Mathematics

Math 8

2023-2024

Aligned with Ohio's Learning Standards for Mathematics (2017)

Department of Academic Services
Office of Teaching and Learning
Curriculum Division

COLUMBUS CITY SCHOOLS

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Math 8

Curriculum Map

Year-at-a-Glance

The Year-at-a-Glance provides a high-level overview of the course by grading period, including:

- Units;
- Standards/Learning Targets; and
- Timeframes.

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Scope and Sequence

The Scope and Sequence provides a detailed overview of each grading period, including:

- Units;
- Standards/Learning Targets;
- Timeframes;
- Big Ideas and Essential Questions; and
- Strategies and Activities.

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Curriculum and Instruction Guide

The Curriculum and Instruction Guide provides direction for standards-based instruction, including:

- Unpacked Standards / Clear Learning Targets;
- Content Elaborations;
- Sample Assessments;
- Instructional Strategies;
- Instructional Resources; and
- ODE Model Curriculum with Instructional Supports.

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Math 8

Year-at-a-Glance

Grading Period	Reporting Category	Weeks
Grading Period 1	**Reporting Category: Equations and Expressions & The Number System**	**14 weeks**
	1. Exponents & Scientific Notation - 3 weeks 2. Real Numbers - 3 weeks 3. Solve Equations with Variables on Each Side - 4 weeks 4. Linear Relationships and Slope - 3 weeks 5. Systems of Linear Equations (solve by graphing only) - 1 week	
Grading Period 2	**Reporting Category: Functions**	**4 weeks**
	1. Identifying Functions 2. Function Tables 3. Constructing and Comparing Functions 4. Qualitative Graphs	
Grading Period 3	**Reporting Category: Statistics (Statistics falls under the Equations and Expressions category)**	**5 weeks**
	1. Scatter Plots and Lines of Best Fit 2. Two-Way Tables	
Grading Period 4	**Reporting Category: Geometry**	**13 weeks**
	1. Triangles and the Pythagorean Theorem - 4 weeks 2. Transformations - 3 weeks 3. Congruence & Similarity - 3 weeks 4. Volume - 3 weeks	

Standards for Mathematical Practice

The Standards for Mathematical Practice (SMP) describe skills that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The design of each item on Ohio's state tests encourages students to use one or more Standards for Mathematical Practice.

Modeling and Reasoning are included in the eight Standards for Mathematical Practice within Ohio's Learning Standards. Each grade's blueprint identifies modeling and reasoning as an independent reporting category that will account for a minimum of 20 percent of the overall points on that grade's test.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Practice - Grade 8

Modeling and Reasoning on Ohio's State Tests in Mathematics

Math 8

Scope and Sequence

Students should be assessed using teacher-based resources and the ALEKS program. Students are automatically enrolled in the ALEKS course Middle School Course 3. Teachers can move students into RTI 8 if students show a need for remediation. Refer to our guide at https://tinyurl.com/CCS-ALEKS-GUIDE

Textbook information

McGraw-Hill - Reveal Math Course 3

Grading Period 1	Module 1: Exponents and Scientific Notation 3 weeks			
	Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**
	1.1 Powers and Exponents 1.2 Multiply and Divide Monomials 1.3 Powers of Monomials 1.4 Zero and Negative Exponents	8.EE.1 Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions.	How do patterns show the properties of exponents? How do I create equivalent numerical expressions using the properties of integer exponents?	● Although students begin using whole-number exponents in Grades 5 and 6, it is in Grade 8 when students are first expected to understand, explain, and apply the properties of exponents and to extend the meaning beyond counting-number exponents. ● Students should not be told these properties but rather should derive them through experience and reason. Many students who simply "memorize" the rules without understanding may confuse the rules when trying to apply them at later times. Instead students should be encouraged to discover the rules using tables, patterns, and expanded notation. As they have multiple experiences simplifying numerical expressions with exponents, these properties become natural and obvious. ● Students should use multiplicative reasoning and expanded notation to gain understanding of non-positive exponents. ● Another way to view the meaning of 0 and negative exponents is by applying the following principle: The properties of counting-number exponents should continue to work for integer exponents. (See instructional strategies) ● In Grade 8 students should also have practice using negative

				numbers as bases. ● Have students identify the bases before solving problems as many students incorrectly only attribute the exponent to the nearest number. ● Students should also have practice using non-integer bases such as $(1.5)^3$ or $\left(\frac{3}{4}\right)^{-2}$
	1.5 Scientific Notation	8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. 8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology.	How do I identify numbers in scientific notation? How do I convert numbers from scientific notation to decimal notation? (and vice versa) How do I compare numbers in scientific notation?	● The meanings of integer exponents, especially with respect to 0 and negatives, can be further explored in a place-value chart: Thus, integer exponents support writing any decimal in expanded form like the following: $3247.568 = 3 \cdot 10^3 + 2 \cdot 10^2 + 4 \cdot 10^1 + 7 \cdot 10^0 + 5 \cdot 10^{-1} + 6 \cdot 10^{-2} + 8 \cdot 10^{-3}$. ● Expanded form and the connection to place value is important for helping students make sense of scientific notation, which allows very large and very small numbers to be written concisely, enabling easy comparison. Students can make sense of scientific notation and negative exponents by using patterns. ● To develop familiarity, go back and forth between standard notation and scientific notation for numbers. Compare numbers, where one is given in scientific notation and the other is given in standard notation. Have students place numbers written in scientific notation on a number line and order them without converting them to standard form. Students should come to the conclusion that when determining value, the power is more important than the coefficient.
	1.6 Compute with Scientific Notation	8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. 8.EE.4 Perform operations with numbers expressed in scientific notation, including problems	How are operations performed on numbers where both scientific notation and decimal notation are used? How do different technology devices show scientific notation? What are appropriately sized units when dealing with very large or very small quantities?	● Real-world problems can help students compare quantities and make sense about their relationships. Conversely scientific notation can also help students make sense of really large or small numbers by modeling situations. Students should as often as possible have a real-world situation to model when using scientific notation to help their understanding of the concept.

	where both decimal notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology. Also Addresses: 8.EE.1 Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions.	How do I choose appropriately sized units when dealing with very large or very small quantities?	

Module 2: Real Numbers — **3 weeks**

Lesson	Standards/Learning Targets	Big Ideas/Essential Questions	Strategies/Activities
2.1 Terminating and Repeating Decimals	8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating.	How do I convert rational numbers from fractions to their decimal expansions? When does a rational number in simplest fractional form have a terminating decimal?	• In previous grades, students become familiar with rational numbers called decimal fractions. In Grade 7, students carry out the long division and recognize that the remainders may repeat in a predictable pattern—a pattern creates the repetition in the decimal representation (see 7.NS.2.d). In Grade 8, they explore its occurrence. • Ask students what will happen in long division once the remainder is 0. They can reason that the long division is complete, and the decimal representation terminates. However, if the remainder never becomes 0, then the remainder will repeat in a cyclical pattern. The important understanding is that students can see that the pattern will continue to repeat. • Explore differences between terminating and repeating decimals.
2.2 Roots	8.EE.2 Use square root and cube	How do I solve for square roots	• To help students build a conceptual understanding, connect

	root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	and cube roots? How do I show solutions with square root and cube root symbols? How can I evaluate square roots of perfect squares and cube roots of perfect cubes?	roots to area and volume models where the area and volume are the radicand and the solution is the length of the side of the model. • Another way to explain it is that the area and volume of the square or cube represents n; and the square and cube's side length is represented by $\sqrt{n}$ and $\sqrt[3]{n}$ respectively. • Have students use geoboards, square tiles, graph paper, or unit cubes to build squares and cubes reviewing exponents in the process. Problems such as the Painted Cube Problem can then be modified to extend to square and cube roots. • Also, provide practical opportunities for students to flexibly move between forms of squared and cubed numbers. For example, If $3^2 = 9$ then $\sqrt{9} = 3$. This flexibility should be experienced symbolically and verbally with manipulatives and with drawings.
2.3 Real Numbers	8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating. 8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	How do I know if a real number is rational or irrational?	• It could be appropriate to use Venn diagrams/set diagrams or flowcharts to show the relationships among real, rational, irrational numbers, integers, and natural numbers. The diagram should show that all real numbers are either rational or irrational. • Students should come to the understanding that (1) every rational number has a decimal representation that either terminates or repeats and (2) every terminating or repeating decimal is a rational number. Then, they can use that information to reason that on the number line, irrational numbers must have decimal representations that neither terminate nor repeat. • In previous grades, students learned processes that can be used to locate any rational number on the number line: Divide the interval from 0 to 1 into b equal parts; then, beginning at 0, count out a of those parts. Now they can use similar strategies to locate irrational numbers on a number line. • Use an interactive number line to allow students to see how a number line can be infinitely divided. One resource is Zoomable Number Line by MathisFun. • Although students at this grade do not need to be able to prove that $\sqrt{2}$ is irrational, they minimally need to know that $\sqrt{2}$ is irrational (see 8.EE.2), which means that its decimal
2.4 Estimate Irrational Numbers	8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value	How do I approximate the value of irrational numbers? How do I get more precise approximations of real numbers?	

	of expressions, e.g., π^2. Also Addresses: 8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.		representation neither terminates nor repeats. Nonetheless, they should be able to approximate irrational numbers such as √2 without using the square root key on the calculator. ● The √2 caused Greek Mathematicians many problems, for although they could construct it using tools, but they could not measure it. Integrating math history into the lesson may be interesting for some students. ● Have students do explorations where precision matters. Discuss situations where precision is vital and other situations where reasonableness is more vital than precision. Although learning about significant digits formally takes place in high school, it is appropriate to talk about how intermediate rounding affects precision. The display of irrational numbers on a calculator could also be used for a discussion point. Another discussion could take place comparing results using the pi button compared to the typical approximation of 3.14. ● The concept of precision should also be tied to real-world contexts. For example, we do not buy 3.5 apples, but we may buy 3.5 lbs of ground beef, so rounding to a whole number is a precise number in terms of buying apples. Therefore, being precise should relate to the real-world context of the problems.
2.5 Compare and Order Real Numbers	8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating. 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., π^2.	How do I locate the (approximate) location of an irrational number on a number line? How can I compare and order rational and irrational numbers?	
Module 3: Solve Equations with Variables on Each Side 4 weeks			
Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**
3.1 Solve Equations	8.EE.7 Solve linear equations in one variable. b. Solve linear equations with rational number coefficients, including equations whose	How do I solve multi-step equations with rational coefficients? How can I determine if an equation has no solutions, one solution, or	● In Grade 7, students learned integer operations for the first time. They also applied the properties of operations when solving two-step equations and inequalities. Now students build on the fact that solutions maintain equality and that equations may have only one solution, many solutions, or no

	solutions require expanding expressions using the distributive property and collecting like terms.	infinitely many solutions? How can I graph the solution to a linear equation on a number line?	solutions at all. ● Properties of Operations Table 3 on page 97 of Ohio's Learning Standards in Mathematics states the Properties of Operations and Table 4 states the Properties of Equality. Teachers should be using the correct terminology to justify steps when performing operations and solving equations. ● Students incorrectly think that the variable is always on the left side of the equation. Give students situations where the variable is on the right side of the equation. Emphasize using the Symmetric Property of Equality if students wish to flip the variable to the other side of the equal sign. ● Equation-solving in Grade 8 should involve multi-step problems that require the use of the distributive property, collecting like terms, rational coefficients, and variables on both sides of the equation. ● In Grade 7, students may have used a pan balance, number lines, or algebra tiles to solve two-step equations. Eighth grade students could review these models and build upon them. For example, algebra tiles may help prevent student errors such as incorrectly combining like terms on opposite sides of the equations. ● When not using models, some students benefit from drawing a vertical line through the equals sign to separate the two sides of the equation. ● Connect mathematical analysis with real-life events by using contextual situations when solving equations. Students should experience— • analyzing and representing contextual situations with equations; • identifying whether there is one solution, no solutions, or infinitely many solutions; and then • solving the equations to prove conjectures about the solutions.
3.2 Write and Solve Equations with Variables on Each Side	8.EE.7 Solve linear equations in one variable. b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. Also Addresses: 8.EE.7a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).		
3.3 Solve Multi-Step Equations	8.EE.7 Solve linear equations in one variable. b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.		

Grading Period 2	3.4 Write and Solve Multi-Step Equations	8.EE.7 Solve linear equations in one variable. b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. Also Addresses: 8.EE.7a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).		
	3.5 Determine the Number of Solutions	8.EE.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).		

