	• Distance on the coordinate plane is limited to horizontal and vertical distances only. • When counting distance on the coordinate plane, some students will count the starting point as "1."

	the same first coordinate or the same second coordinate. Also Addresses: 6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. 6.NS.7 Understand ordering and absolute value of rational numbers. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.		Encourage them to count the boxes between points instead.

Reporting Category: Expressions and Equations — **12 weeks**

Module 5: Numerical and Algebraic Expressions — **4 weeks**

Lesson	Standards/Learning Targets	Essential Questions	Strategies/Activities
5.1 Powers and Exponents	6.EE.1 Write and evaluate numerical expressions involving whole number exponents.	What are exponents and how are they used?	• Relate exponents to what students already know about powers of 10. • Make sure students understand that exponents are a notation for repeated multiplication, just as multiplication is notation for repeated addition. • The standard is about evaluating expressions. This section/lesson gives students an understanding of the rules for exponents so they can apply them in the next section/lesson.
5.2 Numerical Expressions	6.EE.1 Write and evaluate numerical expressions involving whole number exponents. Also Addresses: 6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers. c. Evaluate expressions at specific values of their	What is an expression? What is a variable? What is a coefficient? What is a term? How do I use order of operations to evaluate an expression?	• The use of negatives and positives should mirror the level of introduction in Grade 6; students are developing the concept and not generalizing operation rules. • The use of technology can assist in the exploration of the meaning of expressions. Many calculators will allow one to store a value for a

		variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order.		variable and then use the variable in expressions. This enables the student to discover how the calculator deals with expressions like x^2 , 5x, xy, and 2(x + 5). • The emphasis should not be placed on counting terms in an expression. • Provide a variety of expressions and problem situations for students to practice and deepen their skills. They should understand that when evaluating an expression through substitution, like variables will have the same value.
	5.3 Write Algebraic Expressions	6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 − y. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. 6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	How can I translate a word phrase into a mathematical expression? How can we use variables to solve real world problems?	• Include whole-number exponents, fractions, and decimals when writing expressions. • Encourage students to show step-by-step thinking when rewriting an expression with the least number of terms. • Provide opportunities for students to share their work and explain their thinking.
	5.4 Evaluate Algebraic Expressions	6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order.	How can I evaluate an expression?	• Using the mnemonic PEMDAS causes many misconceptions and is highly discouraged. Students get the impression that multiplication must come before division, addition must come before subtraction, operations must be performed left to right, and that calculations in parenthesis must come first. Instead people may solve 5 – 9 + 3 – 5 + 9 by changing the order using the Associative and Additive Inverse properties (5 – 5) + (–9 + 9) + 3 instead of adding and subtracting from left to

				right.
	5.5 Factors and Multiples	6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor.	What is a factor? What are common factors and the greatest common factor (GCF)? How can you use prime factorization?	• Show how the Greatest Common Factor (GCF) is useful in expressing the numbers using the Distributive Property [(36 + 24) = 12(3 + 2), where 12 is the GCF of 36 and 24].
	5.6 Use the Distributive Property	6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. 6.EE.3 Apply the properties of operations to generate equivalent expressions. Also Addresses: 6.EE.2.b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.	How do I use the distributive property? How do I know two expressions are equivalent?	• Students often get confused when they see a variable such as x without a coefficient. Make a connection to the Multiplicative Identity Property and have students write the "invisible" 1 in front of the variables without coefficient. - This will also help students correct the belief that if there is no coefficient, then x = 1. • Have students write out the distribution: 2(x + 3) = 2(x) + 2(3) = 2x + 6; this may reduce errors when students forget to distribute to the second term.
	5.7 Equivalent Algebraic Expressions	6.EE.3 Apply the properties of operations to generate equivalent expressions. 6.EE.4 Identify when two expressions are equivalent, i.e., when the two expressions name the same number regardless of which value is substituted into them. Supporting Standard(s): 6.EE.2	What are equivalent expressions? How can I find out if two expressions are equivalent?	• Provide opportunities for students to write equivalent expressions, both numerically and with variables. • Use manipulatives to model expressions (algebra tiles, counters, unifix cubes, etc). This can help students translate between concrete numerical expressions and abstract symbolic representations.
Module 6: Equations and Inequalities🔗				**4 weeks**

Lesson	Standards/Learning Targets	Essential Questions	Strategies/Activities
6.1 Use Substitution to Solve One-Step Equations	6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. Also Addresses: 6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	What is an equation? What is a solution to an equation? How can I solve an equation? How can substitution be used to find the solution to an equation?	• The skill of solving an equation must be developed conceptually before it is developed procedurally. This means that students should be thinking about what numbers could possibly be a solution to the equation before solving the equation. • Although solving equations in this cluster is limited to one-step equations and inequalities, students should be given more complicated linear equations when using substitution to determine whether numbers in a set make the equation true. • A variety of concrete models such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable. Students should move from concrete models, to pictures, and then equations. • The focus in Grade 6 should be on inverse operations, so the discussion around the models should always go back to inverse operations. • Students need practice with one-step equations with fractions using manipulatives and diagrams instead of just using inverse operations in order to give the opportunity to create understanding. This will help alleviate the misuse of fraction rules in later grades/courses. Numbers that are easily modeled should be used. • The process of translating between mathematical phrases and symbolic notation will also assist students in the writing of equations or inequalities for a situation. This process should go both ways; students should also be able to write a mathematical phrase for an equation. A strategy for assisting with this is to give students an equation and ask them to come up with the situation or story that the equation could be modeling. • Provide multiple situations in which students must determine if a single value is required as a solution, or if the situation allows for multiple
6.2 One-Step Addition Equations 6.3 One-Step Subtraction Equations 6.4 One-Step Multiplication Equations 6.5 One-Step Division Equations	6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. 6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q, and x are all nonnegative rational numbers.	What are inverse operations? How can I use inverse operations to solve one-step equations?	
6.6 Inequalities	6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. 6.EE.8 Write an inequality of the form x > c or x <	What is an inequality? How do inequalities affect the number of solutions? How do I translate words (greater than, less than , at least, etc) into mathematical symbols? (and vice versa)	

	c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams. Also Addresses: 6.NS.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. 6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.		solutions. • Have students write inequalities to represent real-world problems. Writing out inequalities in words is a strategy for assisting students' conceptual understanding. • When graphing inequalities on a number line, do not tell students the arrow points in the direction of the inequality sign because this creates a misconception that is difficult to combat in later grades. Instead ask the student when graphing, which numbers make the inequality true.
Module 7: Relationships Between Two Variables 🔗			**4 weeks**
Lesson	**Standards/Learning Targets**	**Essential Questions**	**Strategies/Activities**
7.1 Relationships Between Two Variables	6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. Also Addresses: 6.EE.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, using the algebraic order of operations when there are no parentheses to	How are variables in an equation related? What makes a variable dependent or independent?	• Provide multiple situations for the student to analyze and determine what unknown is dependent on the other components. • It is important for students to identify the two quantities that are being compared and then choose variables to represent the quantities and finally define what each variable means in the context of the problem. • Multiplying unknowns and variables by fractions is essential for solving equations and understanding functions. According to research using fractions as multipliers on unknown quantities is a significant challenge for students. In order to be successful in later mathematics, students need to develop reciprocal reasoning. For example, if $y = \frac{3}{5}x$ then $x = \frac{5}{3}y$.

		specify a particular order.		○ Start with whole numbers and their inverses, then move oro equations using inverses that are reciprocals to rational numbers beyond whole numbers. ● Students should use multiple representations to explore how one variable changes in relation to the other. They need to be able to translate freely among the story, words (mathematical phrases), models, tables, graphs, and equations. In addition students need to be flexible enough to start with any of the representations and use the initial representation to develop the others.
	7.2 Write Equations to Represent Relationships Represented in Tables	6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. Also Addresses: 6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. 6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q, and x are all nonnegative rational numbers.	How can I use a table to represent a relationship? How can I use a table to write an equation?	
	7.3 Graphs of Relationships	6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. Also Addresses: 6.RP.3a Make tables of equivalent ratios relating quantities with whole number measurements; find missing values in the tables; and plot the pairs of values on the coordinate plane. Use tables to	How can graphing functions better represent information?	

		compare ratios. 6.NS.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. 6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.		
	7.4 Multiple Representations	6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. Also Addresses: 6.RP.3a Make tables of equivalent ratios relating quantities with whole number measurements; find missing values in the tables; and plot the pairs of values on the coordinate plane. Use tables to compare ratios. 6.NS.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. 6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending	What are different ways I can represent relationships?	

	on the purpose at hand, any number in a specified set. 6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q, and x are all nonnegative rational numbers.		
Reporting Category: Geometry and Statistics			**9 weeks**
Module 10: Statistical Measures and Displays 🔗			**3 weeks**
Lesson	**Standards/Learning Targets**	**Essential Questions**	**Strategies/Activities**
10.1 Statistical Questions	6.SP.1 Develop statistical reasoning by using the GAISE model. a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. Also Addresses: 6.SP.5 Summarize numerical data sets in relation to their context. a. Report the number of observations.	How can we determine if a question is a statistical question? What is statistical variability? How can I solve a question which includes a subject that has variability? How can I measure a data set? What are the six steps for solving a statistical question?	• "What is a Statistical Question?" is a lesson plan by the United States Census Bureau has an activity on identifying statistical questions. (Student Version)
10.2 Dot Plots and Histograms	6.SP.4 Display numerical data in plots on a number line, including dot plots (line plots), histograms, and box plots. (GAISE Model, step 3) 6.SP.5 Summarize numerical data sets in relation to their context. a. Report the number of observations.	How can data be plotted? What is a dot plot? What is a histogram?	• In Grade 6, students are at Level A. At Level A, students should have some opportunities to collect data, but it is not necessary in every case. • It is advised to use naturally occurring events in the classroom regarding data such as how many people on average buy lunch at school. When students collect data, the data should be limited to the classroom. • Students should understand the difference between categorical and numerical data. • Although teachers may want to use categorical data to help introduce the GAISE process, standards 6.SP.3-5 focus on numerical data. • Discuss with students the importance of experimental design when conducting simple
10.3 Measures of Center	6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. 6.SP.4 Display numerical data in plots on a number line, including dot plots (line plots), histograms, and	What is the mean, median and mode for a given data set? How are mean, median, and mode related?	

