Mathematics

Math 7

2023-2024

Aligned with Ohio's Learning Standards
for Mathematics (2017)

Department of Academic Services
Office of Teaching and Learning
Curriculum Division

COLUMBUS CITY SCHOOLS

Curriculum Map

Year-at-a-Glance

The Year-at-a-Glance provides a high-level overview of the course by grading period, including:

- Units;
- Standards/Learning Targets; and
- Timeframes.

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Scope and Sequence

The Scope and Sequence provides a detailed overview of each grading period, including:

- Units;
- Standards/Learning Targets;
- Timeframes;
- Big Ideas and Essential Questions; and
- Strategies and Activities.

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Curriculum and Instruction Guide

The Curriculum and Instruction Guide provides direction for standards-based instruction, including:

- Unpacked Standards / Clear Learning Targets;
- Content Elaborations;
- Sample Assessments;
- Instructional Strategies;
- Instructional Resources; and
- ODE Model Curriculum with Instructional Supports.

COLUMBUS CITY SCHOOLS

Math 7

Year-at-a-Glance

Grading Period 1

Reporting Category: Ratios and Proportions — **6 weeks**

1. Unit Rates
2. Proportional Relationships
3. Tables, Graphs, and Equations of Proportional Relationships
4. Solving Problems Involving Proportions
5. Solving Percent Problems

Grading Period 2

Reporting Category: The Number System — **16 weeks**

1. Operations with Integers
2. Operations with Rational Numbers
3. Simplifying Algebraic Expressions
4. Write and Solve Equations
5. Write and Solve Inequalities

Grading Period 3

Reporting Category: Geometry — **8 weeks**

1. Geometric Figures
2. Measure Figures

Grading Period 4

Reporting Category: Statistics and Probability — **6 weeks**

1. Probability
2. Sampling and Statistics

Standards for Mathematical Practice

The Standards for Mathematical Practice (SMP) describe skills that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The design of each item on Ohio's state tests encourages students to use one or more Standards for Mathematical Practice.

Modeling and Reasoning are included in the eight Standards for Mathematical Practice within Ohio's Learning Standards. Each grade's blueprint identifies modeling and reasoning as an independent reporting category that will account for a minimum of 20 percent of the overall points on that grade's test.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Practice - Grade 7

Modeling and Reasoning on Ohio's State Tests in Mathematics

COLUMBUS CITY SCHOOLS

Math 7

Scope and Sequence

Students should be assessed using teacher-based resources and the ALEKS program. Students are automatically enrolled in the ALEKS course Middle School Course 2. Teachers can move students into RTI 7 if students show a need for remediation. Refer to our guide at https://tinyurl.com/CCS-ALEKS-GUIDE

Textbook information

McGraw-Hill - Reveal Math Course 2

Grading Period 1				
	Reporting Category: Ratios and Proportions			**6 weeks**
	Module 1: Proportional Relationships			**3.5 weeks**
	Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**
	1.1 Unit Rates Involving Ratios of Fractions	7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.	What is a unit rate, and how does it relate to ratios and fractions? How can you compute the unit rate of a ratio of fractions? What strategies can be used to simplify ratios of fractions before computing the unit rate? How can unit rates be applied to real-world situations involving lengths, areas, and other quantities?	• In Grade 6 students reasoned about ratios using models such as tables, double number lines, tape diagrams, and graphs. They avoided using fraction notation for ratios and did not set up nor explicitly solve proportions. Now in Grade 7, students should be able to set up proportions using fraction notation. Note: Solving problems using cross products should be avoided. • Applications should now focus on solving unit-rate problems with more sophisticated numbers. Entries in tables and unit rates can be rational numbers including complex fractions. For scaffolding ideas and more information about ratios and rates see Model Curriculum Grade 6.RP.1-3. • Students obtain proportional reasoning when they understand that the ratio of the two quantities remains constant even though the corresponding values of
	1.2 Understanding Proportional Relationships	7.RP.2 Recognize and represent proportional relationships between quantities.	How can you determine whether two quantities are in a proportional relationship? What are the key indicators or patterns to look for? What are equivalent ratios, and how can they be used to test for a proportional relationship? How do you determine if ratios are equivalent? What is the significance of the constant of proportionality being a unit rate? How does it relate to the relationship between	