Module 4: Linear Relationships and Slope 4 weeks

Lesson	Standards/Learning Targets	Big Ideas/Essential Questions	Strategies/Activities
4.1 Proportional Relationships and Slope	8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.	How do I graph proportional relationships? How does the constant of proportionality relate to slope?	● Students in Grade 7 represented proportional relationships as equations such as y = kx or t = pn. They also graphed proportional relationships, discovering that a graph of a proportion must go through the origin, and that in the point (1, r), r is the unit rate. Now in Grade 8, the unit rate of a proportion is used to introduce "the slope" of the line. ● Students need to make connections between the different representations (equations, tables, graphs) in order to come to a unified understanding that the different representations are in essence different ways of modeling the same information. ● Explicit connections need to be made between the multiplicative factor, the slope, scale factor, and an increment in a table. ● To reinforce the relationships between the x and the y, students should continually name quantities for the real-world problem they represent. They should also identify the independent and dependent variables.
4.2 Slope of a Line	Foundational for: 8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b. 8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.		
4.3 Similar Triangles and Slope	8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the	How can I use similar right triangles to find the slope of a line?	● By using coordinate grids and various sets of similar triangles, students can prove that the slopes of the corresponding sides are equal, thus making the unit rate or rate of change equal.

	equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.		
4.4 Direct Variation	8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b. Also Addresses: 8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.	How can I use an equation to determine the relationship between two variables? How are the equations y = mx and y = mx + b related?	• Distance-time problems are common in mathematics. In this cluster, they serve the purpose of illustrating how the rates of two objects can be represented, analyzed, and described in different ways: verbally, graphically, tabularly, and algebraically. Emphasize the creation of representative graphs and the meaning of various points. Then compare the same information when represented in an equation.
4.5 Slope-Intercept Form	8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.		• Use graphing utilities such as Desmos to show the lines in the form of y = mx + b as vertical translations of the equation y = mx.
4.6 Graph Linear Equations	Foundational for: 8.EE.8 Analyze and solve pairs of simultaneous linear equations graphically. b. Use graphs to find or estimate the solution to a pair of two simultaneous linear equations in two variables. Equations should include all three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. 8.F.3 Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.		

Module 11: Scatter Plots and Two-Way Tables — 5 weeks

Lesson	Standards/Learning Targets	Big Ideas/Essential Questions	Strategies/Activities
11.1 Scatter Plots	**8.SP.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering; outliers; positive, negative, or no association; and linear association and nonlinear association. (GAISE Model, steps 3 and 4)	How do I construct a scatterplot? How do I choose appropriate scales, labels and plot points based on my choices? How do I use characteristics (clusters, gaps, outliers) to describe scatterplots? How do I describe a trend (linear, curved, positive, negative, strong association, weak association, no association)?	• Building on the study of statistics using univariate data in Grades 6 and 7, students are now ready to study bivariate data. They will extend their descriptions and understanding of variation to the graphical displays of bivariate data. • Explain to students *uni* means one, *bi* means two, and *variate* means variable, so univariate is data using one variable and bivariate is data using two variables. • Eighth graders can design and conduct nonrandom sample surveys; although they should begin to start thinking informally about random selection and what kind of sample best represents a population. They may also do comparative experiments. • Scatterplots are the most common form of representations displaying bivariate data in Grade 8. Provide scatterplot of linear data and have students practice informally finding the trend line. Students could be given a scatterplot and a spaghetti noodle to determine the "best fit." Discussion should include "What does it mean for a data point to be above the line?" or "What does it mean for it to be below the line?" • By changing the data slightly, students can have a rich discussion about the effects of the change on the graph. The study of the trend line ties directly to the algebraic study of slope and y-intercept. Students should interpret the slope and y-intercept of the trend line in the context of the data. Then students can make predictions based on the trend line. Give students a variety of data sets that intersect the y-axis at various points, so students do not mistakenly think that all trend lines must go through the origin. • After a trend line is fitted through the data, the slope of the line is approximated and interpreted as a rate of change, in the context of the problem. If the slope is positive, then the two variables are positively associated. Similarly if the slope is negative, then the two variables are negatively associated. Students should also be exposed to data that do not have an
11.2 Draw Lines of Fit	**8.SP.2** Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (GAISE Model, steps 3 and 4)	How do I determine the "center" of the points on a scatterplot and draw a line through that center?	
11.3 Equations for Lines of Fit	**8.SP.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. (GAISE Model, steps 3 and 4) Also Addresses: **8.F.4** Construct a function to model a linear relationship between two quantities.	How do I approximate the slope of a trend line? How do I use a scatterplot and line of best fit to write the equation of a trend line? How do I interpret the rate of change and intercept of the trend line in the context of the problem? How do I interpret data points in relationship to the trend line (above, below, or on the line)?	

	Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	How do I use a trend line and its equation to answer questions and make predictions? How do changes in data change the trend line? How do I measure the association between two quantitative variables using the Quadrant Count Ratio (QCR)?	association. ● Students should create and interpret scatterplots, focusing on outliers, positive, or negative association, linearity, or curvature. Assuming the data are linear, students should informally draw a trend line on the scatterplot and informally evaluate the strength of fit. They should be able to interpret visually how well the trend line fits the "cloud" of points. ● To move students from Level A to Level B, questions should move from "Is there an association?" to "How strong is the association?" The Quadrant Count Ratio (QCR) can help students informally determine the strength between two variables. This is an important building block in building the conceptual understanding of the correlation coefficient in high school.
11.4 Two-Way Tables 11.5 Associations in Two-Way Tables	**8.SP.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.	How do I construct a two-way frequency table? How do I calculate relative frequencies? How do I determine possible association between the two variables using row (or column) relative frequencies?	● Students may believe bivariate data is only displayed in a scatterplot. The standard 8.SP.4 provides the opportunity to display bivariate, categorical data in a table. ● Types of Frequencies: ○ Frequency Table ○ Relative Frequency Table ○ Marginal Frequency ○ Joint Frequency ○ Conditional Frequency
Module 6: Systems of Linear Equations			**1 week**
Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**
6.1 Solving Systems of Equations by Graphing	**8.EE.8** Analyze and solve pairs of simultaneous linear equations graphically. a. Understand that the solution to a pair of linear equations in	How do I find or estimate which points are solutions to a pair of linear equations in two variables? How can I use graphing to solve a	● This cluster builds on the informal understanding of slope, students gained from graphing unit rates and proportional relationships in grades 6 and 7. It also builds upon the stronger, more formal understanding of slope and the relationship between two variables from 8.EE.5-6 and 8.F.4-5.

Grading Period 3

		two variables corresponds to the point(s) of intersection of their graphs, because the point(s) of intersection satisfy both equations simultaneously. b. Use graphs to find or estimate the solution to a pair of two simultaneous linear equations in two variables. Equations should include all three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to pairs of linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. (Limit solutions to those that can be addressed by graphing.)	pair of linear equations? How do I represent the solution to a pair of linear equations in two variables?	● Students will use graphing to solve pairs of simultaneous linear equations. Beginning work should involve pairs of equations with solutions that are ordered pairs of integers, making it easier to locate the point of intersection. Although students should also be able to approximate solutions that do not fall evenly onto the intersection of grid squares. ● Provide opportunities for students to see and compare simultaneous linear equations in forms other than slope-intercept form (y = mx + b). Students may solve pairs of simultaneous linear equations by inspection, by graphing using slope-intercept form, or by graphing using tables of values. ● Students should be able to solve simple cases by inspection. For example, x + y = 3 and x + y = 5 has no solution because x + y cannot equal both 3 and 5. ● Students should have practice working with graphs that have a variety of scales including fractions and decimals. ● Students have the tendency to see the intersection point of a graphed system of equations and round it to the nearest grid line cross-section. Emphasize to students that they can sometimes find intersections in the middle of the grid squares that allow for approximations to be more precise. ● Students could also investigate pairs of simultaneous equations using graphing calculators or online graphing resources. They could be asked to explain verbally and in writing what, in the equation and situation, makes lines shift to different locations on the graph.
	6.2 Determine Number of Solutions		How can I determine if a pair of linear equations has no solutions, one solution, or infinitely many solutions?	
	6.5 Write and Solve Systems of Equations		How can I solve real-world problems using pairs of linear equations? What kinds of real-world problems can be solved using pairs of linear equations? How do I use two different scales when graphing pairs of linear equations in two variables?	● Graphing pairs of linear equations should be introduced through contextual situations relevant to eighth graders, so students can create meaning. They should explore many tasks for which they must write and graph pairs of equations with different slopes and y-intercepts. This should lead to the generalization that finding one point of intersection is the single solution to the pair of linear equations. ● Students should relate the solution to the context of the problem, commenting on the reasonableness of their solution. ● Emphasize that the solution must satisfy both equations.

Module 5: Functions			**4 weeks**
Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**
5.1 Identify Functions	8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. *Function notation is not required.	How do I determine if a table, graph, equation, or verbal description represents a linear or nonlinear function?	• Students should be expected to reason from a context, a graph, or a table, after first being clear which set represents the input (e.g., independent variable) and which set is the output (e.g., dependent variable). When a relationship is not a function, students should produce a counterexample: an "input value" with at least two "output values." If the relationship is a function, the students should explain how they verified that for each input there was exactly one output. • In Grade 6 students explored independent and dependent variables, and how the dependent variable changes in relation to the independent variable. In Grade 8 students need to continue identifying the independent and dependent variables in functions. Students need practice justifying the relationship between the independent and dependent variable. • In Grade 6 students explored independent and dependent variables, and how the dependent variable changes in relation to the independent variable. In Grade 8 students need to continue identifying the independent and dependent variables in functions. Students need practice justifying the relationship between the independent and dependent variable.
5.2 Function Tables	8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. *Function notation is not required.	How do I identify a set of input and output values for a function?	
5.3 Construct Linear Functions	8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	How do I interpret the slope/rate of change and y-intercept/initial value of a linear function?	

	5.4 Compare Functions	8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	How do properties of two functions compare when represented differently?	• The standards explicitly call for exploring functions numerically, graphically, verbally, and algebraically. For fluency and flexibility in thinking, students need experiences translating among these different representations. • Students need experience translating among the different representations using different functions. For example, they should be able to determine which function has a greater slope by comparing a table and a graph. • Students need to compare functions using the same representation. For example, within a real-world context, students compare two graphs of linear functions and relate the graphs back to its meaning within the context and its quantities. Students should work with graphs that have a variety of scales including rational numbers.
	5.5 Nonlinear Functions	8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. *Function notation is not required. 8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. 8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	What are the characteristics of linear and nonlinear functions? How do I use characteristics of functions to determine if they are linear or nonlinear? What are examples of functions that are nonlinear?	• In Grade 8, the focus is on linear functions, and students begin to recognize a linear function from its form $y = mx + b$ knowing that $y = mx$ as a special case of a linear function. Students also need experiences with nonlinear functions. This includes functions given by graphs, tables, or verbal descriptions but for which there is no formula for the rule.

5.6 Qualitative Graphs	8.F.3 Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.	How do I determine if it is reasonable to "connect the points" on a graph based on context?	• When plotting points and drawing graphs, students should develop the habit of determining, based upon the context, whether it is reasonable to "connect the dots" on the graph. In some contexts, the inputs are discrete, and connecting the dots is incorrect. For example, if a function is used to model the height of a stack of n paper cups, it does not make sense to have 2.3 cups.

Module 7: Triangles and the Pythagorean Theorem — **4 weeks**

Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**
7.2 Angle Relationships and Triangles	8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	What is the sum of the interior angles of a triangle? What is the relationship between the exterior angle of a triangle and the two remote (non-adjacent) interior angles?	• In Grade 7 students should have had some practice exploring that the sum of the angles inside a triangle equal 180 degrees. Now students use transformations to prove it. • Students can create a triangle and use rotations and transformations to line up all the angles to prove that the sum of the interior angles of a triangle equals 180 degrees. They need to be able to demonstrate and explain why the sum of the interior angles equals 180 degrees. • Students should build on this activity to explore exterior angle relationships in triangles. They can also extend this model to explorations involving other parallel lines, angles, and parallelograms formed. Students should be able to explain why two angles in a triangle have to be less than 180 degrees. • Investigations should lead to the Angle-Angle criterion for similar triangles. For instance, groups of students should explore two different triangles with one, two, and three given angle measurements. Students observe and describe the relationship of the resulting triangles. As a class, conjectures lead to the generalization of the Angle-Angle criterion.
7.3 The Pythagorean Theorem	8.G.6 Analyze and justify an informal proof of the Pythagorean Theorem and its converse. 8.G.7 Apply the Pythagorean Theorem to determine unknown	How do I identify the legs and hypotenuse of a right triangle? What is the relationship between the areas of the squares created using the side lengths of a right triangle? How can I justify the Pythagorean Theorem and its converse?	• Students should understand the Pythagorean theorem as an area relationship between the sum of the squares on the lengths of the legs and the square on the length of the hypotenuse. This can be represented as $(\text{leg a})^2 + (\text{leg b})^2 = \text{hypotenuse}^2$. The Pythagorean Theorem only applies to right triangles. Exclusively using $a^2 + b^2 = c^2$ frequently leads to student errors in identifying the parts of the triangle. Use

		side lengths in right triangles in real-world and mathematical problems in two and three dimensions. Also Addresses: 8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	How can Pythagorean Theorem determine if a triangle is a right triangle? How do I use Pythagorean Theorem to find the missing side length of a right triangle? What are the relationships of a triangle plotted on a coordinate plane? How does the distance between two points on a coordinate plane relate to a right triangle? How can I use Pythagorean Theorem to calculate the distance between two points on a coordinate plane? How can I find real-world lengths that cannot be measured directly using Pythagorean Theorem?	words like leg a, leg b, and hypotenuse c. ● It is important for students to see right triangles in different orientations. ● Students should be given the opportunity to explore right triangles to determine the relationships between the measures of the legs and the measure of the hypotenuse. Experiences should involve using square grid paper to draw right triangles from given measures and representing and computing the areas of the squares on each side. Students can physically cut the squares on the legs apart and rearrange them, so they fit on the square along the hypotenuse. Data should be recorded in tables, allowing for students to conjecture about the relationship among the areas. ● Students can apply the Pythagorean Theorem to real-world situations involving two- and three-dimensions. Some examples of this may include designing roofs, ramp dimensions, etc. Students should sketch right triangles to model real-world situations. Challenge students to identify additional ways that the Pythagorean Theorem is or can be used in real-world situations or mathematical problems, such as finding the height of something that is difficult to physically measure, or the right triangle formed by the diagonal of a prism.
	7.4 Converse of the Pythagorean Theorem	8.G.6 Analyze and justify an informal proof of the Pythagorean Theorem and its converse.		● Previously, students have discovered that not every combination of side lengths will create a triangle. Now they need to explore situations that involve the Pythagorean Theorem to test whether or not side lengths represent right triangles. This is an opportunity to remind students that the longest side is the only possibility for the hypotenuse. Students should be able to explain why a triangle is or is not a right triangle using the converse of the Pythagorean Theorem. This might be an opportunity for students to explore Pythagorean triples.
	7.5 Distance on the Coordinate Plane	8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.		● Students in Grade 8 should extend the use of the Pythagorean Theorem to find the distance between two points. Understanding how to determine distance by using vertical and horizontal lengths as legs of a right triangle is more important than deriving or memorizing a formula.