		box plots. (GAISE Model, step 3) 6.SP.5 Summarize numerical data sets in relation to their context. a. Report the number of observations. b. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range) as well as informally describe the shape and the presence of clusters, gaps, peaks, and outliers in a distribution. Also Addresses: 6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.		experiments. A simple experiment consists of taking measurements on a particular condition or group. • Highlight the importance of consistency on the controls of the experiment to gain more comparable data, which could be done directly or could be done indirectly leveraging students' competitive natures. • To find the median visually and kinesthetically, students could reorder the data in ascending or descending order, then place a finger on each end of the data and continue to move toward the center by the same increments until the fingers touch. • The concept of mean as "fair share" can be demonstrated visually and kinesthetically by using stacks of linking cubes or blocks. The blocks are redistributed among the towers so that all towers have the same number of blocks. It may be useful initially to present problems that work out as whole numbers, so students can use concrete models such as blocks. However, students should move towards finding the mean as a decimal or fraction, reinforcing the idea that the mean is a number representing the arithmetic average but not necessarily an actual number in the data set. • When summarizing data, students should be able to describe what a "typical" piece of data represents and connect this concept to measures of center. Students should not only determine measures of center and measures of variability, but also use these numbers to answer the statistical question asked in the context of the problem. The distribution should be described in terms of its center (mean, median, mode), spread (range, interquartile range), and shape (symmetry, skewness, clusters, gaps, outliers). • Continue to have students connect contextual situations to data to describe the data set in words prior to computation.
	10.4 Interquartile Range and Box Plots	6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. 6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. 6.SP.4 Display numerical data in plots on a number line, including dot plots (line plots), histograms, and box plots. (GAISE Model, step 3) 6.SP.5 Summarize numerical data sets in relation to	What is a box plot? What is range? What is interquartile range?	

		their context. c. Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range) as well as informally describe the shape and the presence of clusters, gaps, peaks, and outliers in a distribution.		
	10.5 Mean Absolute Deviation	OMIT [Standard moved to 7th Grade]		
	10.6 Outliers	6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. 6.SP.4 Display numerical data in plots on a number line, including dot plots (line plots), histograms, and box plots. (GAISE Model, step 3) 6.SP.5 Summarize numerical data sets in relation to their context. c. Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range) as well as informally describe the shape and the presence of clusters, gaps, peaks, and outliers in a distribution. d. Choose the measures of center and variability, based on the shape of the data distribution and the context in which the data were gathered.	What is an outlier? How does an outlier affect the data?	
	10.7 Interpret Graphical Displays	6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	How do you decide which graph to use to display your data?	• Students need multiple opportunities to look at data to determine if a question is a statistical question. Numerical data should be analyzed using different tools, such as organized lists, box-plots,

		6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. 6.SP.4 Display numerical data in plots on a number line, including dot plots (line plots), histograms, and box plots. (GAISE Model, step 3) 6.SP.5 Summarize numerical data sets in relation to their context. a. Report the number of observations. b. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range) as well as informally describe the shape and the presence of clusters, gaps, peaks, and outliers in a distribution. d. Choose the measures of center and variability, based on the shape of the data distribution and the context in which the data were gathered. Also Addresses: 6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. 6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.		dot plots, histograms, and stem-and-leaf plots; categorical data should be analyzed using lists and bar graphs. After students have analyzed the data, it is important to think about what may have caused the data to look like it does. This should lead to a discussion on variability.

Module 8: Area 🔗			3 weeks
Lesson	**Standards/Learning Targets**	**Essential Questions**	**Strategies/Activities**
8.1 Area of Parallelograms 8.2 Area of Triangles 8.3 Area of Trapezoids	6.G.1 Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems. 6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order.	How can I find the area of a two dimensional figure? How can I find the area of a rectangle? How can I find the area of a parallelogram? How can I find the area of a triangle? How can area be applied in real world situations?	• When exploring area and volume it is very important for students to continue to physically manipulate materials and create drawings in order to make connections to the symbolic and more abstract aspects of geometry. • Although unit squares and unit cubes are the most common units used in area and volume problems, units can be anything including sheets of paper or boxes. These are oftentimes the type of units used in real-life area and volume problems. It might be helpful to extend students' thinking by exposing them to problems with different units besides unit squares. For example, have students measure the area table using a piece of paper as a unit, and then have them measure the same table using standard units such as square feet or square inches. Make the connection that although the area is represented with different numbers and units, the area itself remains the same. • To build the concept of conservation of area, have students do activities where the area is cut up and rearranged. Tangrams can also be useful for exploring this concept. Have a discussion about how pieces can have the same size (area) but different shape.
8.4 Area of Regular Polygons	6.G.1 Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems.	How can I find the area of a regular polygon?	
8.5 Polygons on the Coordinate Plane	6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. Also Addresses: 6.G.1 Through composition into rectangles or decomposition into triangles, find the area of right	What is coordinate geometry? How can I draw a polygon?	• The coordinate plane can help students see movement and change in both geometry and algebra. Given the coordinates for vertices, students will draw polygons in the coordinate plane. They will determine the lengths of vertical and/or horizontal sides of the polygon by subtraction or counting. Students should discover that for a vertical line, the x-coordinates are the same and for a horizontal line the y-coordinates are the same. Include shapes whose bases are not

	triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems.		parallel to the x-axis. Some activities that can be done on the coordinate plane are as follows: ○ Give students coordinates of the vertices of a shape and have students make inferences about the shape before plotting it. ○ Describe a polygon by giving all the coordinates of the vertices except one. ○ Given the coordinates of a shape, give new coordinates of a shape that is identical in shape. ○ Create a shape given a vertex and dimensions. How many other identical shapes can you create with the same criteria?
Module 9: Volume and Surface Area 🔗			**3 weeks**
Lesson	**Standards/Learning Targets**	**Essential Questions**	**Strategies/Activities**
9.1 Volume of Rectangular Prisms	6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = \ell \cdot w \cdot h$ and $V = B \cdot h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. Also Addresses: 6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	What is volume (V)? How can I measure volume? What is a rectangular prism? How can I measure the volume of a rectangular prism? How can volume be applied in real world situations?	● Volume is the three-dimensional space inside a solid. It can be thought of as "filling" a solid. Focus on what volume means conceptually and move students away from the formulaic definition length × width × height which only holds true for rectangular prisms. Instead have the students think about layers, so they discover the formula B × h which is more inclusive. ● In order for a student to understand the location of a unit cube in an array of a prism, they must "see" the cube in a three-dimensional coordinate system consisting of rows and columns and layers. (Layers may be vertical or horizontal.) ● It is a difficult concept for students to extend this visualization to fractional edge lengths. Before moving towards fractional edge lengths, it may be helpful to have students calculate volume with units other than the unit cube. ● See the Ohio Model Curriculum for 6th grade,pages 191-193, for ideas on handling fractional edge lengths.

	9.2 Surface Area of Regular Prisms 9.3 Surface Area of Triangular Prisms 9.4 Surface Area of Pyramids	6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	What is a net? What is a face? What is a vertex? What are the attributes used to identify prisms, cones, cylinders, and pyramids? What is surface area? How can I find the surface area of a solid? How can surface area be applied in real world situations?	• Introduce surface area as the idea of "wrapping." Have students build prisms using unit cubes. Then have them count the number of squares on each face to connect the idea of area to surface area. Remind them that they also must count the bottom of the figure. • Exploring possible nets could be done by taking apart (unfolding) three-dimensional objects such as Kleenex boxes. This process is foundational for the study of surface area of prisms. Have students cut apart the faces and rearrange them to illustrate that there are many different nets for the same object. • Both the composition and decomposition of rectangular prisms should be explored. Understanding that there are multiple nets for the same object may be difficult for some to visualize; provide concrete examples of nets for the object.

ODE Model Curriculum

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is a model curriculum that supports the standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards. The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, possible connections between topics, and some common misconceptions.

Mathematics Grade 6 Model Curriculum with Instructional Supports

Curriculum and Instruction Guide

Module 1: Ratios and Rates

Unpacked Standards / Clear Learning Targets		
Learning Target - RATIO AND PROPORTIONAL RELATIONSHIPS Understand ratio concepts and use ratio reasoning to solve problems. **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes." **6.RP.2** Understand the concept of a unit rate a /b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3 /4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements; find missing values in the tables; and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	**Essential Understanding** • Understand the difference between part to part and part to whole and how those relationships are represented as ratios. • Understand a ratio relationship is a multiplicative comparison of two quantities in which both quantities change by the same factor. • Understand a rate is a set of infinitely many equivalent ratios. • Understand reasoning with ratios involves attending to and coordinating two quantities • Understand forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest. • Ratios can be written in three ways: 2 to 3, 2:3, 2/3. • A rate is a ratio that compares two different units. • A unit rate is a ratio that compares two different units where one of the measurements is one. • I can use graphs of rate or ratio to show patterns. • I can use graphs to convert from one unit to another in either the US Customary or metric system.	**Academic Vocabulary** Double number line Equivalent ratio Part-to-part ratio Part-to-whole ratio Rate Ratio Ratio table Scaling Unit price Unit rate Unit ratio Coordinate plane Double number line Equivalent ratio Ordered pair Origin Scaling X-axis Y-axis

COLUMBUS CITY SCHOOLS

Math 6

I Can Statements

- I can give examples of ratios as fractions and use ratios to compare quantities.
- I can give examples of rates and write rates as unit rates.
- I can use tables to solve problems involving ratios and rates.
- I can use graphs to represent problems involving ratios and rates.
- I can find equivalent ratios and rates by using unit rates and equivalent fractions.
- I can solve real world problems using ratios and rates.

Priority Standards:

Proficient:

- Write a ratio to describe a relationship between two quantities
- Use models to solve simple problems involving ratios
- Complete simple ratio tables including finding missing values in tables of equivalent ratios
- Understand the concept of unit rates
- Solve a wide variety of routine problems involving ratios and rates
- Use ratio tables to solve routine real-world problems
- Solve routine mathematical and real-world unit rate problems (including unit pricing and constant speed)
- Convert measurement units within the same system using ratio reasoning

Accomplished (all of Proficient +):

- Select appropriate representations and strategies to solve mathematical and real-world ratio and rate problems
- Solve a wide variety of problems involving ratios and rates
- Apply ratio reasoning to convert measurement units within the same system

Advanced (all of Proficient + all of Accomplished +):

- Select efficient representations and strategies to solve mathematical and real-world ratio and rate problems
- Solve a wide variety of real-world problems involving ratios and rates, including where a ratio is associated with a rate
- Solve a variety of non-routine problems requiring conversion of measurement units

Prior Standard(s)

4.MD.1 Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

5.NF.5 Interpret multiplication as scaling (resizing).

Future Standard(s)

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.

7.RP.2 Recognize and represent proportional relationships between quantities.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.

N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.	

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 6, Number 1, page 36
- Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5
- Ohio's K-8 Learning Progressions, Number and Operations--Fractions, pages 6-7
- Ohio's K-8 Learning Progressions, Ratio and Proportional Relationships, page 15

Instructional Strategies

6.RP.1
Have students work with models to develop their understanding of ratios. (MP.2, MP.6) Help students see that initially they do not express ratios using fraction notation so that ratios can be differentiated from fractions and from rates. Later, students understand that ratios can be expressed in fraction notation, but that ratios are different from fractions in several ways.

Introduce ratios and rates with real-world experiences such as taste.