			the two quantities?	the quantities may change (y = kx). In other words, the relationship of the first quantity compared to the amount of the second quantity is always the same regardless if the quantities increase or decrease. • It is important that students are able to differentiate between situations that are directly proportional and those that are not. Otherwise, they may haphazardly apply proportional techniques to nonproportional situations. That means they need to carefully attend to the relationships in the problem. • One way to view and reason with proportions is to use within and between relationships. Within relationships focus on making comparisons within the same units/measure-space such as 180 miles: 60 miles = 6 gallons: 2 gallons. Whereas between relationships focus on making comparisons between different units/measure-space such as 180 miles: 6 gallons = 60 miles: 2 gallons. • Have students explore graphs that are proportions and those that are not. Given various graphs, they may make tables using three points on the graph and decide whether they are proportional or not. Ask students what all proportional graphs have in common. Students should come to the conclusion that a proportional graph is a straight line that goes through the origin.
	1.3 Tables of Proportional Relationships	7.RP.2 Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	What is the constant of proportionality, and how can you identify it in tables?	
	1.4 Graphs of Proportional Relationships	7.RP.2 Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	What role does graphing on a coordinate plane play in determining whether a relationship is proportional? What characteristics should you observe in the graph? How does a proportional relationship appear on a graph? What does it mean for the graph to be a straight line through the origin? What is the constant of proportionality, and how can you identify it in graphs?	
	1.5 Equations of Proportional Relationships	7.RP.2 Recognize and represent proportional relationships between quantities. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations.	What is the constant of proportionality, and how can you identify it in various representations such as tables, graphs, equations, diagrams, and verbal descriptions?	
	1.6 Solve Problems Involving Proportional Relationships	7.RP.2 Recognize and represent proportional relationships between quantities. 7.RP.3 Use proportional relationships to solve	How can you identify and apply proportional relationships to solve multistep problems involving ratios and percentages?	

	multistep ratio and percent problems.	What strategies can be used to set up and solve multistep problems using proportional reasoning? How can you determine whether a problem requires the use of proportional relationships versus other mathematical operations or strategies?	

Module 2: Solve Percent Problems — 2.5 weeks

Lesson	Standards/Learning Targets	Big Ideas/Essential Questions	Strategies/Activities
2.1 Percent of Change	7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Also Addresses: 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	How can I represent an increase or decrease though addition and multiplication? How does increase/decrease using addition compare to increase/decrease using multiplication? What does it mean to have a percent greater than 100%? What does it mean to have a percent less than 1%? How can I represent large or small percents?	• Because percents have been introduced as rates in Grade 6, the work with percents should continue to follow the thinking involved with rates and proportions. Solutions to problems can be found by using the same strategies for solving rates, such as looking for equivalent ratios or based upon understandings of decimals. Previously, percents have focused on those between 0 and 100, now percents above 100 and less than 1 are introduced. • Use examples when the final amount is greater than the original amount. • Proportion bars, tape diagrams, and double number lines can be useful to visualize percent increase and decrease problems. Discuss that there are two perspectives in solving percent increase and percent decrease problems. The first perspective is finding the increase or decrease and the adding/subtracting it to/from the original to get the new price. The second perspective is to combine the percents first (20% off would be 80% of the original price or a meal with a 15% tip would be 115% of the original bill), and then set up and