				• An extension could be having students understand how to find the midpoint as well as there is an intuitive connection to finding the distance between two points.
Grading Period 4	**Module 8: Transformations & Module 9: Congruence and Similarity**			**3 weeks**
	Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**
	8.1 Translations 8.2 Reflections 8.3 Rotations	8.G.1 Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). a. Lines are taken to lines, and line segments are taken to line segments of the same length. 8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	How can I use physical models, transparencies, or geometry software to explore transformations and verify their properties? What are the effects of transformations on two-dimensional figures using coordinates?	• Transformations should include those done both with and without coordinates. • Students should be able to appropriately label figures, angles, lines, line segments, congruent parts, and images (primes or double primes). • Students are expected to use logical thinking, expressed in words using correct terminology. They should also be using informal arguments, which are justifications based on known facts and logical reasoning. However, they are not expected to use theorems, axioms, postulates or a formal format of proof such as two-column proofs. • Students should solve mathematical and real-life problems based on understandings related to this cluster. Investigation, discussion, justification of their thinking, and application of their learning will assist them in the more formal learning of geometry standards in high school. • Initial work should be presented in such a way that students understand the concept of each type of transformation and the effects that each transformation has on an object before working within the coordinate system. • Provide opportunities for students to physically manipulate figures to discover properties of similar and congruent figures involving appropriate manipulatives, such as tracing paper, rulers, Miras, transparencies, and/or dynamic geometric software. Time should be allowed for students to explore the figures for each step in a series of transformations, e.g., cutting out and tracing. • Discussion should include the description of the relationship between the preimage (original figure) and image(s) in regards to their corresponding parts (length of sides and measure of angles) and the description of the movement, (line of

				reflection, distance, and direction to be translated, center of rotation, angle of rotation, and the scale factor of dilation). ● Students should be able to provide a sequence of transformations required to go from a preimage to its image. Provide opportunities for students to discuss the procedure used, whether different procedures can obtain the same results, and if there is a more efficient way that can be used instead. They need to learn to describe transformations using words, numbers, drawings, and expressions. ● Although computer software is encouraged to be used in this cluster, it should not be used prematurely. Students need time to develop these geometric concepts with hands-on materials such as transparencies. ● Work in the coordinate plane follows an intuitive understanding of the transformations and should involve the mapping of various polygons by changing the coordinates using addition, subtraction, and multiplication.
	9.1 Congruence and Transformations	8.G.1 Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). a. Lines are taken to lines, and line segments are taken to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. 8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	Which transformations preserve angle measures? Which transformations preserve side lengths?	● In Grade 6 students learned that when two shapes match exactly they have the same area. In Grade 7 they learn that two figures that “match up” or are put on top of each other are the same. In Grade 8, they learn the formal term of congruence and define it by using transformations. Students should also become familiar with the symbol for congruence (≅). ● Students should observe and discuss which properties of the polygons remained the same and which properties changed. Understandings should include generalizations about which transformations maintain size or maintain shape, as well as which transformations do not. ● A discussion can be had about the meaning of congruence. Initially one can use the informal definition of congruence being the same size and shape, but the discussion should eventually move toward the definition in 8.G.2 “a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.” Use the word “mapping” when discussing the overlay of two figures.

		(Include examples both with and without coordinates.)		
	9.2 Congruence and Corresponding Parts	8.G.1 Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). a. Lines are taken to lines, and line segments are taken to line segments of the same length. b. Angles are taken to angles of the same measure.		
	7.1 Angle Relationships and Parallel Lines	8.G.5 Use informal arguments to establish facts about ~~the angle sum and exterior angle of triangles, about~~ the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	What are the relationships between angles formed by parallel lines and a transversal?	● In Grade 7, students develop an understanding of the special relationships of angles and their measures (complementary, supplementary, adjacent, and vertical). Now in 8.G.5 the focus is on learning about the sum of the measures of the interior angles of a triangle and exterior angle of triangles by using transformations. ● This might be a good time to introduce vocabulary of the types of angles, such as interior, exterior, alternate interior, alternate exterior, corresponding, same side interior, and same side exterior. Students are expected to recognize but not memorize this vocabulary.
	8.4 Dilations	8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	Which transformations create proportional side lengths? What are the relationships between coordinates of figures before and after transformations? How can I use a sequence of transformations to describe two similar figures?	★ See 8.1-8.3 above. ● Introduce dilation by discussing a topic such as "How do we double the size of a wiggly curve?" After much discussion, lead students toward assigning an arbitrary point, O, on the plane, and pushing every point on the squiggly line twice as far away from O. Explain to students that this is a dilation. A dilation pushes out (or pulls in) every point of the figure from its center of dilation proportionally by the same amount. This can be easily modeled by pushing in or pulling out an image on an overhead projector or an image drawn on a flashlight. In this case the center of dilation is O, and it can be anywhere on the plane. This can also be done by copying and pasting line segments using technology such as Microsoft Word, Powerpoint, or Smartboard.

			• A dilation also has a scale factor. When doubling the size of a wiggly curve, the scale factor is 2, but a scale factor could be any number such as 1⁄2 or 3. Explain that in Grade 8 scale factors always have to be positive. Discuss what happens to a figure when the scale factor is less than one, compared to when the scale factor is greater than one. • Although students should have experiences with the center of dilation being anywhere either inside or outside the figure, the expectation for Grade 8 is that they be proficient using centers of dilations at the origin and at a vertex of an image.
9.3 Similarity and Transformations 9.4 Similarity and Corresponding Parts	8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Include examples both with and without coordinates.)		• Review that in Grade 7 students learned that similar figures have sides that are proportional and angles that are congruent. Also, in 7th grade students used to say that figures are similar if they have the same shape but different size. Now they will be defining similarity in terms of transformations. Students should also become familiar with the symbol for similarity (~).
9.5 Indirect Measurement	OMIT Lesson		
Module 10: Volume			**3 weeks**
10.1 Volume of Cylinders	8.G.9 Solve real-world and mathematical problems involving volumes of cones, cylinders, and spheres.	What are the components of formulas for volume/ How do volume formulas for different figures compare? How do I use dimensions to find the volume of a figure? How can I find missing measures of a figure when given the volume? What are the differences between linear, square, and cubic units? How can I use volume to model real-world situations?	• In Grade 7 students explored circles and the surface area and volume of right prisms. In Grade 8, they are putting the two concepts together to explore right cylinders, right cones, and spheres. The focus in this cluster should be on relationships between solids and the real-world application of volume. Not only do students need to find the volume, but they should also be able to find a missing dimension given the volume. • To develop students' spatial skills, they need practice learning how to draw three-dimensional solids such as cones, cylinders, pyramids, and spheres. • There are many times in life, where people need to represent three-dimensional solids as two-dimensional figures in

	10.2 Volume of Cones 10.3 Volume of Spheres 10.4 Finding Missing Dimensions 10.5 Volume of Composite Solids		What is the relationship between the volume of a cube and a pyramid with the same base and height? What is the relationship between the volume of a cylinder and a cone with the same base and height?	presentations using technology. Have students practice creating three-dimensional solids on technology platforms. ● Most students can be readily led to the understanding that the volume of a right rectangular prism can be thought of as the area of a "Base" times the height, and so because the area of the base of a cylinder is a circle whose area equals $\pi\pi rr2$ the volume of a cylinder is Vcylinder = $\pi r^2 h$ or $V = Bh$. Foam layers that have the height of 1 unit can be used to show how to build a cylinder of h height and reinforce Base × height as well. ● To explore the formula for the volume of a cone, use cylinders and cones with the same radius and height. Fill the cone with rice or water and pour it into the cylinder. Students will discover/experience that 3 full cones are needed to fill the cylinder. This non-mathematical demonstration of the formula for the volume of a cone, Vcone = $\frac{1}{3}\pi r^3 h$ or Vcone = $\frac{1}{3}Bh$, will help students make sense of the formula. ● Make sure to differentiate between the height of an object and slant height. ● To explore the formula for the sphere, use spheres, cylinders, and cones with the same radius, whereas the height of the cone and the cylinder must be the same as the radius, but the height of the sphere will be twice the radius. Discuss the relationships between the solids. Fill the sphere with rice or water and pour into the cylinder. Students will discover/experience that there is water remaining in the sphere. This water/rice will fill the cone. The students should see that the volume of a sphere = the volume of a cylinder + volume of a cone. Because r = h (radius = height), by using substitution, the volume of the cylinder is $\frac{3}{3}\pi r^3$, and the volume of the cone is $\frac{1}{3}\pi r^3$, so the volume of the sphere is Vsphere = $\frac{4}{3}\pi r^3$, This non-mathematical demonstration of the formula for the volume of a sphere, Vsphere = $\frac{1}{3}\pi r^3$, will help students make sense of the formula. ● Students should experience many types of real-world applications using these formulas. They should be expected to explain and justify their solutions. Some examples include the

				following: finding the amount of space left over in a can with 3 tennis balls; finding total volume in a silo; finding how much ice cream in a cone, etc.

ODE Model Curriculum

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is a model curriculum that supports the standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards. The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, possible connections between topics, and some common misconceptions.

Mathematics Grade 8 Model Curriculum with Instructional Supports

Curriculum and Instruction Guide

Module 1: Exponents and Scientific Notation

Unpacked Standards / Clear Learning Targets		
Learning Target 8.EE.1 Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions. 8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. 8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology.	**Essential Understanding** Exponents Note: In 8th Grade all exponents are integers, so all notes refer to cases using integer exponents. • $x^0 = 1$, when $x \neq 0$. • $x^{-n} = \frac{1}{x^n}$, when $x \neq 0$. • A coefficient is different than an exponent. For example, n^2 is different than 2n because n^2 means n × n and 2n means 2 × n, and n^3 is different than 3n because n^3 means n × n × n and 3n means 3 × n. • A positive exponent denotes repeated multiplication of the base. • A negative exponent denotes repeated division of the base. • A negative exponent does not change the sign of the base. Scientific Notation • Scientific notation is a mathematical expression written as a decimal number greater than or equal to one but less than 10 multiplied by a power of ten, e.g. 3.1×10^4 . • A number expressed in scientific notation that has a negative exponent is between negative one and positive one. • A number expressed in scientific notation that has a positive exponent is greater than one or less than negative one. • Powers of ten can be used to compare numbers written in scientific notation.	**Academic Vocabulary** Base Cube Exponent Power Square Decimal notation Scientific notation Standard form

I Can Statements:

- I can explain why a zero exponent produces a value of one.
- I can explain how a number raised to an exponent of I is the reciprocal of that number.
- I can explain the properties of integer exponents to generate equivalent numerical expressions.
- I can express numbers as a single digit times an integer power of 10.
- I can use scientific notation to estimate very large and/or very small quantities.
- I can compare quantities in scientific notation to express how much larger one is compared to the other.
- I can perform operations using numbers expressed in scientific notations and decimals.
- I can use scientific notation to express very large and very small quantities.
- I can interpret scientific notation that has been generated by technology.
- I can choose appropriate units when using scientific notation.

Performance Level Descriptors:

Proficient:

- Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large quantities.
- Use scientific notation to represent and compare very large and very small quantities.
- Express how many times a number written as an integer power of 10 is than another number written as an integer power of 10.
- Solve routine problems that require performing operations with numbers expressed in scientific notation, including numbers written in both decimal and scientific notation and interprets scientific notation that has been generated by technology.
- Apply the properties of integer exponents to solve mathematical problems.