Journal / Quick Write Prompts: Compare and contrast expressing a relationship between quantities as a ratio, fraction and percent. Create a ratio problem for your classmates using a different context (situation) than the ones you have worked on in class. The most important thing to remember when solving ratio and percent problems is…. Some good test questions for ratio and percent are…

Use tape diagrams (bar model) to model problems where both quantities have the same units.

6.RP.2
As students move from additive reasoning to multiplicative reasoning, draw attention to the common ratio and connect it to the unit rate. This is the foundation for proportional reasoning that will be more solidified in Grade 7 and which extends to slope in Grade 8.

Explain to students that the same or fixed ratio is associated with a quality such as steepness, flavor, or speed remains the same or fixed as the variable changes together.

One method for solving problems involving ratios is by finding the unit rate first. Students ask themselves "How many for one?". They recognize that a relationship exists, and then they calculate the rate so that one of the quantities is one. They can use this strategy to compare two rates, or they can multiply the unit rate by one of the quantities to find a missing value.

6.RP.3
Recognize ratios as multiplicative relationships.

Use models to solve problems involving ratios and unit rates such as ratio tables, tape diagrams, and double number lines.

Identify or create equivalent ratios.

Make tables of equivalent ratios relating quantities with whole number measurements to do the following:
- Find missing values
- Compare ratios
- Plot pairs of values in the first quadrant of the coordinate plane
- Compare ratios
- Develop the concept of proportion without solving proportions explicitly.

Building-Up and Breaking-Down are strategies where students take a ratio and either build it up using addition or take it down using subtraction to get a new equivalent ratio. Although it is a good strategy to introduce ratios, it is not truly proportional reasoning because it primarily uses additive reasoning which does not take into account

Use double number lines to model problems where both quantities have different units. Use the multiplication table to help students find equivalent ratios. Have students scale quantities up or down by using a rate table. Use tables and graphs to point out to students the additive and multiplicative structure of ratios.		the constant ratio between the two quantities. However, it can be an important benchmark in understanding. To move students from additive reasoning to more multiplicative reasoning, ask students for more efficient ways to move across the ratio table stressing multiples. Apply ratio reasoning to convert measurement units within the same system. Solve real-life problems involving measurement units that need to be converted. Students should solve real-life problems involving measurement units that need to be converted. Representing these measurement conversions with tools such as ratio tables, t-charts, double number line diagrams, or tape diagrams/bar models will help students internalize the size relationships between same system measurements.

Sample Assessments and Performance Tasks

Reporting Category:
Ratios and Proportions

Standards:
6.RP.1, 2, and 3

Approximate Portion of Test:
24% - 33%; 13 - 17 points

OST Test Specs:
- Numbers in a ratio will be expressed as whole numbers or decimals.
- Ratios can be expressed as a fraction (1/5), with a colon (1:5), or with words, e.g., per, to, each, for each, for every.
- Numbers in a rate will be expressed as whole numbers or decimals.
- Numbers will be expressed as whole numbers or decimals.
- For 3c, benchmark percents (1%, 5%, 10%, 20%, 25%, 50%, and 100%) will be used.
- For 3c, items may require students to find the part, the whole, and/or the percent in a real-world or mathematical problem.

Instructional Resources

Illustrative Math Tasks for 6.RP

Better Lesson → 6.RP.1, 6.RP.2, 6.RP.3

Other Math Tasks:

Finals Week - Figure out which drink has the strongest caffeine concentration

Which Carrots Should You Buy? - Determine which size buys the better deal

Which Ticket Option is the Best Deal? - Compare ticket packages

Sharing Costs Equitably - Traveling to School - Determine how much each person should pay towards the cost of gasoline

Adopted Resource

Reveal:

Lesson 1-1: Understanding Ratios
Lesson 1-2: Tables of Equivalent Ratios
Lesson 1-3: Graphs of Equivalent Ratios
Lesson 1-4: Compare Ratio Relationships
Lesson 1-5: Solve Ratio Problems
Lesson 1-6: Convert Customary Measurement Units
Lesson 1-7: Understand Rates and Unit Rates
Lesson 1-8: Solve Rate Problems

ALEKS :

Ratios, Proportions, and Measurement

- Ratios and Unit Rates
- Equivalent Fractions
- Ordered Pairs
- U.S. Customary Units of Measurement

Return to Scope and Sequence

Module 2: Fractions, Decimals and Percents

Unpacked Standards / Clear Learning Targets

Learning Target -

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

c. Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means 30/100 times the quantity; solve problems involving finding the whole, given a part and the percent.

Essential Understanding

- Percents are used to compare part-to-whole relationships.
- Percents can be found using different sized wholes.
- Percents are out of a 100.
- A percent is a specific type of ratio, which can be represented as a fraction, with a denominator of 100.
- All percent problems involve a part and a whole (100) measured in some unit and the same part and whole measured in hundredths.
- Benchmark percents can be used to estimate and calculate other percents
- Percent means out of each hundred and represents the parts of a whole that are divided into 100 equal parts.
- A percent is a ratio per 100.
- If I know the percent and the whole, I can find the part of the whole that the percent represents.

Academic Vocabulary

Percent
Benchmark Percent
Part
Whole
Rate per 100
Circle Graph
Equivalent Ratios

I Can Statements

- I can explain that percent is out of 100.
- I can explain how ratio and percent are related.
- I can find a percent of a quantity as a rate per 100 using ratios.
- I can solve problems involving finding the whole given a part and the percent by reasoning about tables or equivalent ratios, tape diagrams, double line diagrams, or equations.
- I can solve problems involving finding the whole given a part and the percent by reasoning about tables or equivalent ratios, tape diagrams, double line diagrams, and equations.
- I can convert units using multiplication and division in multiple ways (proportions, multiplication, division, and moving decimals).
- I can convert units to solve real-world problems using multiplication and division in multiple ways (proportions, multiplication, division, and moving decimals).
- I can convert measurement units using ratio reasoning within customary units.
- I can convert measurement units using ratio reasoning within metric units.
- I can convert measurement units using ratio reasoning between customary and metric units.

:Priority Standards:

Proficient:
- Find the percent of a quantity using 100 grids
- Find a percent of a quantity as a rate per 100 using 100 grids
- Find a percent of a quantity as a rate per 100

Accomplished (all of Proficient +):
- Solve problems involving finding the whole, given a part and the percent

Advanced (all of Proficient + all of Accomplished +):

Prior Standard(s)	Future Standard(s)
6.RP.2 Understand the concept of a unit rate a /b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3 /4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."	**6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q, and x are all nonnegative rational numbers.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 6, Number 1, page 36
- Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5
- Ohio's K-8 Learning Progressions, Number and Operations--Fractions, pages 6-7
- Ohio's K-8 Learning Progressions, Ratio and Proportional Relationships, page 15

Instructional Strategies

6.RP.3c
Represent percents using models, such as 100 grids, tape diagrams, and double number lines.

Use ratio reasoning to relate a percent of a quantity as a rate per 100.

Use language to explain percent: ask students what "cent" actually means (100). What is a century? How many cents in a dollar?

Use examples when the final amount is greater than the original amount.

Explain percents greater than 100% and less than 1% in context.

Use 100 grids to compare percents to fractions.

Compare different ratios by changing to percents.

Use benchmark percents (1%, 5%, 10%, 20%, 25%, 50%, and 100%) to compute other percents of a given whole number both mentally and with a model.

Use bar diagrams, equivalent ratios, double number lines, and ratio tables to find the percent of a number.

Use benchmark percents to estimate a percent problem.

- Round the whole to a convenient number (to the nearest 100?) and round the percent to a convenient benchmark fraction. Then, multiply the rounded whole by the benchmark fraction.

The use of bar diagrams and equivalent ratios is beneficial when estimating the percent of a number.

Sample Assessments and Performance Tasks

Reporting Category:
Ratios and Proportions

Standards:
6.RP.3

Approximate Portion of Test:
24% - 33%; 13 - 17 points

OST Test Specs:

- Numbers in a ratio will be expressed as whole numbers or decimals.
- Ratios can be expressed as a fraction (1/5), with a colon (1:5), or with words, e.g., per, to, each, for each, for every.
- For 3c, benchmark percents (1%, 5%, 10%, 20%, 25%, 50%, and 100%) will be used.
- For 3c, items may require students to find the part, the whole, and/or the percent in a real-world or mathematical problem

Instructional Resources

Better Lesson
Shmoop
Khan Academy Videos
3-Act Math:
Voting Booth
Illustrative Mathematics:
Anna in D.C.
Exam scores
Overlapping Squares
Shirt Sale

Adopted Resource	
Reveal: Lesson 2-1: Understand Percents Lesson 2-2: Percents Greater Than 100% and Less Than 1% Lesson 2-3: Relate Fractions, Decimals, and Percents Lesson 2-4: Find the Percent of a Number (focus on benchmark percents only) Lesson 2-5: Estimate the Percent of a Number Lesson 2-6: Find the Whole	**ALEKS:** Fractions, Decimals, and Percents • Converting Between Fractions and Decimals • Understanding Percents • Percents, Decimals, and Fractions • Percent of a Number • Percent Equations

Return to Scope and Sequence

Module 3: Compute with Multi-Digit Numbers and Fractions

Unpacked Standards / Clear Learning Targets		
Learning Target - **6.NS.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. **6.NS.2** Fluently divide multi-digit numbers using a standard algorithm. **6.NS.3** Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.	**Essential Understanding** • There are two meanings of division: partitive and measurement. • Partitive problems are sharing problems and rate problems. (Rate problems are not always partitive problems.) • Measurement problems are repeated subtraction or equal groups. • There is a relationship between multiplication and division that can be seen using visual models.	**Academic Vocabulary** Denominator Fraction Numerator Quotient Visual model Divisor Dividend Multi-digit Remainder Whole numbers Difference Product Sum

I Can Statements
• I can compute quotients of fractions divided by fractions (including mixed numbers). • I can interpret quotients of fractions. • I can figure out how to solve division problems with fractions in a real-world situation. • I can solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

- I can divide multi-digit numbers using the standard algorithm with speed and accuracy, without any math tools (i.e., calculator, multiplication chart).
- I can fluently add multi-digit decimals using the standard algorithm for each operation with speed and accuracy.
- I can subtract multi-digit decimals using the standard algorithm for each operation with speed and accuracy.
- I can divide multi-digit decimals using the standard algorithm for each operation with speed and accuracy.
- I can multiply multi-digit decimals using the standard algorithm for each operation with speed and accuracy.

:Priority Standards:

Proficient:
- Divide simple multi-digit whole numbers
- Divide multi-digit whole numbers
- Recognize a visual model for division of a fraction by a fraction
- Interpret a visual model for division of a fraction by a fraction
- Divide a fraction by a fraction using a visual model
- Add, subtract, and multiply multi-digit whole numbers and decimals to hundredths using strategies and algorithms
- Add and subtract multi-digit decimal numbers
- Divide multi-digit decimals by whole number divisors
- Add, subtract, multiply, and divide multi-digit decimals

Accomplished (all of Proficient +):
- Divide a fraction by a fraction using visual models and equations
- Add, subtract, multiply, and divide multi-digit decimals to solve real-world problems

Advanced (all of Proficient + all of Accomplished):
- Divide a fraction by a fraction

Prior Standard(s)

3.OA.6 Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. In general, students able to multiply fractions can develop strategies to divide fractions, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade.