Accomplished (all of Proficient +):

- Solve problems involving the conversion between decimal notation and scientific notation and the comparison of numbers written in different notations.

Advanced (all of Proficient + all of Accomplished +):

- Calculate and interpret values written in scientific notation within new and unfamiliar contexts.
- Use properties of integer exponents to order or evaluate multiple numerical expressions with integer exponents.

Prior Standard(s)		Future Standard(s)
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. 5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10. 5.NBT.7 Solve real-world problems by adding, subtracting, multiplying, and dividing decimals using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, or multiplication and division; relate the strategy to a written method and explain the reasoning used.	6.EE.1 Write and evaluate numerical expressions involving whole number exponents. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. A.APR.1 Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 8, Number 4, page 55
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

Instructional Strategies

Although students begin using whole-number exponents in Grades 5 and 6, it is in Grade 8 when students are first expected to understand, explain, and apply the properties of exponents and to extend the meaning beyond counting-number exponents.

Students should not be told these properties but rather should derive them through experience and reason. Many students who simply "memorize" the rules without understanding may confuse the rules when trying to apply them at later times. Instead students should be encouraged to discover the rules using tables, patterns, and expanded notation. As they have multiple experiences simplifying numerical expressions with exponents, these properties become natural and obvious.

Students should use multiplicative reasoning and expanded notation to gain understanding of non-positive exponents.

Another way to view the meaning of 0 and negative exponents is by applying the following principle: The properties of counting-number exponents should continue to work for integer exponents.

- Therefore, the properties for exponents can also be used to help students understand $x^0 = 1$. For example, consider the following expression and simplification: $3^0 \cdot 3^5 = 3^{0+5} = 3^5$. This computation shows that when 3^0 is multiplied by 3^5, the result should be 3^5, which implies that 3^0 must be 1. Because this reasoning holds for any base other than 0, it can be reasoned that $a^0 = 1$ for any nonzero number a.
- The properties of exponents can also help students make sense of negative exponents. To make a judgment about the meaning of 3−4, the approach is similar: $3^{-4} \cdot 3^4 = 3^{-4+4} = 3^0 = 1$. This computation shows that 3^{-4} should be the reciprocal of 3^4, because their product is 1. And again, this reasoning holds for any nonzero base. Thus, we can reason that $a^{-n} = \frac{1}{a^n}$.

In Grade 8 students should also have practice using negative numbers as bases. For example, it is very difficult for students to differentiate between -3^2 and $(-3)^2$. To help students differentiate between the two forms, encourage them to write it out in expanded form: $(-3)^2 = -3 \cdot -3$ and $-3^2 = -1 \cdot 3 \cdot 3$.

Have students identify the bases before solving problems as many students incorrectly only attribute the exponent to the nearest number. For example, instead of realizing that $(4 + 2)^3 = (4 + 2) \cdot (4 + 2) \cdot (4 + 2) = 216$, they incorrectly calculate $4 + 2^3 = 12$.
Students should also have practice using non-integer bases such as $(1.5)^3$ or $\left(\frac{3}{4}\right)^{-2}$

Sample Assessments and Performance Tasks

Reporting Category: The Number System	OST Test Specs:
Standards: 8.EE.1, 3, and 4 **Approximate Portion of Test: 20% - 25%; 11 - 13 points**	Exponents will be integers. Bases will be whole number, fractions, or decimals. Variables will be used only for unknown exponents (*Ex:* $3^2 \bullet 3^x = 3^9$) All numbers will be able to be written tin the form $a \times 10^b$ where a and b are integers and $1 \leq a < 10$. Exponents may be positive or negative. Items may use all types of rational numbers. Exponents may be positive or negative.

Instructional Resources		
8.EE.1 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Ants versus humans Extending the Definitions of Exponents, Variation 1 Raising to the zero and negative powers	8.EE.3 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Ant and Elephant Orders of Magnitude Pennies to heaven	8.EE.4 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Ants versus humans Choosing appropriate units Giantburgers Pennies to heaven

Adopted Resource	
Reveal: Lesson 1-1: Powers and Exponents Lesson 1-2: Multiply and Divide Monomials Lesson 1-3: Powers of Monomials Lesson 1-4: Zero and Negative Exponents Lesson 1-5: Scientific Notation Lesson 1-6: Compute with Scientific Notation	**ALEKS:** Exponents, Polynomials, and Radicals (ALEKS TOC): ● Product, Power, and Quotient Rules ● Negative Exponents ● Scientific Notation Whole Numbers and Integers (ALEKS TOC): ● Exponents and Order of Operations

Return to Scope and Sequence

Module 2: Real Numbers

Unpacked Standards / Clear Learning Targets		
Learning Target 8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating. 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., π^2. 8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	**Essential Understanding** Roots • The equation $x^2 = p$ has two solutions for x: $\sqrt{p}$ and $-\sqrt{p}$. For example, in describing the solutions to $x^2 = 36$, students can write $x = \pm\sqrt{36} = \pm 6$. • The $\sqrt{p}$ is defined to be the positive solution to the equation $x^2 = p$. For example, it is **not** correct to say that $\sqrt{36} = \pm 6$, instead it should be $\sqrt{36} = 6$. • $x^3 = p$ has only one solution for x: $\sqrt[3]{p}$ • The square root of 2 is irrational. Real Numbers • Every real number can be classified as repeating, terminating, or non-repeating, nonterminating. • Real numbers are either rational or irrational. • A rational number is any number that can be written as the quotient or fraction of two integers, $\frac{p}{q}$, where p is the numerator and q is the non-zero denominator. • Rational numbers when written as a decimal expansion are repeating or terminating. • Irrational numbers when written as a decimal expansion are non-repeating and nonterminating. • A number is classified by its simplest form, e.g., $\sqrt{25}$ is rational because 5 is rational. • Square roots may be negative, e.g., Both 6 and −6 are square roots of 36, and $\sqrt{36}$ means only the positive square root whereas $-\sqrt{36}$ means only the negative square root. • A negative sign cannot be inside a square root number at this grade level. • The roots of perfect squares and perfect cubes of whole numbers are rational. • Square roots of whole numbers that are not perfect squares are irrational.	**Academic Vocabulary** Cube root Irrational Rational Root Square root Decimal Irrational number Rational number Repeating decimal Square root Terminating decimal

	• Cube roots of whole numbers that are not perfect cubes are irrational. • Given two distinct numbers, it is possible to find both a rational and an irrational number between them.	

I Can Statements:

- I can use square root and cube root symbols as inverse operations to represent solutions to equations of the form x^2= p and x^3 =p, where p is a positive rational number.
- I can evaluate square roots of small perfect squares through $12^2 = 144$.
- I can evaluate cube roots of small perfect cubes: cube root of 1 through the cube root of 125.
- I can understand that the square root of 2 is irrational.
- I can define rational and irrational numbers.
- I can show that the decimal expansion of rational numbers repeats eventually.
- I can convert a decimal expansion, which repeats eventually into a rational number.
- I can show that every number has a decimal expansion.
- I can approximate irrational numbers as rational numbers.
- I can approximately locate and order irrational numbers on a number line.
- I can estimate the value of expression involving irrational numbers using rational approximations without a calculator.

Performance Level Descriptors:

Proficient:
- Evaluate square roots of small perfect squares
- Calculate the cube root of small perfect cubes
- Use square root and cube root symbols to represent solutions of the form x^2 = p and x^3 = p, where p is a positive rational number.
- Use the properties of natural number exponents to generate equivalent numerical expressions.
- Apply the properties of natural number exponents to solve simple mathematical problems.
- Identify square roots of non-square numbers and pi as irrational numbers.

Accomplished (all of Proficient +):
- Place irrational numbers on a number line in an abstract setting using variables.
- Use square root and cube root symbols to represent solutions to real-world problems resulting from equations of the form x^2 = p and x^3 = p.

Advanced (all of Proficient + all of Accomplished +):
- Explain how square roots and cube roots relate to each other and to their radicands.
- Notice and explain the patterns that exist when writing rational numbers (repeating decimals) as fractions.
- Explain how to get more precise approximations of square roots.

- Identify between which two whole number values a square root of a non-square number is located.
- Identify rational and irrational numbers and convert less familiar rational numbers (repeating decimals) to fraction form
- Place irrational numbers on a number line

Prior Standard(s)	Future Standard(s)	
6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. 7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.	8.G.9 N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. A.REI.4ab 4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. b. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 8, Number 4, page 55
- Ohio's K-8 Learning Progressions, The Number System, pages 16-17
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

Instructional Strategies

In previous grades, students become familiar with rational numbers called decimal fractions. In Grade 7, students carry out the long division and recognize that the remainders may repeat in a predictable pattern—a pattern creates the repetition in the decimal representation (see 7.NS.2.d). In Grade 8, they explore its occurrence.

Ask students what will happen in long division once the remainder is 0. They can reason that the long division is complete, and the decimal representation terminates.

However, if the remainder never becomes 0, then the remainder will repeat in a cyclical pattern. The important understanding is that students can see that the pattern will continue to repeat.

Explore differences between terminating and repeating decimals.

To help students build a conceptual understanding, connect roots to area and volume models where the area and volume are the radicand and the solution is the length of the side of the model.

Another way to explain it is that the area and volume of the square or cube represents n; and the square and cube's side length is represented by $\sqrt{n}$ and $\sqrt[3]{n}$ respectively. Have students use geoboards, square tiles, graph paper, or unit cubes to build squares and cubes reviewing exponents in the process. Problems such as the Painted Cube Problem can then be modified to extend to square and cube roots.

Also, provide practical opportunities for students to flexibly move between forms of squared and cubed numbers. For example, If $3^2 = 9$ then $\sqrt{9} = 3$. This flexibility should be experienced symbolically and verbally with manipulatives and with drawings.

It could be appropriate to use Venn diagrams/set diagrams or flowcharts to show the relationships among real, rational, irrational numbers, integers, and natural numbers. The diagram should show that all real numbers are either rational or irrational.

Students should come to the understanding that (1) every rational number has a decimal representation that either terminates or repeats and (2) every terminating or repeating decimal is a rational number. Then, they can use that information to reason that on the number line, irrational numbers must have decimal representations that neither terminate nor repeat.

In previous grades, students learned processes that can be used to locate any rational number on the number line: Divide the interval from 0 to 1 into b equal parts; then, beginning at 0, count out a of those parts. Now they can use similar strategies to locate irrational numbers on a number line.

Use an interactive number line to allow students to see how a number line can be infinitely divided. One resource is Zoomable Number Line by MathisFun.

Although students at this grade do not need to be able to prove that $\sqrt{2}$ is irrational, they minimally need to know that $\sqrt{2}$ is irrational (see 8.EE.2), which means that its decimal representation neither terminates nor repeats. Nonetheless, they should be able to approximate irrational numbers such as $\sqrt{2}$ without using the square root key on the calculator.

The $\sqrt{2}$ caused Greek Mathematicians many problems, for although they could construct it using tools, but they could not measure it. Integrating math history into the lesson may be interesting for some students.

Have students do explorations where precision matters. Discuss situations where precision is vital and other situations where reasonableness is more vital than precision. Although learning about significant digits formally takes place in high school, it is appropriate to talk about how intermediate rounding affects precision. The display of irrational numbers on a calculator could also be used for a discussion point. Another discussion could take place comparing results using the pi button compared to the typical approximation of 3.14.

The concept of precision should also be tied to real-world contexts. For example, we do not buy 3.5 apples, but we may buy 3.5 lbs of ground beef, so rounding to a whole number is a precise number in terms of buying apples. Therefore, being precise should relate to the real-world context of the problems.

Sample Assessments and Performance Tasks

Reporting Category: The Number System **Standards: 8.NS.1 and 2; 8.EE.2** **Approximate Portion of Test: 20% - 25%; 11 - 13 point**	**OST Test Specs:** ● Irrational numbers are limited to expressions involving π or radicals. ● Irrational expressions will only use one operation. ● Items will include square roots and cube roots. ● Radicands will be positive rational numbers. ● Items may require the identification of both solutions of the equation $x^2 = p$.

Instructional Resources

8.EE.2 Better Lesson Shmoop Khan Academy Videos Painted Cube or Birthday Problem • Painted Cubes from the Mathematics Centre • Painted Sides of a Cube from The University of Georgia • Painted Cube from NRICH Enriching Mathematics • The Painted Cube by CollecEDNY	8.NS.1 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Converting Decimal Representations of Rational Numbers to Fraction Representations Converting Repeating Decimals to Fractions Identifying Rational Numbers Repeating or Terminating?	8.NS.2 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Comparing Rational and Irrational Numbers Irrational Numbers on the Number Line Placing a square root on the number line

Adopted Resource	
Reveal: Lesson 2-1: Terminating and Repeating Decimals Lesson 2-2: Roots Lesson 2-3: Real Numbers Lesson 2-4: Estimate Irrational Numbers Lesson 2-5: Compare and Order Real Numbers	**ALEKS:** Exponents, Polynomials, and Radicals (ALEKS TOC): ● Square Roots and Irrational Numbers ● Higher Roots and Nonlinear Equations Decimals (ALEKS TOC): ● Venn Diagrams and Sets of Rational Numbers

Return to Scope and Sequence

Module 3: Solve Equations with Variables on Each Side

Unpacked Standards / Clear Learning Targets		
Learning Target 8.EE.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	**Essential Understanding** • Linear equations can have no solutions, one solution, or infinitely many solutions. • Linear equations are solved by using inverse operations. • Linear equations that are equivalent to x = a have one solution. • Linear equations that are equivalent to a = a have infinitely many solutions. • Linear equations that are equivalent to a = b have no solutions.	**Academic Vocabulary** Distributive property Infinite solutions Inverse operation Like terms No solution One variable equation

I Can Statements:

- I can solve equations in one variable with variables on both sides of the equation.
- I can give examples of linear equations in one variable with one solution.
- I can give examples of linear equations in one variable with infinitely many solutions.
- I can give examples of linear equations in one variable with no solution.
- I can solve linear equations with rational number coefficients.
- I can solve equations whose solutions require expanding expressions using the distributive property and/or collecting like terms.

Performance Level Descriptors:		
Proficient: • Solve straightforward one or two step linear equations with integer coefficients. • Solve straightforward multi-step linear equations with rational coefficients • Solve routine multi-step linear equations with rational coefficients and variables on both sides and provide examples of equations that have one solution, infinitely many solutions, or no solutions	Accomplished (all of Proficient +): • Strategically choose and use procedures to solve linear equations in one variable. • Justify why an equation has one solution, infinitely many solutions, or no solution.	Advanced (all of Proficient + all of Accomplished +): •

Prior Standard(s)	Future Standard(s)
7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers.	**A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **A,REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 8, Number 1, pages 50-51
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

Instructional Strategies

In Grade 7, students learned integer operations for the first time. They also applied the properties of operations when solving two-step equations and inequalities. Now students build on the fact that solutions maintain equality and that equations may have only one solution, many solutions, or no solutions at all.

Properties of Operations Table 3 on page 97 of Ohio's Learning Standards in Mathematics states the Properties of Operations and Table 4 states the Properties of Equality. Teachers should be using the correct terminology to justify steps when performing operations and solving equations.

Students incorrectly think that the variable is always on the left side of the equation. Give students situations where the variable is on the right side of the equation. Emphasize using the Symmetric Property of Equality if students wish to flip the variable to the other side of the equal sign.

Equation-solving in Grade 8 should involve multi-step problems that require the use of the distributive property, collecting like terms, rational coefficients, and variables on both sides of the equation.

In Grade 7, students may have used a pan balance, number lines, or algebra tiles to solve two-step equations. Eighth grade students could review these models and build upon them. For example, algebra tiles may help prevent student errors such as incorrectly combining like terms on opposite sides of the equations.

When not using models, some students benefit from drawing a vertical line through the equals sign to separate the two sides of the equation.

Connect mathematical analysis with real-life events by using contextual situations when solving equations. Students should experience—

- analyzing and representing contextual situations with equations;
- identifying whether there is one solution, no solutions, or infinitely many solutions; and then
- solving the equations to prove conjectures about the solutions.

Sample Assessments and Performance Tasks

Reporting Category: Expressions and Equations **Standards: 8.EE.7** **Approximate Portion of Test: 20% - 29%; 11 - 15 points**	**OST Test Specs:** ● Items may use all types of rational numbers. ● Equations can be more complex than the forms $px + r = q$ and $p(x + r) = q$.

Instructional Resources	
Better Lesson Shmoop Khan Academy Videos Dan Meyer Activity Ditch Diggers	Illustrative Mathematics Coupon versus discount Sammy's Chipmunk and Squirrel Observations Solving Equations The Sign of Solutions

Adopted Resource	
Reveal: Lesson 3-1: Solve Equations with Variables on Each Side Lesson 3-2: Write and Solve Equations with Variables on Each Side Lesson 3-3: Solve Multi-Step Equations Lesson 3-4: Write and Solve Multi-Step Equations Lesson 3-5: Determine the Number of Solutions	**ALEKS:** Equations and Inequalities (ALEKS TOC): ● Equations with Variables on Both Sides ● Applications of Equations ● Multi-Step Equations ● The Distributive Property ● Simplifying Algebraic Expressions

Return to Scope and Sequence

Module 4: Linear Relationships and Slope

Unpacked Standards / Clear Learning Targets		
Learning Target **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. **8.EE.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.	**Essential Understanding** • The slope is a constant ratio between the rise and the run for any two points on a line. • A graph of a proportional relationship is a line that passes through the origin. • Only the slope, m, of the equation y = mx represents a proportional relationship. • Slope is represented by m in the equation y = mx or y = mx + b. • Corresponding angles in similar right triangles are equal. • Corresponding sides of similar triangles are proportional. • A line in the form y = mx and intersects the origin. • A line in the form y = mx + b intersects the y-axis at (0, b) with b	**Academic Vocabulary** Constant of proportionality Constant of variation Direct variation Initial value Linear relationship Rate of change Proportional relationship Slope Y-intercept Origin Corresponding angles Corresponding sides Similar triangles

	being the y-intercept. *Note: A linear function has neither a slope nor a y-intercept. But the graph of a linear function has both.* • A relationship between two variables can be represented as a graph, table, equation, or verbal description.	Intersect

I Can Statements:

- I can determine the rate of change by the definition of slope.
- I can graph proportional relationships from data or equations.
- I can compare/contrast slope and rate of change.
- I can compare two different proportional relationships represented in different ways.
- I can interpret the unit rate of proportional relationships as the slope of a graph.

Performance Level Descriptors:

Proficient:

- Graph proportional relationships, interpreting the unit rate as the slope
- Graph proportional relationships, interpreting the unit rate as the slope and compare two different proportional relationships using the same representation
- Graph proportional relationships, interpreting the unit rate as the slope and compare two different proportional relationships using different representations
- Determine the slope of a line given a graph

Accomplished (all of Proficient +):

- Apply understanding of slope to solve routine problems graphically and algebraically

Advanced (all of Proficient + all of Accomplished +):

- Apply understanding of slope to solve non-routine problems graphically and algebraically

Prior Standard(s)	Future Standard(s)
7.RP.2 Recognize and represent proportional relationships between quantities.	**8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 8, Number 1, page 50-51
- Ohio's K-8 Critical Areas of Focus, Grade 8, Number 3, pages 53-54
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

Instructional Strategies

Students in Grade 7 represented proportional relationships as equations such as $y = kx$ or $t = pn$. They also graphed proportional relationships, discovering that a graph of a proportion must go through the origin, and that in the point (1, r), r is the unit rate. Now in Grade 8, the unit rate of a proportion is used to introduce "the slope" of the line.

Students need to make connections between the different representations (equations, tables, graphs) in order to come to a unified understanding that the different representations are in essence different ways of modeling the same information.

Explicit connections need to be made between the multiplicative factor, the slope, scale factor, and an increment in a table.

To reinforce the relationships between the x and the y, students should continually name quantities for the real-world problem they represent. They should also identify the independent and dependent variables.

By using coordinate grids and various sets of similar triangles, students can prove that the slopes of the corresponding sides are equal, thus making the unit rate or rate of change equal.

Use graphing utilities such as Desmos to show the lines in the form of $y = mx + b$ as vertical translations of the equation $y = mx$.

Sample Assessments and Performance Tasks

Reporting Category: Equations and Expressions **Standards: 8.EE.5 and 6** **Approximate Portion of Test: 20% - 29%; 11 - 15 points**	**OST Test Specs:** Items may use all types of rational numbers. Items pertain only to direct proportional relationships. All triangles will be right triangles and in a coordinate grid.

Math 8

Instructional Resources	
8.EE.5 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Coffee by the Pound Comparing Speeds in Graphs and Equations Peaches and Plums Sore Throats, Variation 2 Stuffing Envelopes Who Has the Best Job?	**8.EE.6** Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Slopes Between Points on a Line
Adopted Resource	
Reveal: Lesson 4-1: Proportional Relationships and Slope Lesson 4-2: Slope of a Line Lesson 4-3: Similar Triangles and Slope Lesson 4-4: Direct Variation Lesson 4-5: Slope-Intercept Form Lesson 4-6: Graph Linear Equations	**ALEKS:** Graphs, Functions, and Sequences (ALEKS TOC): ● Proportional Relationships ● Slope ● Direct and Inverse Variation ● Equations of Lines ● Tables and Graphs of Lines ● Ordered Pairs Ratios, Proportions, and Measurement (ALEKS TOC): ● Proportions ● Ratios and Unit Rates ● Similar Figures

Return to Scope and Sequence

Module 5: Functions

Unpacked Standards / Clear Learning Targets		
Learning Target **8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. *Function notation is not required. **8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **8.F.3** Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	**Essential Understanding** • A function is a rule that assigns each input exactly one output. • The graph of a function is a set of ordered pairs consisting of an input and a corresponding output. • Functions can be represented as an equation, graph, table, and verbal description. • Properties of graphs of linear functions include slope/rate of change, y-intercept/initial value, x-intercept, where the slope is increasing, constant, or decreasing. • A vertical line has an undefined slope, where y is not a function of x. • A graph of a linear function is a non-vertical straight line. • A nonlinear function is a function whose graph is not a straight line. • A table represents a linear function when constant differences between input values produce constant differences between output values. • Linear functions have a constant rate of change. • Some functions are not continuous	**Academic Vocabulary** Domain Input Output Range Initial value Linear function Rate of change Slope Y-intercept

I Can Statements:

- I can define a function.
- I can determine if an equation represents a function.
- I can apply a function rule for any input that produces exactly one output.
- I can generate a set of ordered pairs from a function and graph the function.
- I can recognize the equation y=mx+b is the equation of a function whose graph is a straight line where m is the slope and b is the y-intercept
- I can provide examples of nonlinear functions using multiple representations (tables, graphs, and equations).
- I can compare the characteristics of linear and nonlinear functions using various representations.
- I can determine the rate of change (slope) and initial value (y-intercept) from two (x,y) values, a verbal description, values in a table, or graph.
- I can construct a function to model a linear relationship between two quantities.

- I can relate the rate of change and initial value to real world quantities in a linear function in terms of the situation modeled and in terms of its graph or a table of values.

Performance Level Descriptors:

Proficient:
- Identify whether a relation is a function from a graph or a mapping
- Given tables of ordered pairs, determine if the relation is a function
- Complete a table to show a relation that is or is not a function
- Compare properties (i.e. slope, y-intercept, values) of two functions in a graph
- Compare properties (i.e. slope, y-intercept, values) of two functions represented in the same way (algebraically, graphically, or verbal descriptions)
- Compare properties (i.e. slope, y-intercept, values) of two functions each represented in a different way (algebraically, graphically, numerically in tables, or verbal descriptions)
- Given a straight forward qualitative description of a functional relationship between two quantities, sketch a graph
- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values
- Construct a function to model a linear relationship between two quantities

Accomplished (all of Proficient +):
- Justify whether two functions represented in different ways are equivalent or not by comparing their properties

Advanced (all of Proficient + all of Accomplished +):
- Explain why a function is linear or nonlinear
- Interpret qualitative features of a function in a context
- Strategically and efficiently choose different ways to represent functions in solving a variety of problems

Prior Standard(s)	Future Standard(s)		
7.RP.2 Recognize and represent proportional relationships between quantities. **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. **8.EE.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.	**A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **F.BF.1** Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from context. **F.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. **F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).	**F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions. **F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	**F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically **F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context. **S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **G.CO.2** 2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 8, Number 2, page 52
- Ohio's K-8 Learning Progressions, Functions, page 20

Instructional Strategies

Students should be expected to reason from a context, a graph, or a table, after first being clear which set represents the input (e.g., independent variable) and which set is the output (e.g., dependent variable). When a relationship is not a function, students should produce a counterexample: an "input value" with at least two "output values." If the relationship is a function, the students should explain how they verified that for each input there was exactly one output.

In Grade 6 students explored independent and dependent variables, and how the dependent variable changes in relation to the independent variable. In Grade 8 students need to continue identifying the independent and dependent variables in functions. Students need practice justifying the relationship between the independent and dependent variable.

In Grade 6 students explored independent and dependent variables, and how the dependent variable changes in relation to the independent variable. In Grade 8 students need to continue identifying the independent and dependent variables in functions. Students need practice justifying the relationship between the independent and dependent variable.

The standards explicitly call for exploring functions numerically, graphically, verbally, and algebraically. For fluency and flexibility in thinking, students need experiences translating among these different representations.

Students need experience translating among the different representations using different functions. For example, they should be able to determine which function has a greater slope by comparing a table and a graph.

Students need to compare functions using the same representation. For example, within a real-world context, students compare two graphs of linear functions and relate the graphs back to its meaning within the context and its quantities. Students should work with graphs that have a variety of scales including rational numbers.

In Grade 8, the focus is on linear functions, and students begin to recognize a linear function from its form $y = mx + b$ knowing that $y = mx$ as a special case of a linear function. Students also need experiences with nonlinear functions. This includes functions given by graphs, tables, or verbal descriptions but for which there is no formula for the rule.