5.NBT.5 Fluently multiply multi-digit whole numbers using a standard algorithm.

5.NBT.6 Find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties

Future Standard(s)

6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q, and x are all nonnegative rational numbers.

7.SP.3 Describe and analyze distributions.

7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NBT.7** Solve real-world problems by adding, subtracting, multiplying, and dividing decimals using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, or multiplication and division; relate the strategy to a written method and explain the reasoning used.	

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 6, Number 2, pages 37-38
- Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5
- Ohio's K-8 Learning Progressions, Number and Operations--Fractions, pages 6-7
- Ohio's K-8 Learning Progressions, The Number System, pages 16-17

Instructional Strategies

6.NS.1
The use of models is highly encouraged to give students a visual understanding of fraction division.

- A variety of models are available to students. It is important for students to be exposed to many different types of models, and they should be allowed to use the tool that best models the problem and makes sense to them. It may be helpful for students to use manipulatives such as snap cubes or fraction bars to begin exploring fraction division. Using grid paper for modeling or folding strips could also be helpful for students to make the mathematics visible to them.

Methods for Dividing Fractions

- Dividing across fractions
- Common Denominator Method
- Dividing Fractions by Multiplying the Inverse

Please see the Ohio Model Curriculum, pages 51-78, for more details on strategies for teaching fraction division.

6.NS.2
It may be beneficial to expose students to multiple standard algorithms and have them try each. After exposure to several algorithms, students can choose which algorithm makes sense to them. Use class discussion to help students create understanding explaining why their preferred algorithm works.

- Partial Quotients
- Area Model
- Explicit-Trade Method
- Traditional Division

Check answers with multiplication.

Grid paper helps students organize their computational work. It is highly encouraged that all students have access to grid paper.

6.NS.3
Make sure to intentionally connect decimal notation and computation with fraction notation and computation.

When adding and subtracting decimals continue to emphasize place value.

For students struggling with addition and subtracting with decimals, connect the algorithm with models such as base-ten blocks, decimal squares, or number lines. Models help reinforce why it is important to line up the place value positions before adding and subtracting.

One strategy for solving equations with decimals is to clear the equation of decimals by creating an equivalent equation without decimals. For example, $5 - 0.17 = x$ is equivalent to $500 - 17 = 100x$ by multiplying each side of the equation by 100.
Different Algorithms:

- Multiplication

		○ Area Model ○ Partial Products ○ Lattice Algorithm ○ Traditional Multiplication ● Division ○ Partial Quotients ○ Explicit-Trade ○ Traditional Division

Sample Assessments and Performance Tasks

Reporting Category:
The Number System
Standards:
6.NS.1, 2, and 3
Approximate Portion of Test:
20% - 25%; 11 - 13 points

OST Test Specs:
●Either the divisor or the dividend must be a non-unit fraction.
●The focus will be on interpreting and using visual models.
●Language such as "reduce", "simplify", or "lowest terms" will not be used
●For items with a context, division is limited to 2 digits by 1, 3 digits by 1, 3 digits by 2, 4 digits by 2, 5 digits by 2, or 3 digits by 3.
●For items without a context, division is limited to 2 digits by 1, 3 digits by 1, 3 digits by 2, 4 digits by 2, 5 digits by 2.
●Items may require the use of the order of operations
●Students will never have a calculator for items aligned to this standard (6.NS.2 and 3).

Instructional Resources

6.NS.1:
Better Lesson
Shmoop
Khan Academy Videos
Dan Meyer Activity
Nana's Lemonade
Illustrative Mathematics
Baking Cookies
Cup of Rice
Dan's Division Strategy
Drinking Juice, Variation 2
Drinking Juice, Variation 3
How many x_are in. . . ?

6.NS.2:
Better Lesson
Shmoop
Khan Academy Videos
Illustrative Mathematics
Interpreting a Division Computation
How Many Staples?
Batting Average

6.NS.3:
Better Lesson
Shmoop
Khan Academy Videos
Illustrative Mathematics
12 Rectangular Units
2 Units Wide and 3 Units Long
Adding Base Ten Numbers, Part 1
Adding Base Ten Numbers, Part 2
Adding Base Ten Numbers, Part 3
Buying Gas
Changing Currency
Gifts from Grandma, Variation 3

How Many Containers in One Cup / Cups in One Container? Making Hot Cocoa, Variation 1 Making Hot Cocoa, Variation 2 Running to School, Variation 2 Running to School, Variation 3 Standing in Line Traffic Jam Video Game Credits	Jayden's Snacks Movie tickets Pennies to heaven Reasoning about Multiplication and Division and Place Value, Part 1 Reasoning about Multiplication and Division and Place Value, Part 2 Setting Goals Tenths of Tenths and Hundredths of Hundredths What is the Best Way to Divide?

Adopted Resource	
Reveal: Lesson 3-1: Divide Multi-Digit Whole Numbers Lesson 3-2: compute with Multi-Digit Decimals Lesson 3-3: Divide Whole Numbers by Fractions Lesson 3-4: Divide Fractions by Fractions Lesson 3-5: Divide with Whole and Mixed Numbers	**ALEKS:** Whole Numbers, Decimals, Fractions: • Whole Numbers: Division • Decimals: Addition, Subtraction, Multiplication, Division Place Value and Ordering • Fractions: Multiplication with Fractions Division with Fractions

Return to Scope and Sequence

Module 4: Integers, Rational Numbers, and the Coordinate Plane

Unpacked Standards / Clear Learning Targets		
Learning Target **6.NS.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6.NS.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.	**Essential Understanding** Integers are whole numbers, their opposites and zero. Negative numbers are numbers less than zero. I can find the opposite of a number by using a number line to determine the distance from zero to the original number and then from zero on the opposite side to the same number. The opposite of a positive number is its negative, and the opposite of a negative number is its positive. Some key words for recognizing positive integers are: gain, increase, rise, above, more and up. Some key words for negative integers are loss, decrease, drop, below, less and down.	**Academic Vocabulary** Elevation Negative number Opposite (as in direction) Positive number Quantities Sea level Coordinate plane Ordered pair Rational number Plane Points

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., − (−3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **6.NS.7** Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. d. Distinguish comparisons of absolute value from statements about order. **6.NS.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	A rational number can be expressed as a fraction and has an exact location on a number line. A system of two number lines is called a coordinate plane: the horizontal line is called the x-axis and the vertical line is called the y-axis. The 2 axes intersect at the origin, written as the point (0, 0). A point on the coordinate plane can be described by its distance along both number lines. An ordered pair (x, y) is used to locate that point. When the axes are extended in both directions, they divide the coordinate plane into 4 parts, called quadrants. I can use coordinate geometry to determine the distance between two points. Absolute value is the distance of a number from zero. I can use absolute value to determine the distance between two points.	Quadrant Absolute value Magnitude Ordering Rational number Distance

I Can Statements

- I can identify an integer and its opposite.
- I can use integers to represent quantities in real-world situations (above/below sea level, etc.).
- I can explain where zero fits into a situation represented by integers.
- I can identify a rational number as a point on a number line.
- I can identify a rational number as a point in the coordinate plane.
- I can place numbers on a horizontal number line.
- I can understand quadrants of a coordinate plane.
- I can understand that 0 is its own opposite.
- I can interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
- I can write, interpret, and explain statements of order for rational numbers in real-world contexts.

- I can identify the absolute value of rational numbers.
- I can interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
- I can distinguish comparisons of absolute value from statements about order and apply to real world contexts.
- I can calculate absolute value.
- I can graph points in all four quadrants of the coordinate plane.
- I can solve real-world problems by graphing points in all four quadrants of a coordinate plane.
- I can calculate the distances between two points with the same first coordinate or the same second coordinate using absolute value, given only coordinates.

:Priority Standards:

Proficient:
- Can identify or locate a positive or negative whole number on a number line
- Find and position positive and negative rational numbers on a horizontal or vertical number line and on a coordinate plane
- Use positive and negative numbers to represent quantities in real world contexts
- Use number lines to compare and order positive and negative numbers
- Represent real-world quantities with positive and negative numbers
- Plot pairs of positive values on the coordinate plane
- Locate points in all four quadrants of the coordinate plane
- Locate points and ordered pairs in all four quadrants of the coordinate plane

Accomplished (all of Proficient +):
- Use number lines to compare and order positive and negative numbers
- Represent real-world quantities with positive and negative numbers
- Locate points and ordered pairs in all four quadrants of the coordinate plane (**understand the signs in ordered pairs**)

Advanced (all of Proficient + all of Accomplished +):
- Represent real-world quantities with positive and negative numbers
- Graph ordered pairs in all four quadrants of the coordinate plane
- Write, interpret, and explain statements of order in real-world contexts

Prior Standard(s)

3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.
5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far

Future Standard(s)

6.EE.8 Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or

to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond, e.g., x-axis and x-coordinate, y-axis and y-coordinate. **5.G.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	vertical number line diagram. **8.SP.1** Construct and interpret scatter plots for bivariateG measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering; outliers; positive, negative, or no association; and linear association and nonlinear association.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 6, Number 2, pages 37-38
- Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5
- Ohio's K-8 Learning Progressions, Number and Operations--Fractions, pages 6-7
- Ohio's K-8 Learning Progressions, The Number System, pages 16-17

Instructional Strategies

6.NS.5 The negative sign can mean several things: ○ A sign attached to a number to form negative numbers; ○ A subtraction sign; or ○ An indication to take the opposite of.	**6.NS.6** This is the first time that students see the number line extending in both directions. Since a number line shows both quantity and direction, there are now two different numbers that have the same quantity such as 4 and −4. Therefore, students shift from using line segments to show value on the number line to using arrows which show both quantity (magnitude) and direction. Make sure students are exposed to both horizontal and vertical number lines. Scales are not limited to 1.
6.NS.7 Students should learn that the absolute value of a number does not take into account sign or direction; it is only a measure of distance (magnitude) from 0. Discourage students from saying that the "answer is always positive or 0" since that will lead to misconceptions when students encounter problems such as $\|4x-2\| = 18$ in high school. Continue to use number lines to give students a visual for ordering integers. ○ Please contact math curriculum leaders if you are interested in an classroom empty number line and middle school number cards.	**6.NS.8** Students are seeing all 4 quadrants for the first time in 6th grade. It might be helpful to label the ends of each axis with their respective + or −. Scales should not be limited to 1. Students should realize that when two ordered pairs differ only by signs, they are reflections across one or both of the axes. Distance on the coordinate plane is limited to horizontal and vertical distances only. When counting distance on the coordinate plane, some students will count the starting point as "1." Encourage them to count the boxes between points instead.