When plotting points and drawing graphs, students should develop the habit of determining, based upon the context, whether it is reasonable to "connect the dots" on the graph. In some contexts, the inputs are discrete, and connecting the dots is incorrect. For example, if a function is used to model the height of a stack of n paper cups, it does not make sense to have 2.3 cups.

Sample Assessments and Performance Tasks

Reporting Category: Functions **Standards: 8.F.1 - 5** **Approximate Portion of Test: 20% - 29%; 11 - 15 points**	**OST Test Specs:** Function notation is not permitted. Nonlinear relations may be included for the purpose of identifying a function. Functions will be linear. Context may require the graphing of discrete linear functions. Axes can be numbered with scales other than 1.

	Graphs may display linear and/or nonlinear relationships. Graphs are described from left to right. Graphs may refer to quantitative or qualitative measures (e.g., the axes of graphs may or may not have scales). Functional relationships will be continuous.

Instructional Resources		
Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Foxes and Rabbits Function Rules Introducing Functions Pennies to heaven The Customers US Garbage, Version 1	Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Battery Charging	Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Introduction to Linear Functions

Better Lesson Shmoop Khan Academy Videos Dan Meyer Activity 25 Billion Apps Illustrative Mathematics Baseball Cards Chicken and Steak, Variation 1 Chicken and Steak, Variation 2 Delivering the Mail, Assessment Variation Distance across the channel Downhill High School Graduation Video Streaming	Better Lesson Shmoop Khan Academy Videos Dan Meyer Activity Joulies Illustrative Mathematics Bike Race Distance Riding by the Library Tides

Adopted Resource	
Reveal: Lesson 5-1: Identify Functions Lesson 5-2: Function Tables Lesson 5-3: Construct Linear Functions Lesson 5-4: Compare Functions Lesson 5-5: Nonlinear Functions Lesson 5-6: Qualitative Graphs	**ALEKS:** Graphs, Functions, and Sequences (ALEKS TOC): ● Introduction to Functions ● Tables and Graphs of Lines ● Graphs of Functions ● Applications

Return to Scope and Sequence

Module 11: Scatter Plots and Two-Way Tables

Unpacked Standards / Clear Learning Targets		
Learning Target **8.SP.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering; outliers; positive, negative, or no association; and linear association and nonlinear association. (GAISE Model, steps 3 and 4) **8.SP.2** Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (GAISE Model, steps 3 and 4) **8.SP.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. (GAISE Model, steps 3 and 4) **8.SP.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.	**Essential Understanding** **Quantitative (numerical) variables** • Scatterplots are used for bivariate quantitative data. • When two variables are represented on a scatterplot, an association may exist. • An association between two variables can be seen in the pattern created by the data: o clusters; o positive, negative, or no association; and/or o linear or nonlinear association. • Outliers are bivariate points that do not fit the trend. • When a scatterplot suggests a linear association, a line can be informally fitted to the data. • Closeness of data points to the line can be judged visually. • When looking for a linear association, a line takes all of the points into consideration, and the prediction is based on an overall pattern rather than just one or two points. • The slope and y-intercept describe the linear association between two variables. • Linear functions can be used to describe contextual problems	**Academic Vocabulary** Bivariate data Scatter plot Line of best fit Linear association Measurement data Negative association Nonlinear association Positive association Clustering Outlier Slope Causal factors Causation Correlation Frequency Two-way table

	through the following: o interpreting slope and intercept; and/or o making predictions. **Categorical variables** • Two-way tables are used for bivariate categorical data. • Two-way tables are used to display frequencies and relative frequencies. • Relative frequencies can be used to describe possible associations. • If row (or column) relative frequencies in the table are the same, there is little or no association. • If row (or column) relative frequencies in the table are different, there is some evidence of association.	

I Can Statements:

- I can describe patterns such as clustering, outliers, positive or negative association, and nonlinear association.
- I can construct and interpret scatter plots for bivariate (two different variables such as distance and time) measurement data to investigate patterns of association between two quantities.
- I can show how straight lines are used to model relationships between two quantitative variables and assess how well they fit the data.
- I can interpret the meaning of the slope and intercept of a linear equation in terms of the situation.
- I can recognize patterns shown in comparison of two sets of data.
- I can show how to construct a two-way table.
- I can interpret the data in the two-way table to recognize patterns.
- I can use relative frequencies of the data to describe relationships (positive, negative, or no correlation).

Performance Level Descriptors:

Proficient:	Accomplished (all of Proficient +):	Advanced (all of Proficient + all of Accomplished +):
• Construct a scatter plot • Construct a scatter plot and describe the pattern as positive, negative or no relationship • Describe patterns in scatterplots for routine contexts, such as: clustering, outliers, positive or negative association, linear association, and/or nonlinear association	• Compare more than one trendline for the same scatter plot • Create and use a linear model based on a set of bivariate data to solve a problem in a routine context	• Compare more than one trendline for the same scatter plot and justify the best one • Construct and interpret scatter plots for bivariate measurements data to investigate patterns of association between two quantities • Create and use a linear model based in a set of bivariate data to solve problems in a variety of non-routine contexts

• Recognize a straight line can be used to describe a linear association on a scatter plot • Identify the slope and y-intercept of a linear model on a scatter plot • Draw a straight line on a scatter plot that closely fits the data points • Construct a two-way table of categorical data • Interpret and describe relative frequencies for possible associations from a two-way table representing a routine situation	• Interpret, describe, and compare relative frequencies to identify patterns of association in given contexts

Prior Standard(s)	Future Standard(s)	
6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	**S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. **S.ID.6bc** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. b. Informally assess the fit of a function by discussing residuals. c. Fit a linear function for a scatterplot that suggests a linear association.	**S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **S.ID.8** Compute (using technology) and interpret the correlation coefficient of a linear fit. **S.ID.9** Distinguish between correlation and causation. **S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 8, Number 1, pages 50-51
- Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22-23
- GAISE Model, pages 14 – 15
    - o Focus of 7th grade is Level A – B, pages 22-59

Instructional Strategies

Building on the study of statistics using univariate data in Grades 6 and 7, students are now ready to study bivariate data. They will extend their descriptions and understanding of variation to the graphical displays of bivariate data.

Explain to students *uni* means one, *bi* means two, and *variate* means variable, so univariate is data using one variable and bivariate is data using two variables.

Eighth graders can design and conduct nonrandom sample surveys; although they should begin to start thinking informally about random selection and what kind of sample best represents a population. They may also do comparative experiments.

Scatterplots are the most common form of representations displaying bivariate data in Grade 8. Provide scatterplot of linear data and have students practice informally finding the trend line. Students could be given a scatterplot and a spaghetti noodle to determine the "best fit." Discussion should include "What does it mean for a data point to be above the line?" or "What does it mean for it to be below the line?"

By changing the data slightly, students can have a rich discussion about the effects of the change on the graph. The study of the trend line ties directly to the algebraic study of slope and y-intercept. Students should interpret the slope and y-intercept of the trend line in the context of the data. Then students can make predictions based on the trend line. Give students a variety of data sets that intersect the y-axis at various points, so students do not mistakenly think that all trend lines must go through the origin.

After a trend line is fitted through the data, the slope of the line is approximated and interpreted as a rate of change, in the context of the problem. If the slope is positive, then the two variables are positively associated. Similarly if the slope is negative, then the two variables are negatively associated. Students should also be exposed to data that do not have an association.

Students should create and interpret scatterplots, focusing on outliers, positive, or negative association, linearity, or curvature. Assuming the data are linear, students should informally draw a trend line on the scatterplot and informally evaluate the strength of fit. They should be able to interpret visually how well the trend line fits the "cloud" of points.

To move students from Level A to Level B, questions should move from "Is there an association?" to "How strong is the association?" The Quadrant Count Ratio (QCR) can help students informally determine the strength between two variables. This is an important building block in building the conceptual understanding of the correlation coefficient in high school.

Students may believe bivariate data is only displayed in a scatterplot. The standard 8.SP.4 provides the opportunity to display bivariate, categorical data in a table.

Types of Frequencies:

- Frequency Table
- Relative Frequency Table
- Marginal Frequency
- Joint Frequency
- Conditional Frequency

Sample Assessments and Performance Tasks	
Reporting Category: Expressions and Equations **Standards: 8.SP.1 - 4** **Approximate Portion of Test:20% - 29%; 11 - 15 points**	**OST Test Specs:** Items may use all types of rational numbers. Axes can be numbered with scales other than 1. The trend/association will be linear. Judgment about the association and linear fit will be based solely on visual inspection; calculations will not be required. Items may use all types of rational numbers. Only linear equations are used. Data are required for all items. Only categorical variables are used. Rows and columns are limited to 2 categories each. Total and subtotal cells may or may not be given.

Instructional Resources	
8.SP.1 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Animal Brains Birds' Eggs Hand span and height Texting and Grades I	**8.SP.2** Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Animal Brains Birds' Eggs Laptop Battery Charge
8.SP.3 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics US Airports, Assessment Variation	**8.SP.4** Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Music and Sports What's Your Favorite Subject?

Adopted Resource	
Reveal: Lesson 11-1: Scatter Plots Lesson 11-2: Draw Lines of Fit Lesson 11-3: Equations for LInes of Fit Lesson 11-4: Two-Way Tables Lesson 11-5: Associations in Two-Way Tables	**ALEKS:** Data Analysis and Probability (ALEKS TOC): ● Scatter Plots and Lines of Best Fit ● Frequency Tables Graphs, Functions, and Sequences (ALEKS TOC): ● Tables and Graphs of Lines ● Equations of Lines

Return to Scope and Sequence

Module 6: Systems of Linear Equations

Unpacked Standards / Clear Learning Targets		
Learning Target **8.EE.8** Analyze and solve pairs of simultaneous linear equations graphically. a. Understand that the solution to a pair of linear equations in two variables corresponds to the point(s) of intersection of their graphs, because the point(s) of intersection satisfy both equations simultaneously. b. Use graphs to find or estimate the solution to a pair of two simultaneous linear equations in two variables. Equations should include all three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to pairs of linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. (Limit solutions to those that can be addressed by graphing.)	**Essential Understanding** • Pairs of linear equations can have no solutions, one solution, or infinitely many solutions. • Pairs of linear equations in two variables that intersect at one point have one solution. • Pairs of linear equations in two variables that are parallel have no solutions. • Pairs of linear equations in two variables that have all points in common have infinitely many solutions. • The solution(s) to a pair of linear equations in two variables make both equations true. • A solution to a pair of linear equations in two variables is often written as an ordered pair.	**Academic Vocabulary** Linear equation Linear function Solution System of equations

I Can Statements:

- I can identify the solution(s) to a system of two linear equations in two variables as the point(s) of intersection of their graphs.
- I can describe the point(s) of intersection between two lines as the point(s) that satisfy both equations simultaneously.
- I can use Demos to solve and verify the solution to a system of equations.

Performance Level Descriptors:

Proficient:	Accomplished (all of Proficient +):	Advanced (all of Proficient + all of Accomplished +):
• Solve a system of simple linear equations by inspection and graphically • Find or estimate the solution to a system of linear equations by graphing	• Use the graph of a system of linear equations to represent, analyze and solve a variety of problems	• Strategically and efficiently use graphs of systems of linear equations to represent, analyze and solve a variety of problems

Prior Standard(s)	Future Standard(s)	
6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.	**A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. **A.REI.5** Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **A.REI.6** Solve systems of linear equations algebraically and graphically. **A.REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	**A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. **G.GPE.5** Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.

Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 8, Number 1, pages 50-51]
- [Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19]

Instructional Strategies

This cluster builds on the informal understanding of slope, students gained from graphing unit rates and proportional relationships in grades 6 and 7. It also builds upon the stronger, more formal understanding of slope and the relationship between two variables from 8.EE.5-6 and 8.F.4-5.

Students will use graphing to solve pairs of simultaneous linear equations. Beginning work should involve pairs of equations with solutions that are ordered pairs of integers, making it easier to locate the point of intersection. Although students should also be able to approximate solutions that do not fall evenly onto the intersection of grid squares.

Provide opportunities for students to see and compare simultaneous linear equations in forms other than slope-intercept form ($y = mx + b$). Students may solve pairs of simultaneous linear equations by inspection, by graphing using slope-intercept form, or by graphing using tables of values.

Students should be able to solve simple cases by inspection. For example, $x + y = 3$ and $x + y = 5$ has no solution because $x + y$ cannot equal both 3 and 5.

Students should have practice working with graphs that have a variety of scales including fractions and decimals.

Students have the tendency to see the intersection point of a graphed system of equations and round it to the nearest grid line cross-section. Emphasize to students that they can sometimes find intersections in the middle of the grid squares that allow for approximations to be more precise.

Students could also investigate pairs of simultaneous equations using graphing calculators or online graphing resources. They could be asked to explain verbally and in writing what, in the equation and situation, makes lines shift to different locations on the graph.

Graphing pairs of linear equations should be introduced through contextual situations relevant to eighth graders, so students can create meaning. They should explore many tasks for which they must write and graph pairs of equations with different slopes and y-intercepts. This should lead to the generalization that finding one point of intersection is the single solution to the pair of linear equations.

Students should relate the solution to the context of the problem, commenting on the reasonableness of their solution.

Emphasize that the solution must satisfy both equations.

Sample Assessments and Performance Tasks

Reporting Category: Equations and Expressions	**OST Test Specs:** ● Items may use all types of rational numbers.

Standards: 8.EE.8 Approximate Portion of Test: 20% - 29%; 11 - 15 points	● For 8b and 8c, items are not required to have a graph, but the equations will be easily graphable ● Axes can be numbered with scales other than 1

Instructional Resources	
Better Lesson Shmoop Khan Academy Videos Dan Meyer Activities Coin Counting Playing Catch Up	Illustrative Mathematics Cell Phone Plans Fixing the Furnace Folding a Square into Thirds How Many Solutions? Kimi and Jordan
Adopted Resource	
Reveal: Lesson 6-1: Solve Systems of Equations by Graphing Lesson 6-2: Determine Number of Solutions Lesson 6-5: Write and Solve Systems of Equations	**ALEKS:** Graphs, Functions, and Sequences (ALEKS TOC): ● Systems of Equations (graphing and solutions ONLY - no substitution, no elimination) ● Tables and Graphs of Lines

Return to Scope and Sequence

Module 7: Triangles and Pythagorean Theorem

Unpacked Standards / Clear Learning Targets		
Learning Target 8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, ~~about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.~~ 8.G.6 Analyze and justify an informal proof of the Pythagorean Theorem and its converse. 8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in	**Essential Understanding** Angle Relationships • The sum of the measure of the interior angles of a triangle is 180 degrees. • Any exterior angle of a triangle is congruent to the sum of the measures of the two remote interior angles of the triangle. The Pythagorean Theorem • Side lengths need not be represented by rational numbers.	**Academic Vocabulary** Adjacent angle Angle Sum Converse Hypotenuse Leg Pythagorean Theorem Square root

two and three dimensions. 8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	• The Pythagorean Theorem states that in a right triangle the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. • The Pythagorean Theorem is represented symbolically by $(\text{leg a})^2 + (\text{leg b})^2 = \text{hypotenuse}^2$ • The Pythagorean Theorem only applies to right triangles. • The hypotenuse is the longest side of a right triangle and opposite the right angle. • The legs of a right triangle are perpendicular. • The distance between two non-vertical or non-horizontal points in the coordinate plane can be determined by creating a right triangle with vertical and horizontal legs and applying the Pythagorean Theorem. • Converse is the reverse order of a hypothesis. • The converse of the Pythagorean Theorem states that if the sum of the squares of the legs is equal to the square of the hypotenuse then the triangle is a right triangle • The converse is used to determine whether or not a triangle is a right triangle • The converse of the Pythagorean Theorem works because of the uniqueness of a triangle.	

I Can Statements:

- I can define and identify transversals.
- I can identify angles created when a parallel line is cut by transversal (alternate interior, alternate exterior, corresponding, vertical, supplementary, etc.).
- I can justify that the sum of the interior angles equals 180. *For example, arrange three copies of the same triangle so that the three angles appear to form a line.*
- I can use Angle Angle Criterion to prove similarity among triangles. (Give an argument in terms of transversals why this is so).
- I can identify the legs and hypotenuse of a right triangle.
- I can explain a proof of the Pythagorean Theorem.
- I can explain a proof of the converse of the Pythagorean Theorem.
- I can recall the Pythagorean Theorem and its converse in order to apply it to real world and mathematical problems (2, 3 dimensional).
- I can recall the Pythagorean Theorem and its converse and relate it to any two distinct points on a graph.

Performance Level Descriptors:

Proficient:	Accomplished (all of Proficient +):	Advanced (all of Proficient + all of Accomplished +):
• Use the Pythagorean Theorem to calculate the hypotenuse in mathematical problems • Calculate unknown side lengths using the Pythagorean Theorem given a picture of a right triangle • Apply the Pythagorean Theorem to real-world situations that can be modeled in two dimensions to determine unknown side lengths • Apply the Pythagorean Theorem to find the distance between two points in a coordinate system with the right triangle drawn • Understand and explain a proof of the Pythagorean Theorem and its converse • Determine missing angle measures in triangles with exterior angles and/or angles formed by parallel lines cut by a transversal	• Apply the Pythagorean Theorem in multi-step mathematical and real-world problems in two and three dimensions • Understand and explain the proof of the Pythagorean Theorem and its converse in multiple ways • Give an informal argument that a triangle can only have one 90° angle	• Solve a variety of real-world and mathematical problems involving the angles in triangles and those formed by when parallel lines are cut by a transversal, and give informal arguments

Prior Standard(s)	Future Standard(s)		
6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **7.G.6** Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	**F.TF.3** Use special triangles to determine geometrically the values of sine, cosine, tangent for π /3, π /4, and π /6, and use the unit circle to express the values of sine, cosine, and tangent for π − x, π + x, and 2π − x in terms of their values for x, where x is any real number. **F.TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. **G.CO.9** Prove and apply theorems about lines and angles. **G.CO.10** Prove and apply theorems about triangles.	**G.SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **G.SRT.4** Prove and apply theorems about triangles. **G.SRT.8** Solve problems involving right triangles. **G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **G.GPE.2** Derive the equation of a parabola given a focus and directrix	**G.GPE.3** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **G.GPE.4** Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. **G.GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 8, Number 3, pages 53-54]
- [Ohio's K-8 Learning Progressions, 6-8 Geometry, page 21]

Instructional Strategies

In Grade 7, students develop an understanding of the special relationships of angles and their measures (complementary, supplementary, adjacent, and vertical). Now in 8.G.5 the focus is on learning about the sum of the measures of the interior angles of a triangle and exterior angle of triangles by using transformations.

This might be a good time to introduce vocabulary of the types of angles, such as interior, exterior, alternate interior, alternate exterior, corresponding, same side interior, and same side exterior. Students are expected to recognize but not memorize this vocabulary.

In Grade 7 students should have had some practice exploring that the sum of the angles inside a triangle equal 180 degrees. Now students use transformations to prove it.

Students can create a triangle and use rotations and transformations to line up all the angles to prove that the sum of the interior angles of a triangle equals 180 degrees. They need to be able to demonstrate and explain why the sum of the interior angles equals 180 degrees.

Students should build on this activity to explore exterior angle relationships in triangles. They can also extend this model to explorations involving other parallel lines, angles, and parallelograms formed. Students should be able to explain why two angles in a triangle have to be less than 180 degrees.

Investigations should lead to the Angle-Angle criterion for similar triangles. For instance, groups of students should explore two different triangles with one, two, and three given angle measurements. Students observe and describe the relationship of the resulting triangles. As a class, conjectures lead to the generalization of the Angle-Angle criterion.

Students should understand the Pythagorean theorem as an area relationship between the sum of the squares on the lengths of the legs and the square on the length of the hypotenuse. This can be represented as $(\text{leg a})^2 + (\text{leg b})^2 = \text{hypotenuse}^2$. The Pythagorean Theorem only applies to right triangles. Exclusively using $a^2 + b^2 = c^2$ frequently leads to student errors in identifying the parts of the triangle. Use words like leg a, leg b, and hypotenuse c.

It is important for students to see right triangles in different orientations.

Students should be given the opportunity to explore right triangles to determine the relationships between the measures of the legs and the measure of the hypotenuse. Experiences should involve using square grid paper to draw right triangles from given measures and representing and computing the areas of the squares on each side. Students can physically cut the squares on the legs apart and rearrange them, so they fit on the square along the hypotenuse. Data should be recorded in tables, allowing for students to conjecture about the relationship among the areas.

Students can apply the Pythagorean Theorem to real-world situations involving two- and three-dimensions. Some examples of this may include designing roofs, ramp dimensions, etc. Students should sketch right triangles to model real-world situations. Challenge students to identify additional ways that the Pythagorean Theorem is or can be used in real-world situations or mathematical problems, such as finding the height of something that is difficult to physically measure, or the right triangle formed by the diagonal of a prism.

Previously, students have discovered that not every combination of side lengths will create a triangle. Now they need to explore situations that involve the Pythagorean Theorem to test whether or not side lengths represent right triangles. This is an opportunity to remind students that the longest side is the only possibility for the hypotenuse. Students should be able to explain why a triangle is or is not a right triangle using the converse of the Pythagorean Theorem. This might be an opportunity for students to explore Pythagorean triples.

Students in Grade 8 should extend the use of the Pythagorean Theorem to find the distance between two points. Understanding how to determine distance by using vertical and horizontal lengths as legs of a right triangle is more important than deriving or memorizing a formula.

An extension could be having students understand how to find the midpoint as well as there is an intuitive connection to finding the distance between two points.

Sample Assessments and Performance Tasks

Reporting Category: Geometry **Standards: 8.G.5, 6, 7, and 8** **Approximate Portion of Test: 28% - 37%; 15 - 19 points**	**OST Test Specs:** ● Facts are limited to angle sum, exterior angles of triangles, angles created when parallel lines are cut by a transversal and the angle-angle criterion for similarity of triangles. ● For the converse of the Pythagorean Theorem, only perfect squares are used. ● Items that apply the Pythagorean Theorem are aligned to 8.G.7 or 8.G.8. ● If the triangle is part of a three-dimensional figure, a graphic of the three-dimensional figure will be included. ● Points must either be at the intersection of two grid lines or their coordinates must be given.

Instructional Resources

8.G.5 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics A Triangle's Interior Angles Congruence of Alternate Interior Angles via Rotations Find the Angle Find the Missing Angle Rigid motions and congruent angles Similar Triangles I	8.G.6 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Converse of the Pythagorean Theorem

Similar Triangles II Street Intersections Tile Patterns II: hexagons Tile Patterns I: octagons and squares	
8.G.7 Better Lesson Shmoop Khan Academy Videos Dan Meyer Activity Taco Cart Illustrative Mathematics Area of a Trapezoid Areas of Geometric Shapes with the Same Perimeter Circle Sandwich Glasses Points from Directions Running on the Football Field Spiderbox Two Triangles' Area	8.G.8 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Finding isosceles triangles Finding the distance between points
Adopted Resource	
Reveal: Lesson 7-2: Angle Relationships and Triangles Lesson 7-3: The Pythagorean Theorem Lesson 7-4: Converse of the Pythagorean Theorem Lesson 7-5: Distance on the Coordinate Plane	**ALEKS:** Lines, Angles, and Polygons (ALEKS TOC): • Angle Relationships • Parallel Lines • Classifying and Measuring angles • Classifying Triangles • Angles of Triangles Exponents, Polynomials, and Radicals (ALEKS TOC): • Applying the Pythagorean Theorem

Return to Scope and Sequence

COLUMBUS CITY SCHOOLS

Math 8

Module 8: Transformations & Module 9: Congruence and Similarity

Unpacked Standards / Clear Learning Targets		
Learning Target 8.G.1 Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). a. Lines are taken to lines, and line segments are taken to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. 8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Include examples both with and without coordinates.) 8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. 8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Include examples both with and without coordinates.) 8.G.5 Use informal arguments to establish facts about ~~the angle sum and exterior angle of triangles, about~~ the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	**Essential Understanding** • Identify corresponding sides and angles of transformed figures. • Reflections, rotations, and translations preserve angle measures and side lengths. • Reflections and rotations change location and orientation. • Translations change only location. • Reflections require a line of reflection. • Rotations require a point of rotation, a degree of rotation, and a direction of rotation. • Translations require distance and direction. • Dilations require a center of dilation and a scale factor. • Two figures are congruent if there is a sequence of reflections, rotations, and translations that maps one figure precisely to the other. • Dilations preserve angle measures while corresponding side lengths are proportional • A sequence of transformations, including a dilation that transforms one figure to another, results in figures that are similar. Angle Relationships • Parallel lines cut by a transversal create relationships, either congruent or supplementary, between pairs of angles. • The sum of the measure of the interior angles of a triangle is 180 degrees. • Any exterior angle of a triangle is congruent to the sum of the measures of the two remote interior angles of the triangle. • If two angles in one triangle are congruent to two angles in another, then the triangles are similar.	**Academic Vocabulary** Image Preimage "Prime" (e.g., A, A') Congruence Coordinate plane Coordinates Corresponding Parallel lines Corresponding angle Exterior angle Interior angle Vertical angle Alternate Interior angle Alternate exterior angle Same side Interior angle Same side exterior angle Rigid motion Rotation Transformation Translation Center of dilation Dilation Reflection Rotation Similar figures Translation Sequence Mapping

I Can Statements:

- I can define and identify rotations, reflections, and translations.
- I can identify corresponding sides and corresponding angles of similar figures.
- I can understand prime notation to describe an image after a translation, reflection, or rotation.
- I can identify the center of rotation.
- I can identify direction and degree of rotation.
- I can identify lines of reflection.
- I can define congruence.
- I can identify symbols for congruence.
- I can identify the scale factor of the dilation.
- I can describe the effects of dilations, translations, rotations, and reflections on 2D figures using coordinates.
- I can identify corresponding sides and corresponding angles of similar figures.
- I can reason that a 2D figure is congruent to another if the second can be obtained by a sequence of rotation, reflections, and translation.
- I can define similar figures as corresponding angles are congruent and corresponding side lengths are proportional.