Sample Assessments and Performance Tasks

Reporting Category:
The Number System
Standards:
6.NS.5, 6, 7, and 8
Approximate Portion of Test:
20% - 25%; 11 - 13 points

OST Test Specs:
- Items may use all types of rational numbers.
- Items will not require the student to perform an operation.
- Items may use all types of rational numbers.
- Plotting points in the coordinate plane will include negative values (not jusst first quadrant).
- Students should not be expected to know the quadrant names (I, II, III, IV).
- Items can refer to the quadrant names but should include a diagram which labels the quadrants by number.
- Items will use positive and negative rational numbers.
- Items will only use strict inequalities (not $x \leq c$ or $x \geq c$).
- In items involving finding the distance between two points, either the first coordinates or the second coordinates will be the same.
- Items will not involve multiplication or division of negative numbers.
- Axes can be numbered with scales other than 1.
- In items involving finding distances between two points, either the first coordinates or the second coordinates will be the same.

Instructional Resources

6.NS.5:
Better Lesson
Shmoop
Khan Academy Videos
Illustrative Mathematics
It's Warmer in Miami
Mile High

6.NS.6:
Better Lesson
Shmoop
Khan Academy Videos
Illustrative Mathematics
Extending the Number Line
Locations in the Coordinate Plane
Integers on the Number Line 2
Reflecting points over coordinate axes
Plotting points in the coordinate plane

6.NS.7:
Better Lesson
Shmoop
Khan Academy Videos
Illustrative Mathematics
Above and Below Sea Level
Jumping Flea
Fractions on the Number Line
Integers on the Number Line 1
Comparing Temperatures

6.NS.8:
Better Lesson
Shmoop
Khan Academy Videos
Illustrative Mathematics
Distance Between Points
Nome, Alaska

Adopted Resource

Reveal:
Lesson 4-1: Represent Integers
Lesson 4-2: Opposites and Absolute Value

ALEKS:
Integers & Rational Numbers, Whole Numbers, Fractions, Graphs & Functions:
- Integers & Rational Numbers: Plotting and Comparing Signed Numbers

Lesson 4-3: Compare and Order Integers Lesson 4-4: Rational Numbers Lesson 4-5: The Coordinate Plane Lesson 4-6: Graph Reflections of Points Lesson 4-7: Absolute Value and Distance	• Whole Numbers: Ordering and Estimation • Fractions: Plotting and Ordering Fractions • Graphs and Functions: Ordered Pairs

Return to Scope and Sequence

Module 5: Numerical and Algebraic Expressions

Unpacked Standards / Clear Learning Targets

Learning Target	Essential Understanding	Academic Vocabulary
6.EE.1 Write and evaluate numerical expressions involving whole number exponents. **6.EE.2** Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 − y. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order. **6.EE.3** Apply the properties of operations to generate equivalent expressions. **6.EE.4** Identify when two expressions are equivalent, i.e., when the two expressions name the same number regardless of which value is substituted into them. **6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.NS.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to	A variable is a letter that represents an unknown number: when the value of one variable depends on the value of another, it is called a dependent variable; when the value of one variable does not depend on the value of the other, it is called an independent variable. A table can show the relationship between a dependent and independent variable. The number multiplied by the variable is called the coefficient. A term is a number or a variable; like terms have the same variable raised to the same exponent. The factor of a whole number is any whole number that divides the first number evenly. The factors of a number are less than or equal to the number. A number that is a factor of two or more numbers is called the common factor of those numbers. The greater common factor (GCF) is the greatest number that is a common factor. I need to identify the operations by their key words in order to translate a word expression into a mathematical expression. I can evaluate an expression with a variable or symbols by substituting the given number for the variable or symbol; then following the order of operations. Two expressions are equivalent when they look different but represent the same information. I can use the distributive, commutative and associative properties to find out if two expressions are equivalent.	Base Exponent Numeric expression Power Factor Distributive property Equivalent Associative Identity Substitution Constant Set Unknown Variable Difference Dividend Divisor Factor Product Quotient Sum

express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor.		

I Can Statements

- I can write expressions involving whole number exponents.
- I can evaluate expressions involving whole number exponents.
- I can solve order of operation problems that include exponents.
- I can write expressions that record operations with numbers and with letters standing for numbers.
- I can identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient)
- I can view one or more parts of an expression as a single entity.
- I can evaluate expressions at specific values of their variables.
- I can perform arithmetic operations, including those involving whole-number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order.
- I can apply the properties of operations to create equivalent expressions.
- I can simplify expressions using the different properties.
- I can recognize when two expressions are equivalent to each other.
- I can prove (using various strategies) that two expressions are equivalent no matter what number is substituted.
- I can use numbers and variables to evaluate expressions.
- I can recognize that a variable can represent an unknown number, or, depending on the scenario/situation, any number in a specific set.
- I can relate variables to a context.
- I can write expressions when solving a real-world or mathematical problem.
- I can identify the factors of two whole numbers less than or equal to 100 and determine the Greatest Common Multiple.
- I can identify the multiples of two whole numbers less than or equal to 12 and determine the Least Common Multiple.
- I can apply the Distributive Property to rewrite addition problems by factoring out the Greatest Common Factor.

:Priority Standards:

Proficient:
- Evaluate numerical expressions with two operations
- Write and evaluate numerical expressions with up to two operations including those with exponents of 2 and 3
- Write and evaluate numerical expressions including those with whole number exponents
- Understand the use of variables in simple mathematical expressions

Accomplished (all of Proficient +):
- Write expressions and equations that correspond to given situations
- Evaluate complex algebraic expressions including exponents
- Apply the properties of operations to write equivalent expressions
- Use least common multiples and greatest common factors to solve routine real-world problems

Advanced (all of Proficient + all of Accomplished +):
- Write expressions for complex mathematical and real-world situations
- Explain why two expressions are equivalent using precise mathematical language
- Efficiently use least common multiples and greatest common factors to solve real-world problems

- Identify expressions that correspond to given routine situations
- Identify one-and two-step expressions that correspond to given familiar situations
- Write expressions that correspond to given routine situations
- Evaluate one-step expressions
- Evaluate algebraic expressions with up to two operations
- Evaluate algebraic expressions
- Apply the understanding of equivalent expressions to identify equivalent expressions
- Identify up to two-step equivalent expressions
- Find common factors of two numbers less than or equal to 100
- Find common multiples of two number less than or equal to 100
- Find the greatest common factor of two numbers less than or equal to 100
- Find the least common multiple of two whole numbers less than or equal to 12

Prior Standard(s)	Future Standard(s)
1.OA.3 Apply properties of operations as strategies to add and subtract. **3.OA.5** Apply properties of operations as strategies to multiply and divide. **4.OA.4** Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite. **5.OA.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them **5.OA.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of	**7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **8.EE.1** Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions. **A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. **5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10.	

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 6, Number 3, pages 39-40
- Ohio's K-8 Learning Progressions, Operations and Algebraic Thinking, pages 8-10
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

Instructional Strategies

6.EE.1
Relate exponents to what students already know about powers of 10.

Make sure students understand that exponents are a notation for repeated multiplication, just as multiplication is notation for repeated addition.

The standard is about evaluating expressions. This section/lesson gives students an understanding of the rules for exponents so they can apply them in the next section/lesson.

The use of negatives and positives should mirror the level of introduction in Grade 6; students are developing the concept and not generalizing operation rules.

The use of technology can assist in the exploration of the meaning of expressions. Many calculators will allow one to store a value for a variable and then use the variable in expressions. This enables the student to discover how the calculator deals with expressions like x^2 , 5x, xy, and 2(x + 5).

6.EE.2
Using the mnemonic PEMDAS or "Please Excuse My Dear Aunt Sally" causes many misconceptions and is highly discouraged. Students get the impression that multiplication must come before division, addition must come before subtraction, operations must be performed left to right, and that calculations in parenthesis must come first. Instead people may solve 5 – 9 + 3 – 5 + 9 by changing the order using the Associative and Additive Inverse properties (5 – 5) + (–9 + 9) + 3 instead of adding and subtracting from left to right.

Include whole-number exponents, fractions, and decimals when writing expressions.

Encourage students to show step-by-step thinking when rewriting an expression with the least number of terms.

Provide opportunities for students to share their work and explain their thinking.

6.EE.3
Students often get confused when they see a variable such as x without a coefficient. Make a connection to the Multiplicative Identity Property and have students write the "invisible" 1 in front of the variables without coefficient. - This will also help students correct the belief that if there is no coefficient, then x = 1.

Have students write out the distribution: 2(x + 3) = 2(x) + 2(3) = 2x + 6; this may reduce errors when students forget to distribute to the second term.

The emphasis should not be placed on counting terms in an expression. Provide a variety of expressions and problem situations for students to practice and deepen their skills. They should understand that when evaluating an expression through substitution, like variables will have the same value.		
6.EE.4 Provide opportunities for students to write equivalent expressions, both numerically and with variables. Use manipulatives to model expressions (algebra tiles, counters, unifix cubes, etc). This can help students translate between concrete numerical expressions and abstract symbolic representations.	**6.EE.6** It is not advised to used mnemonic variables for expressions. For example, if you were buying bananas for $0.49 a pound, using 0.49b might be interpreted as $0.49 per banana.	**6.NS.4** Show how the Greatest Common Factor (GCF) is useful in expressing the numbers using the Distributive Property [(36 + 24) = 12(3 + 2), where 12 is the GCF of 36 and 24].

Sample Assessments and Performance Tasks

Reporting Category: Expressions and Equations; The Number System **Standards:** 6.EE.1, 2, 3, 4, and 6; 6.NS.4 **Approximate Portion of Test:** Expressions and Equations: 31% - 44%; 17-23 points The Number System: 20% - 25%; 11 - 13 points	**OST Tech Specs:** ●Positive rational number bases will be used. ●Only whole number exponents will be used. ●Expressions must contain at least one exponent. ●Items will not require the student to calculate unknown bases of exponents. ●Items may require the use of the order of operations. ●Items will use positive rational numbers. ●In 2a, real-world problems will be avoided. ●In 2c, items may require the use of the order of operations. ●Multiplication may be represented by a raised dot, parentheses, or a coefficient and a variable. ●Division may be represented by a fraction bar or a division sign ●Items will use nonnegative rational numbers. ●Expressions may include exponents. ●Expressions must contain variables. ●Items may require students to recognize the formal names of properties. ●Property names will be used sparingly. ●Items will use nonnegative rational numbers. ●Expressions may include exponents. ●Expressions must contain variables. ●Items may require the use of the order of operations

	●Items will use nonnegative rational numbers. ●Expressions must contain variables. ●Items will not require students to evaluate expression ●When finding the greatest common factor, whole numbers less than or equal to 100 will be used. ●When finding the least common multiple, whole numbers less than or equal to 12 will be used. ●Students may need to know the name of the distributive property. ●Students need to recognize the formal names of properties.

Instructional Resources

6.EE.1: Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Exponent Experimentation 1 Exponent Experimentation 2 Exponent Experimentation 3 Seven to the What?!? Sierpinski's Carpet The Djinni's Offer	6.EE.2: Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Distance to School Rectangle Perimeter 1 Families of Triangles	6.EE.3: Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Anna in D.C.
6.EE.4: Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Rectangle Perimeter 2 Equivalent Expressions	6.EE.6 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Firefighter Allocation Pennies to Heaven:	6.NS.4: Better Lesson Shmoop Khan Academy Videos Dan Meyer Activity Shipping Routes Illustrative Mathematics Adding Multiples Bake Sale Factors and Common Factors Multiples and Common Multiples The Florist Shop

Adopted Resource	
Reveal: Lesson 5-1: Powers and Exponents Lesson 5-2: Numerical Expressions Lesson 5-3: Write Algebraic Expressions Lesson 5-4: Evaluate Algebraic Expressions Lesson 5-5: Factors and Multiples Lesson 5-6: Use the Distributive Property Lesson 5-7: Equivalent Algebraic Expressions	**ALEKS:** Whole Numbers; Integers and Rational Numbers • Whole Numbers: Exponents and Order of Operations Evaluating and Writing Expressions Prime Numbers, Factors, and Multiples • Integers and Rational Numbers: The Distributive Property Simplifying Algebraic Expressions

Return to Scope and Sequence

Module 6: Equations and Inequalities

Unpacked Standards / Clear Learning Targets		
Learning Target **6.EE.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q, and x are all nonnegative rational numbers. **6.EE.8** Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	**Essential Understanding** I can solve an equation by isolating the variable on one side of the equal sign by applying inverse operations. The solution can be checked by substituting it into the original equation to see that it makes a true statement. Not every equation has a solution. An inequality is a mathematical sentence that compares two expressions; the symbols <, ≤, >and ≥ are used. To solve an inequality, I can follow the same rules for solving equations.	**Academic Vocabulary** Equation Inequality Set Substitution True (equation/inequality) Constant Expression Unknown Variable Inverse operations Nonnegative Operation Rational number Subtraction Property of Equality Constraint Greater than Infinite solutions Less than Line diagrams One-variable equations

COLUMBUS CITY SCHOOLS

Math 6

I Can Statements

- I can recognize solving an equation or inequality as a process of answering "which values from a specified set, if any, make the equation or inequality true?".
- I can use the solution to an equation or inequality to prove that the answer is correct.
- I can use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- I can recognize that a variable can represent an unknown number, or, depending on the scenario/situation, any number in a specific set.
- I can relate variables to a context.
- I can write expressions when solving a real-world or mathematical problem.
- I can define an inverse operation.
- I can use inverse operations to solve one-step variable equations.
- I can apply rules of the form x + p = q and px = q, for cases in which p, q and x are all nonnegative rational numbers, to solve real world and mathematical problems (there is only one unknown quantity).
- I can develop a rule for solving one-step equations using inverse operations with nonnegative rational coefficients.
- I can solve and write equations for real-world mathematical problems containing one unknown.
- I can identify the constraint or condition in a real-world or mathematical problem in order to set up an inequality.
- I can recognize that inequalities of the form x>c or x<c have infinitely many solutions.
- I can write an inequality of the form x>c or x<c to represent a constraint or condition in a real-world or mathematical problem.
- I can represent solutions to inequalities or the form x>c or x<c, with infinitely many solutions, on the number line diagrams.

:Priority Standards:

Proficient:

- Identify expressions and equations that correspond to given routine situations
- Solve simple one-step equations involving addition and subtraction
- Use substitution to determine whether a given number makes an equation true
- Identify one-and two-step expressions and equations that correspond to given **familiar** situations
- Solve one-step equations with positive integer coefficients
- Write equations that correspond to given **routine** situations
- Write and solve one-step equations with positive integer coefficients

Accomplished (all of Proficient +):

- Write equations that correspond to given situations
- Write and solve equations with positive rational coefficients
- Given a situation, write an inequality and graph solutions on a number line

Advanced (all of Proficient + all of Accomplished +):

- Write equations for complex mathematical and real-world situations
- Given a complex situation, write an inequality and graph solutions on a number line

- Write and graph solutions to inequalities on a number line
- Determine the solution set for a given inequality
- Use substitution to determine whether a given number in a specified set makes an inequality true

Prior Standard(s)	Future Standard(s)
5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. **5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **6.NS.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction modelsG and equations to represent the problem. **6.NS.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **6.NS.7** Understand ordering and absolute value of rational numbers. **6.EE.2** Write, read, and evaluate expressions in which letters stand for numbers.	**7.EE.4** 4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **8.EE.8** Analyze and solve pairs of simultaneous linear equations graphically. **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. **A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 6, Number 3, pages 39-40
- Ohio's K-8 Learning Progressions, Operations and Algebraic Thinking, pages 8-10
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

Instructional Strategies

6.EE.5-8

The skill of solving an equation must be developed conceptually before it is developed procedurally. This means that students should be thinking about what numbers could possibly be a solution to the equation before solving the equation.

Although solving equations in this cluster is limited to one-step equations and inequalities, students should be given more complicated linear equations when using substitution to determine whether numbers in a set make the equation true.

A variety of concrete models such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable. Students should move from concrete models, to pictures, and then equations.

The focus in Grade 6 should be on inverse operations, so the discussion around the models should always go back to inverse operations.

Students need practice with one-step equations with fractions using manipulatives and diagrams instead of just using inverse operations in order to give the opportunity to create understanding. This will help alleviate the misuse of fraction rules in later grades/courses. Numbers that are easily modeled should be used.

The process of translating between mathematical phrases and symbolic notation will also assist students in the writing of equations or inequalities for a situation. This process should go both ways; students should also be able to write a mathematical phrase for an equation. A strategy for assisting with this is to give students an equation and ask them to come up with the situation or story that the equation could be modeling.

Provide multiple situations in which students must determine if a single value is required as a solution, or if the situation allows for multiple solutions.

Have students write inequalities to represent real-world problems. Writing out inequalities in words is a strategy for assisting students' conceptual understanding.

When graphing inequalities on a number line, do not tell students the arrow points in the direction of the inequality sign because this creates a misconception that is difficult to combat in later grades. Instead ask the student when graphing, which numbers make the inequality true.

Sample Assessments and Performance Tasks

Reporting Category: Expressions and Equations **Standards:** 6.EE. 5, 6, 7, and 8 **Approximate Portion of Test:** 31% - 44%; 17-23 points	OST Test Specs: ●Items will use nonnegative rational numbers. ●Only one-variable linear equations and inequalities will be used. ●An equation or inequality will be given. ●Items will not require solving an equation or inequality. ●Items will only use strict inequalities (not $x \leq c$ or $x \geq c$). ●Items will use nonnegative rational numbers. ●Expressions must contain variables. ●Items will not require students to evaluate expressions ●Items will use nonnegative rational numbers.

	●Items will use one-step equations of the form x + p = q, x – p = q, or px =q, where p, x, and q are nonnegative rational numbers using models or algebraically. ●Items will use one-step equations of the form $xx\ pp = qq$ where x and q are nonnegative rational numbers and p is a positive integer using models and algebraically. ●Items will use nonnegative rational numbers. ●Items will only use strict inequalities (not x ≤ c or x ≥ c).

Instructional Resources

6.EE.5:	6.EE.6:	6.EE.7:	6.EE.8:
Better Lesson	Better Lesson	Better Lesson	Better Lesson
Shmoop	Shmoop	Shmoop	Shmoop
Khan Academy Videos	Khan Academy Videos	Khan Academy Videos	Khan Academy Videos
Illustrative Mathematics	Illustrative Mathematics	Illustrative Mathematics	Illustrative Mathematics
Exponent Experimentation 3	Firefighter Allocation	Fruit Salad	Fighting Adventures 1
Log Ride	Pennies to Heaven	Morning Walk	Height Requirements
Make Use of Structure			

Adopted Resource

Reveal:

Lesson 6-1: Use Substitution to Solve One-Step Equations
Lesson 6-2: One-Step Addition Equations
Lesson 6-3: One-Step Subtraction Equations
Lesson 6-4: One Step Multiplication Equations
Lesson 6-5: One-Step Division Equations
Lesson 6-6: Inequalities

ALEKS:

Whole Numbers; Equations and Inequalities:

- Whole Numbers: Introduction to One-Step Equations
- Equations and Inequalities: One-Step Equations
  Applications of Equations
  Writing and Graphing Inequalities

Return to Scope and Sequence

COLUMBUS CITY SCHOOLS

Math 6

Module 7: Relationships Between Two Variables

Unpacked Standards / Clear Learning Targets

Learning Target	Essential Understanding	Academic Vocabulary
6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	• Expressions on both sides of the equal sign have the same value. • The value of the dependent variable is determined by the value of the independent variable. • The relationship between two quantities can be represented as a table, graph, and/or equation.	dependent variable independent variable nonnegative quantity relationship representation variable

I Can Statements

- I can define independent and dependent variables.
- I can use variables to represent two quantities in a real-world problem that change in relationship to one another.
- I can write an equation to express one quantity (dependent) in terms of the other quantity(independent).
- I can analyze the relationship between the dependent variable and independent variable using tables and graphs.
- I can relate the data in a graph and table to the corresponding equation.

:Priority Standards:

Proficient:
- Write a one-variable equation to express one quantity in terms of the other quantity
- Use variables to represent two quantities in a real-world problem that change in relationship to one another

Accomplished (all Proficient +):
- Use tables and graphs to analyze the relationship between dependent and independent variables and relate these to the equation

Advanced (all of Proficient + all of Accomplished +):
- Analyze the relationship between dependent and independent variables represented in tables and graphs, and then write an appropriate equation

Prior Standard(s)	Future Standard(s)
5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.	**7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. **A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 6, Number 3, pages 39-40
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

Instructional Strategies

6.EE.9

Provide multiple situations for the student to analyze and determine what unknown is dependent on the other components.

It is important for students to identify the two quantities that are being compared and then choose variables to represent the quantities and finally define what each variable means in the context of the problem.

Multiplying unknowns and variables by fractions is essential for solving equations and understanding functions. According to research using fractions as multipliers on unknown quantities is a significant challenge for students. In order to be successful in later mathematics, students need to develop reciprocal reasoning. For example, if $y = \frac{3}{5}x$ then $x = \frac{5}{3}y$.

Start with whole numbers and their inverses, then move oro equations using inverses that are reciprocals to rational numbers beyond whole numbers.

Students should use multiple representations to explore how one variable changes in relation to the other. They need to be able to translate freely among the story, words (mathematical phrases), models, tables, graphs, and equations. In addition students need to be flexible enough to start with any of the representations and use the initial representation to develop the others.