Performance Level Descriptors:

Proficient:
- Create a single translation of a geometric figure
- Create an image of a geometric figure using a reflection over an axis and/or multiple translations
- Describe a sequence of rigid transformations between two congruent figures
- Identify if two figures are related by a dilation, translation, rotation, or reflection
- Create dilations of figures by a given whole number scale factor
- Recognize that a dilation produces a similar figure.
- Identify two congruent figures
- Identify pairs of equivalent angles when parallel lines are cut by a transversal

Accomplished (all of Proficient +):
- Explain why a dilation produces a similar figure and that rigid transformations maintain angle measure and side lengths

Advanced (all of Proficient + all of Accomplished +):
- Justify why two figures are congruent and/or similar

Prior Standard(s)	Future Standard(s)	
6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **7.G.2** Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions. **7.G.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	**G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch. **G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	**G.CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **G.SRT.1** Verify experimentally the properties of dilations given by a center and a scale factor. **G.SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 8, Number 3, pages 53-54
- Ohio's K-8 Learning Progressions, 6-8 Geometry, page 21

Instructional Strategies

Transformations should include those done both with and without coordinates.

Students should be able to appropriately label figures, angles, lines, line segments, congruent parts, and images (primes or double primes).

Students are expected to use logical thinking, expressed in words using correct terminology. They should also be using informal arguments, which are justifications based on known facts and logical reasoning. However, they are not expected to use theorems, axioms, postulates or a formal format of proof such as two-column proofs.

Students should solve mathematical and real-life problems based on understandings related to this cluster. Investigation, discussion, justification of their thinking, and application of their learning will assist them in the more formal learning of geometry standards in high school.

Initial work should be presented in such a way that students understand the concept of each type of transformation and the effects that each transformation has on an object before working within the coordinate system.

Provide opportunities for students to physically manipulate figures to discover properties of similar and congruent figures involving appropriate manipulatives, such as tracing paper, rulers, Miras, transparencies, and/or dynamic geometric software. Time should be allowed for students to explore the figures for each step in a series of transformations, e.g., cutting out and tracing.

Discussion should include the description of the relationship between the preimage (original figure) and image(s) in regards to their corresponding parts (length of sides and measure of angles) and the description of the movement, (line of reflection, distance, and direction to be translated, center of rotation, angle of rotation, and the scale factor of dilation).

Although computer software is encouraged to be used in this cluster, it should not be used prematurely. Students need time to develop these geometric concepts with hands-on materials such as transparencies.

Work in the coordinate plane follows an intuitive understanding of the transformations and should involve the mapping of various polygons by changing the coordinates using addition, subtraction, and multiplication.

In Grade 6 students learned that when two shapes match exactly they have the same area. In Grade 7 they learn that two figures that "match up" or are put on top of each other are the same. In Grade 8, they learn the formal term of congruence and define it by using transformations. Students should also become familiar with the symbol for congruence (≅).

Students should observe and discuss which properties of the polygons remained the same and which properties changed. Understandings should include generalizations about which transformations maintain size or maintain shape, as well as which transformations do not.

A discussion can be had about the meaning of congruence. Initially one can use the informal definition of congruence being the same size and shape, but the discussion should eventually move toward the definition in 8.G.2 "a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations." Use the word "mapping" when discussing the overlay of two figures.

In Grade 7, students develop an understanding of the special relationships of angles and their measures (complementary, supplementary, adjacent, and vertical). Now in 8.G.5 the focus is on learning about the sum of the measures of the interior angles of a triangle and exterior angle of triangles by using transformations.

This might be a good time to introduce vocabulary of the types of angles, such as interior, exterior, alternate interior, alternate exterior, corresponding, same side interior, and same side exterior. Students are expected to recognize but not memorize this vocabulary.

Introduce dilation by discussing a topic such as "How do we double the size of a wiggly curve?" After much discussion, lead students toward assigning an arbitrary point, O, on the plane, and pushing every point on the squiggly line twice as far away from O. Explain to students that this is a dilation. A dilation pushes out (or pulls in) every point of the figure from its center of dilation proportionally by the same amount. This can be easily modeled by pushing in or pulling out an image on an overhead projector or an image drawn on a flashlight. In this case the center of dilation is O, and it can be anywhere on the plane. This can also be done by copying and pasting line segments using technology such as Microsoft Word, Powerpoint, or Smartboard.

A dilation also has a scale factor. When doubling the size of a wiggly curve, the scale factor is 2, but a scale factor could be any number such as 1/2 or 3. Explain that in Grade 8 scale factors always have to be positive. Discuss what happens to a figure when the scale factor is less than one, compared to when the scale factor is greater than one.

Although students should have experiences with the center of dilation being anywhere either inside or outside the figure, the expectation for Grade 8 is that they be proficient using centers of dilations at the origin and at a vertex of an image.

Review that in Grade 7 students learned that similar figures have sides that are proportional and angles that are congruent. Also, in 7th grade students used to say that figures are similar if they have the same shape but different size. Now they will be defining similarity in terms of transformations. Students should also become familiar with the symbol for similarity (~).

Sample Assessments and Performance Tasks

Reporting Category: Geometry **Standards: 8.G.1, 2, 3, and 4** **Approximate Portion of Test: 28% - 37%; 15 - 19 points**	**OST Test Specs:** ● Dilation may be used as a distractor option in selected response items. However, stating "dilation" is not sufficient for identifying a transformation that does not maintain congruence, since dilation by a factor of 1 does maintain congruence. ● Sequences will be limited to no more than two transformations. ● Figures may or may not be given on a coordinate plane. ● Sequences will be limited to no more than two transformations. ● Figures may or may not be given on a coordinate plane.	● The use of coordinates or the coordinate plane is required. ● Coordinate values of x and y must be integers. ● Sequences will be limited to no more than two transformations. ● In items that require the student to draw a transformed figure using a dilation or a rotation, the center of the transformation will be given ● Limit the center of rotation to the origin or a vertex on the figure. ● Limit the center of dilation to the origin, a defined point inside the shape, or a vertex on the figure. ● When a coordinate grid is given, all original figures and transformations, given or not given, will fit onto that coordinate grid.

Instructional Resources

8.G.1 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Origami Silver Rectangle Reflections, Rotations, and Translations	8.G.2 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Circle Sandwich Congruent Rectangles Congruent Segments Congruent Triangles Cutting a rectangle into two congruent triangles

	Triangle congruence with coordinates	
8.G.3 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Effects of Dilations on Length, Area, and Angles Point Reflection Reflecting reflections	8.G.4 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Are They Similar? Creating Similar Triangles Different Areas?	8.G.5 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics A Triangle's Interior Angles Congruence of Alternate Interior Angles via Rotations Find the Angle Find the Missing Angle Rigid motions and congruent angles Similar Triangles I Similar Triangles II Street Intersections Tile Patterns II: hexagons Tile Patterns I: octagons and squares

Adopted Resource	
Reveal: Lesson 8-1: Translations Lesson 8-2: Reflections Lesson 8-3: Rotations Lesson 9-1: Congruence and Transformations Lesson 9-2: Congruence and Corresponding Parts Lesson 7-1: Angle Relationships and Parallel Lines Lesson 8-4: Dilations Lesson 9-3: Similarity and Transformations Lesson 9-4: Similarity and Corresponding Parts	**ALEKS:** Transformations (ALEKS TOC): • Translations • Reflections • Rotations • Dilations • Congruence and Similarity Rations, Proportions, and Measurement (ALEKS TOC): • Similar Figures

Return to Scope and Sequence

COLUMBUS CITY SCHOOLS

Math 8

Module 10: Volume

Unpacked Standards / Clear Learning Targets		
Learning Target **8.G.9** Solve real-world and mathematical problems involving volumes of cones, cylinders, and spheres.	**Essential Understanding** • The bases of cones and cylinders are circles. • The net of a cylinder is a rectangle with 2 circles. • Cones and pyramids have one base. • The point of a cone and pyramid is called the apex. • The height of a pyramid or cone is the perpendicular distance from the apex to the (possibly extended) base. • The slant height of a pyramid or cone is the distance measured along the lateral face from the apex to the base. • The volume of a pyramid is 1/3 of the volume of a prism with congruent bases and heights. • The volume of a cone is 1/3 of the volume of a cylinder with congruent bases and heights.	**Academic Vocabulary** Cylinder Cone Formula Height Pi Radius/Radii Sphere Volume

I Can Statements:

- I can recognize formulas for volume of cones, cylinders, and spheres.
- I can compare the volume of cones, cylinders, and spheres.
- I can determine and apply appropriate volume formulas in order to solve mathematical and real world problems for the given shape.
- I can, given the volume of a cone, cylinder, or sphere, find the radii, height, or approximate for π.

Performance Level Descriptors:

Proficient:	Accomplished (all of Proficient +):	Advanced (all of Proficient + all of Accomplished +):
• Find the volume of a cone, cylinder, or sphere given the height and/or radius • Solve real-world and mathematical problems involving the volumes of cones, cylinders and spheres	• Solve real-world and mathematical problems involving the volume of a composite solid including a cone, cylinder, or sphere	• Informally explain the derivation of the formulas for cones, cylinders, and spheres

Prior Standard(s)	Future Standard(s)	
8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	**G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. **G.GMD.2** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. **G,GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	**G,MG,1** Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder **G.MG.3** Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 8, Number 3, pages 53-54
- Ohio's K-8 Learning Progressions, 6-8 Geometry, page 21

Instructional Strategies

In Grade 7 students explored circles and the surface area and volume of right prisms. In Grade 8, they are putting the two concepts together to explore right cylinders, right cones, and spheres. The focus in this cluster should be on relationships between solids and the real-world application of volume. Not only do students need to find the volume, but they should also be able to find a missing dimension given the volume.

To develop students' spatial skills, they need practice learning how to draw three-dimensional solids such as cones, cylinders, pyramids, and spheres.

There are many times in life, where people need to represent three-dimensional solids as two-dimensional figures in presentations using technology. Have students practice creating three-dimensional solids on technology platforms.

Most students can be readily led to the understanding that the volume of a right rectangular prism can be thought of as the area of a "Base" times the height, and so because the area of the base of a cylinder is a circle whose area equals $\pi\pi rr2$ the volume of a cylinder is Vcylinder = $\pi r^2 h$ or $V = Bh$. Foam layers that have the height of 1 unit can be used to show how to build a cylinder of h height and reinforce Base × height as well.

To explore the formula for the volume of a cone, use cylinders and cones with the same radius and height. Fill the cone with rice or water and pour it into the cylinder. Students will discover/experience that 3 full cones are needed to fill the cylinder. This non-mathematical demonstration of the formula for the volume of a cone, Vcone = $\frac{1}{3}\pi r^3 h$ or Vcone = $\frac{1}{3}Bh$, will help students make sense of the formula.

Make sure to differentiate between the height of an object and slant height.

To explore the formula for the sphere, use spheres, cylinders, and cones with the same radius, whereas the height of the cone and the cylinder must be the same as the radius, but the height of the sphere will be twice the radius. Discuss the relationships between the solids. Fill the sphere with rice or water and pour into the cylinder. Students will discover/experience that there is water remaining in the sphere. This water/rice will fill the cone. The students should see that the volume of a sphere = the volume of a cylinder + volume of a cone. Because r = h (radius = height), by using substitution, the volume of the cylinder is $\frac{3}{3}\pi r^3$, and the volume of the cone is $\frac{1}{3}\pi r^3$, so the volume of the sphere is Vsphere = $\frac{4}{3}\pi r^3$, This non-mathematical demonstration of the formula for the volume of a sphere, Vsphere = $\frac{1}{3}\pi r^3$, will help students make sense of the formula.

Students should experience many types of real-world applications using these formulas. They should be expected to explain and justify their solutions. Some examples include the following: finding the amount of space left over in a can with 3 tennis balls; finding total volume in a silo; finding how much ice cream in a cone, etc.

Sample Assessments and Performance Tasks

Reporting Category: Geometry

Standards: 8.G.9

Approximate Portion of Test: 28% - 37%; 15 - 19 points

OST Test Specs:
- Items may use all types of rational numbers.
- Items that require the use of π in their calculations should accept answers using approximations of π from 3.14 to $\frac{22}{7}$.

Instructional Resources

Better Lesson
Shmoop
Khan Academy Videos
Dan Meyer Activity

Coca Cola Pool

Illustrative Mathematics
Comparing Snow Cones
Flower Vases
Glasses
Shipping Rolled Oats

Adopted Resource

Reveal:
Lesson 10-1: Volume of Cylinders
Lesson 10-2: Volume of Cones
Lesson 10-3: Volume of Spheres
Lesson 10-4: Find MIssing Dimensions
Lesson 10-5: Volume of Composite Solids

ALEKS:
Perimeter, Area, and Volume (ALEKS TOC):
- Volume of Prisms and Cylinders
- Volume of Pyramids, Cones, and Spheres

Return to Scope and Sequence