Sample Assessments and Performance Tasks

Reporting Category: Expressions and Equations **Standards:** 6.EE.9 **Approximate Portion of Test:** 31% - 44%; 17-23 points	OST Test Specs: ●Items will use nonnegative rational numbers. ●Items will use one-step equations of the form x + p = q, x – p = q, or px =q, where p, x, and q are nonnegative rational numbers using models or algebraically. ●Items will use one-step equations of the form $xx\ pp = qq$ where x and q are nonnegative rational numbers and p is a positive integer using models and algebraically. ●Items may require students to identify which variable is dependent or independent.

Instructional Resources
Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Chocolate Bar Sales Families of Triangles

Adopted Resource	
Reveal: Lesson 7-1: Relationships Between Two Variables Lesson 7-2: Write Equations to Represent Relationships Represented in Tables Lesson 7-3: Graphs of Relationships Lesson 4: Multiple Representations	**ALEKS:** Graphs and Functions: • Ordered Pairs • Function Tables

Return to Scope and Sequence

Module 10: Statistical Measures and Displays

Unpacked Standards / Clear Learning Targets		
Learning Target **6.SP.1** Develop statistical reasoning by using the GAISE model. a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. **6.SP.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **6.SP.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **6.SP.4** Display numerical data in plots on a number line, including dot plots (line plots), histograms, and box plots. (GAISE Model, step 3) **6.SP.5** Summarize numerical data sets in relation to their context. a. Report the number of observations.	**Essential Understanding** • Statistics is the name for the science of collecting, analyzing, and interpreting data. • The GAISE model (outlined in 6.SP.1) is used to analyze and interpret data and has 4 steps: Formulate the Question; Collect Data to Answer the Question; Analyze the Data; and Interpret Results. • Data are not just numbers; they are numbers generated with respect to a particular context and situation. • There are two types of data: categorical and numerical. • Categorical data are sorted into groups and categories. • Numerical data are measurable. • A statistical question anticipates a response that varies, from one individual to the next, and this variability is described in	**Academic Vocabulary** Center Data Distribution Numerical data set Set Shape Spread Variability Clusters Gaps Interquartile range Peaks Observation Outliers

b. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range) as well as informally describe the shape and the presence of clusters, gaps, peaks, and outliers in a distribution. d. Choose the measures of center and variability, based on the shape of the data distribution and the context in which the data were gathered.	terms of spread and overall shape. ● A distribution shows all values of data and how often they occur. ● A set of data has a distribution which can be described by its center, spread, and overall shape. ● The measure of variation describes how data values vary with a single number.	

I Can Statements

- I can analyze data and determine the number of participants/observations.
- I can find the median, mean, and interquartile ranges for a given set of data.
- I can analyze the overall shape of the data and how it might be influenced by quantitative measures of center.
- I can recognize that data has variability.
- I can recognize a statistical question (examples versus non-examples).
- I can identify that a set of data has distribution.
- I can describe a set of data by its center, e.g., mean and median.
- I can describe a set of data by its spread and overall shape, e.g. by identifying data clusters, peaks, gaps and symmetry.
- I can recognize there are measures of central tendency for a data set, e.g., mean, median, mode.
- I can recognize there are measures of variances for a data set, e.g., range, interquartile range, mean absolute deviation.
- I can recognize that measure of central tendency for a data set summarizes the data with a single number.
- I can identify the components of dot plots, histograms, and box plots.
- I can find the median, quartile and interquartile range of a set of data.
- I can analyze a set of data to determine its variance.
- I can create a dot plot to display a set of numerical data.
- I can analyze data and determine the number of participants/observations.
- I can find the median, mean, and interquartile ranges for a given set of data.
- I can analyze the overall shape of the data and how it might be influenced by quantitative measures of center.

:Priority Standards:

Proficient:
- Identify the median of an odd number of whole number data points

Accomplished (all of Proficient +):
- Calculate interquartile range

Advanced (all of Proficient + all of Accomplished +):
- Choose, calculate, and justify the correct measure of center relating to a certain context

- Find the median of an even number of whole number data points; find the mean of whole number data points
- Calculate median, mean, and range
- Display **simple** numerical data using number lines and dot plots
- Display numerical data using number lines and dot plots
- Display numerical data using number lines, dot plots, histograms, and box plots
- Choose the correct measure of center relating to a certain context
- Describe and summarize numerical distributions (data sets) by identifying clusters, peaks, gaps, and symmetry, in relationship to the context in which the data were collected
- Display and interpret numerical data using number lines, dot plots, histograms, and box plots
- Describe numerical distributions (data sets) by identifying cluster, peaks, gaps, and symmetry in relationship to the context in which the data were collected

Prior Standard(s)	Future Standard(s)
5.MD.2 Display and interpret data in graphs (picture graphs, bar graphs, and line plotsG) to solve problems using numbers and operations for this grade, e.g., including U.S. customary units in fractions 1/2, 1/4, 1/8, or decimals.	**7.SP.1** Understand that statistics can be used to gain information about a population by examining a sample of the population. **7.SP.3** Describe and analyze distributions. **S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots) in the context of real-world applications using the GAISE model. **S.ID.2** In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets. **S.ID.3** In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). **S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **S.IC.6** Evaluate reports based on data.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 6, Number 4, page 41
- Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22-23
- GAISE Model, pages 14 – 15
    - Focus of 6th grade is Level A, pages 23-35

Instructional Strategies

6.SP.1-5
In Grade 6, students are at Level A. At Level A, students should have some opportunities to collect data, but it is not necessary in every case.

It is advised to use naturally occurring events in the classroom regarding data such as how many people on average buy lunch at school. When students collect data, the data should be limited to the classroom.

Students should understand the difference between categorical and numerical data.

Although teachers may want to use categorical data to help introduce the GAISE process, standards 6.SP.3-5 focus on numerical data.

Discuss with students the importance of experimental design when conducting simple experiments. A simple experiment consists of taking measurements on a particular condition or group.

Highlight the importance of consistency on the controls of the experiment to gain more comparable data, which could be done directly or could be done indirectly leveraging students' competitive natures.

To find the median visually and kinesthetically, students could reorder the data in ascending or descending order, then place a finger on each end of the data and continue to move toward the center by the same increments until the fingers touch.

The concept of mean as "fair share" can be demonstrated visually and kinesthetically by using stacks of linking cubes or blocks. The blocks are redistributed among the towers so that all towers have the same number of blocks. It may be useful initially to present problems that work out as whole numbers, so students can use concrete models such as blocks. However, students should move towards finding the mean as a decimal or fraction, reinforcing the idea that the mean is a number representing the arithmetic average but not necessarily an actual number in the data set.

When summarizing data, students should be able to describe what a "typical" piece of data represents and connect this concept to measures of center. Students should not only determine measures of center and measures of variability, but also use these numbers to answer the statistical question asked in the context of the problem. The distribution should be described in terms of its center (mean, median, mode), spread (range, interquartile range), and shape (symmetry, skewness, clusters, gaps, outliers).

Continue to have students connect contextual situations to data to describe the data set in words prior to computation.

Students need multiple opportunities to look at data to determine if a question is a statistical question. Numerical data should be analyzed using different tools, such as organized lists, box-plots, dot plots, histograms, and stem-and-leaf plots; categorical data should be analyzed using lists and bar graphs. After students have analyzed the data, it is important to think about what may have caused the data to look like it does. This should lead to a discussion on variability.

Sample Assessments and Performance Tasks

Reporting Category:
Geometry and Statistics
Standards:
6.SP. 1, 2, 3, 4 and 5
Approximate Portion of Test:
20% - 25%; 11 - 13 points

OST Test Specs:
- Items may use all types of rational numbers.
- Items can be aligned to one step of the GAISE model or to multiple steps.
- This standard will focus on steps b and d since steps a and c are assessed in other standards.
- Items will focus on Level A of the GAISE model.
- Items can test knowing the order of the four steps of the model.
- Non negative rational numbers will be used.
- Visual data displays are limited to dot plots (line plots), histograms, and box plots.
- Only numerical data sets will be used.
- Items may require knowledge of mean and median as measures of center.
- Items may require knowledge of range and interquartile range as measures of variation.

Instructional Resources

6.SP.1:
Better Lesson
Shmoop
Khan Academy Videos
Illustrative Mathematics
Buttons: Statistical Questions
Identifying Statistical Questions
Statistical Questions

6.SP.2:
Better Lesson
Shmoop
Khan Academy Videos
Illustrative Mathematics
Average Number of Siblings
Describing Distributions
Electoral College
Is It Center or Is It Variability?
Puppy Weights

6.SP.3:
Better Lesson
Shmoop
Khan Academy Videos
Illustrative Mathematics
Is it Center or Is It Variability?

6.SP.4:
Better Lesson
Shmoop
Khan Academy Videos
Illustrative Mathematics
Average Number of Siblings
Comparing Test Scores
Describing Distributions
Puppy Weights
Puzzle Times

6.SP.5:
Better Lesson
Shmoop
Khan Academy Videos
Illustrative Mathematics
Average Number of Siblings
Comparing Test Scores
Math Homework Problems
Puzzle Times

Adopted Resource	
Reveal: Lesson 10-1: Statistical Questions Lesson 10-2: Dot Plots and Histograms Lesson 10-3: Measures of Center Lesson 10-4: Interquartile Range and Box Plots Lesson 10-5: Mean Absolute Deviation Lesson 10-6: Outliers Lesson 10-7: Interpret Graphical Displays	**ALEKS:** Data Analysis and Probability: • Collecting Data • Graphs of Data • Mean, Median, and Mode • Measures of Variation

Return to Scope and Sequence

Module 8: Area

Unpacked Standards / Clear Learning Targets		
Learning Target **6.EE.2** Write, read, and evaluate expressions in which letters stand for numbers. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order. **6.G.1** Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems. **6.G.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	**Essential Understanding** **Area** • Any side of a triangle can be a base. • The height of a polygon is a perpendicular line segment drawn from a vertex to the opposite side (base) or its extension. • The area of a triangle is half the area of a parallelogram with the same base and height. • Any polygon can be composed or decomposed into known figures to determine area. • The total area of a two-dimensional composite shape is the sum of the areas of all its parts **Coordinates** • The area and side lengths of polygons can be found by plotting coordinates in a coordinate plane. • If both x-coordinates of a line segment are the same, a vertical line segment is formed and the length can be determined. • If both y-coordinates of a line segment are the same, a horizontal line segment is formed and the length can be determined.	**Academic Vocabulary** Area Right triangle Quadrilateral Polygon Coordinate plane Vertices

COLUMBUS CITY SCHOOLS

Math 6

I Can Statements

- I can recognize and know how to compose and decompose polygons into triangles and rectangles.
- I can compare the area of a triangle to the area of the composed rectangle.
- I can apply the techniques of composing and/or decomposing to find the area of triangles, special quadrilaterals and polygons to solve mathematical and real world problems.
- I can draw polygons on the coordinate plane.
- I can use coordinates (with the same x-coordinate or the same y-coordinate) to find the length of a side of a polygon.
- I can apply the technique of using coordinates to find the length of a side of a polygon drawn in the coordinate plane to solve real-world and mathematical problems.

:Priority Standards:

Proficient:

- Evaluate expressions at specific values of their variables as it pertains to formulas used in real-world problems.
- Perform arithmetic operations, including exponents, using the order of operations when there are no parentheses to specify a particular order
- Find areas of right triangles using grid paper
- Find areas of polygons with whole number side lengths by decomposing them into rectangles and triangles
- Find areas of polygons by decomposing them into rectangles and triangles
- Draw polygons in the coordinate plane given coordinates in the first quadrant
- Draw polygons in one quadrant of the coordinate plane
- Draw polygons in the coordinate plane
- Solve routine real-world and mathematical problems by graphing points in the first quadrant
- Solve routine real-world and mathematical problems by graphing points in the first quadrant

Accomplished (all of Proficient +):

- Solve mathematical problems by finding the area of a two-dimensional shape composed of rectangles and triangles
- Draw polygons in the coordinate plane and find lengths of horizontal and vertical sides to solve real-world problems
- Solve real-world and mathematical problems by graphing points in the first quadrant

Advanced (all of Proficient + all of Accomplished +):

- Solve non-routine mathematical problems by finding the area of a two-dimensional shape composed of rectangles and triangles
- Solve real-world and mathematical problems by graphing points and/or polygons in the coordinate plane

Prior Standard(s)	Future Standard(s)
4.MD.3 Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. **5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **5.G.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	**6.G.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real world and mathematical problems. **7.G.1** Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals. **7.G.4** Work with circles. **7.G.6** Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **8.G.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. **8.G.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. **G.MG.1** Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder. **G.SRT.9** Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

Content Elaborations

- Ohio's K-8 Critical Area of Focus, Grade 6, Number 5, page 42
- Ohio's K-8 Learning Progressions, K-5 Geometry, page 11
- Ohio's K-8 Learning Progressions, Measurement and Data, pages 12-14
- Ohio's K-8 Learning Progressions, 6-8 Geometry, page 21

Instructional Strategies

6.G.1, 6.G.3
When exploring area and volume it is very important for students to continue to physically manipulate materials and create drawings in order to make connections to the symbolic and more abstract aspects of geometry.

Although unit squares and unit cubes are the most common units used in area and volume problems, units can be anything including sheets of paper or boxes. These are oftentimes the type of units used in real-life area and volume problems. It might be helpful to extend students' thinking by exposing them to problems with different units besides unit squares. For example, have students measure the area table using a piece of paper as a unit, and then have them measure the same table using standard units such as square feet or square inches. Make the connection that although the area is represented with different numbers and units, the area itself remains the same.

To build the concept of conservation of area, have students do activities where the area is cut up and rearranged. Tangrams can also be useful for exploring this concept. Have

a discussion about how pieces can have the same size (area) but different shape.

The coordinate plane can help students see movement and change in both geometry and algebra. Given the coordinates for vertices, students will draw polygons in the coordinate plane. They will determine the lengths of vertical and/or horizontal sides of the polygon by subtraction or counting. Students should discover that for a vertical line, the x-coordinates are the same and for a horizontal line the y-coordinates are the same. Include shapes whose bases are not parallel to the x-axis. Some activities that can be done on the coordinate plane are as follows:

- Give students coordinates of the vertices of a shape and have students make inferences about the shape before plotting it.
- Describe a polygon by giving all the coordinates of the vertices except one.
- Given the coordinates of a shape, give new coordinates of a shape that is identical in shape.
- Create a shape given a vertex and dimensions. How many other identical shapes can you create with the same criteria?

Sample Assessments and Performance Tasks

Reporting Category:
Expressions and Equations; Geometry and Statistics
Standards:
6.EE.2; 6.G.1 and 3
Approximate Portion of Test:
Expressions and Equations: 31% - 44%; 17 - 23 points
Geometry: 20% - 25%; 11 - 13 points

OST Test Specs:

- Items will use positive rational numbers.
- In 2a, real-world problems will be avoided.
- In 2c, items may require the use of the order of operations.
- Multiplication may be represented by a raised dot, parentheses, or a coefficient and a variable.
- Division may be represented by a fraction bar or a division sign.
- Items will use positive rational numbers.
- Shapes will be limited to polygons that can be decomposed or composed into rectangles and/or triangles.
- Items may use all types of rational numbers.
- The vertices of a polygon may be in different quadrants.
- Students will not be expected to know the quadrant names (I, II, III, IV). Items can refer to the quadrant names but will include a diagram which labels the quadrants by number.
- In items involving finding the distance between two points, either the first coordinates or the second coordinates will be the same.
- Axes can be numbered with scales other than 1.

Instructional Resources

6.G.1:
Better Lesson
Shmoop
Khan Academy Videos
Dan Meyer Activity
Dollar Wall
Bubble Wrap
Illustrative Mathematics
24 Unit Squares
Areas of Right Triangles
Areas of Special Quadrilaterals
Base and Height
Finding Areas of Polygons
Polygons in the Coordinate Plane
Same Base and Height, Variation 1
Same Base and Height, Variation 2
Sierpinski's Carpet
Wallpaper Decomposition

6.G.3:
Better Lesson
Shmoop
Khan Academy Videos
Illustrative Mathematics
Polygons in the Coordinate Plane
Walking the Block

Adopted Resource

Reveal:
Lesson 8-1: Area of Parallelograms
Lesson 8-2: Area of Triangles
Lesson 8-3: Area of Trapezoids
Lesson 8-4: Area of Regular Polygons
Lesson 8-5: Polygons on the Coordinate Plane

ALEKS:
Perimeter, Area, and Volume; Lines, Angles, Polygons:

- P, A, and V: Area of Rectangles
  Area of Parallelograms, Triangles, and Trapezoids
- Lines, Angles, and Polygons: Polygons and Quadrilaterals

Return to Scope and Sequence

COLUMBUS CITY SCHOOLS

Math 6

Module 9: Volume and Surface Area

Unpacked Standards / Clear Learning Targets

Learning Target

6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = \ell \cdot w \cdot h$ and $V = B \cdot h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Essential Understanding

Surface Area

- The surface area of a three-dimensional figure is made up of the sums of the areas of its faces.
- A net is a composite two-dimensional shape of a three-dimensional object used to find the surface area.
- Surface area of a three-dimensional figure includes faces that are visible and not visible from a given viewpoint.

Volume

- Rectangular prisms may have edge lengths that are fractions.
- Volume of a rectangular prism can be determined using the formulas and/or by packing it with unit cubes of the appropriate unit fraction edge lengths.

Academic Vocabulary

Edge
Right rectangular prism
Unit cube
Volume
Net
Surface Area

I Can Statements

- I can calculate the volume of a right rectangular prism.
- I can apply volume formulas for right rectangular prisms to solve real-world and mathematical problems involving rectangular prisms with fractional edge lengths.
- I can model the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths.
- I can recognize that 3-D figures can be represented by nets.
- I can represent three-dimensional figures using nets made up of rectangles and triangles.
- I can apply knowledge of calculating the area of rectangles and triangles to a net.
- I can combine the areas for rectangles and triangles in the net to find the surface area of a 3-dimensional figure.
- I can solve real-world and mathematical problems involving surface area using nets.

:Priority Standards:

Proficient:
- Find the volume of rectangular prisms with whole number sides
- Find volumes of rectangular prisms with whole number side lengths

Accomplished (all of Proficient +):
- Solve mathematical problems by finding the volumes of rectangular prisms with fractional edge lengths

Advanced (all of Proficient + all of Accomplished +):
- Solve real-world problems by finding the volumes of rectangular prisms with fractional edge lengths

- Find volumes of rectangular prisms with fractional edge lengths
- Given nets, find surface areas of rectangular prisms with whole number side lengths
- Use nets to find surface areas of rectangular and triangular prisms and pyramids

Prior Standard(s)	Future Standard(s)
5.MD.5 Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. **6.G.1** Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems.	**7.G.6** Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Content Elaborations

- Ohio's K-8 Critical Area of Focus, Grade 6, Number 5, page 42
- Ohio's K-8 Learning Progressions, K-5 Geometry, page 11
- Ohio's K-8 Learning Progressions, Measurement and Data, pages 12-14
- Ohio's K-8 Learning Progressions, 6-8 Geometry, page 21

Instructional Strategies

Volume is the three-dimensional space inside a solid. It can be thought of as "filling" a solid. Focus on what volume means conceptually and move students away from the formulaic definition length × width × height which only holds true for rectangular prisms. Instead have the students think about layers, so they discover the formula B × h which is more inclusive.

In order for a student to understand the location of a unit cube in an array of a prism, they must "see" the cube in a three-dimensional coordinate system consisting of rows and columns and layers. (Layers may be vertical or horizontal.)

It is a difficult concept for students to extend this visualization to fractional edge lengths. Before moving towards fractional edge lengths, it may be helpful to have students calculate volume with units other than the unit cube.

See the Ohio Model Curriculum for 6th grade,pages 191-193, for ideas on handling fractional edge lengths.

Introduce surface area as the idea of "wrapping." Have students build prisms using unit cubes. Then have them count the number of squares on each face to connect the idea of area to surface area. Remind them that they also must count the bottom of the figure.

Exploring possible nets could be done by taking apart (unfolding) three-dimensional objects such as Kleenex boxes. This process is foundational for the study of surface area of prisms. Have students cut apart the faces and rearrange them to illustrate that there are many different nets for the same object.

Both the composition and decomposition of rectangular prisms should be explored. Understanding that there are multiple nets for the same object may be difficult for some to visualize; provide concrete examples of nets for the object.

Sample Assessments and Performance Tasks

Reporting Category:
Geometry and Statistics
Standards:
6.G.2 and 4
Approximate Portion of Test:
20% - 25%; 11 - 13 points

OST Test Specs:
- At least one side length must be a non-integer.
- Fractional unit cubes will be limited to $\frac{1}{2}$ and $\frac{1}{4}$
- Only positive rational numbers will be used.
- Nets will be composed of rectangles and triangles.
- Solids will include right prisms and pyramids.

Instructional Resources

6.G.2:
Better Lesson
Shmoop
Khan Academy Videos
Dan Meyer Activity
Girl Scout Cookies
Illustrative Mathematics
Banana Bread
Computing Volume Progression 1
Computing Volume Progression 2
Computing Volume Progression 3
Computing Volume Progression 4
Volumes with Fractional Edge Lengths

6.G.4:
Better Lesson
Shmoop
Khan Academy Videos
Dan Meyer Activity
Dandy Candies
Illustrative Mathematics
Nets for Pyramids and Prisms

Adopted Resource

Reveal: Lesson 9-1: Volume of Rectangular Prisms Lesson 9-2: Surface Area of Rectangular Prisms Lesson 9-3: Surface Area of Triangular Prisms Lesson 9-4: Surface Area of Pyramids	**ALEKS:** Perimeter, Area, and Volume: • Volume of Rectangular Prisms • Area of Parallelograms, Triangles, and Trapezoids • Surface Area

Return to Scope and Sequence