				solve the proportion or equation. Students should be able to explain why both situations work. • In percent problems, especially in percent change problems, it is vital to be able to correctly identify the whole (100% amount).
	2.2 Tax	7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. 7.EE.2 In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. Also Addresses: 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	How can proportional relationships be used to calculate the amount of tax, tips, markups, discounts, and interest in a multistep ratio problem? How can you apply proportional reasoning to calculate the total cost of an item including tax, tips, markups, discounts, and interest? How do you determine the proportion or ratio of the tax, tips, markups, discounts, and interest amount to the original price or total cost? How can you interpret and analyze word problems involving tax, tips, markups, discounts, and interest to identify the relevant quantities and relationships? Why would a business need to use percent increase and decrease? Why does a retail shop use discounts and markups? How can simple interest help you make money?	• Students should solve a variety of problems involving percent including tax, interest, tip, mark-ups, mark-downs, commissions, fees, discounts etc. Since the students in Grade 6 do not formally set-up and solve proportions, they were limited to solving percent problems using ratio reasoning with models such as tables and double number line diagrams and 100 grids. Now in Grade 7 students can solve percents more formally using proportions and equations (7.EE.3-4).
	2.3 Tips and Markup			• Discuss how percents can be used to mislead people. They can make big numbers look small and small numbers look big. Which sounds better, a company laid off 5% of its workforce or a company laid off 2,000 people? Another example of deceptive use of percents could be choosing to use an increase instead of percent change or vice versa. For example a shirt that used to sell for $25 dollars now sells for $35, which is only a change of $10, but it is a 40% increase in price.
	2.4 Discounts			
	2.5 Interest	7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Also Addresses:		• Students should solve a variety of problems involving percent including tax, interest, tip, mark-ups, mark-downs, commissions, fees, discounts etc. Since

		7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		the students in Grade 6 do not formally set-up and solve proportions, they were limited to solving percent problems using ratio reasoning with models such as tables and double number line diagrams and 100 grids. Now in Grade 7 students can solve percents more formally using proportions and equations (7.EE.3-4).
	2.6 Commission and Fees	7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. 7.EE.2 In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. Also Addresses: 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	Why would I need to convert a decimal to a percent or a percent to a decimal? How can I represent the total cost of an item with sales tax or commission?	
	2.7 Percent Error	7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Also Addresses: 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as	How can I calculate percent error if I know the approximate and exact values?	• Absolute error is $\lvert approximate\ value - exact\ value \rvert$. Percent error is $\frac{\lvert approximate\ value - exact\ value \rvert}{\lvert exact\ value \rvert} \cdot 100\%$. Discuss the difference between absolute error and percent error. Explain that the percent error is useful for comparing the error to the original amount. The smaller the object is, the more precision

	appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		is needed.
Reporting Category: The Number System			**16 weeks**
Module 3: Operations with Integers			**3 weeks**
Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**
3.1 Add Integers	7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. Also Addresses:	(Addition and Subtraction) Why do opposite numbers have a sum of 0? How can I model addition and subtraction? How can I use properties of operations to solve problems?	• Many students struggle with the negative sign. One way to help students is to talk about values, order, and direction instead of quantities. For example, positive 5 is greater than positive 4, but −4 is greater than −5. ○ Another reason for confusion is that the negative sign can mean several things: • A sign attached to a number to form negative numbers; • A subtraction; or • An indication to take the opposite of Because of the confusion around the negative sign it may be helpful for students to understand the different meanings of the negative sign and identify which meaning is used when in a problem including the meaning shifts. • Using both contextual and numerical problems, students should explore what happens when negatives and positives are combined. Repeated opportunities over time with models will allow students to compare the results of adding and subtracting pairs of numbers, leading to the generalization of the rules. • Two-color counters or colored chips can be used as a physical model for adding and subtracting integers. Integer chips

		7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.		allow the idea of the zero pair (Additive Inverse Property) to be apparent. ● Number lines present a visual image for students to explore and record addition and subtraction results. One of the positive aspects about using a number line model is that it is not limited to integers; it also lends itself toward connections on the coordinate plane. Students can use number lines with arrows and hops. When using number lines, establishing which factor will represent the length, number, and direction of the hops will facilitate understanding.
	3.2 Subtract Integers	7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. Also Addresses: 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		
	3.3 Multiply Integers	7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.	(Multiplication and Division) How can I use properties of operations to	● Multiplying and dividing integers should be thought of as an extension of adding and subtracting integers. Using what

		a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1)(−1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. Also Addresses: 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.	multiply and divide numbers? How does multiplication connect to the distributive property? How do I multiply and divide integers?	students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers. ● In multiplication, the first factor indicates the number of sets, and the second factor indicates the size of the set. This can be easily modeled with situations involving a positive times a positive or a positive times a negative. A negative times a positive can be inferred using the Commutative Property of Multiplication. ● A negative times a negative can be problematic, for students want to know "How can we have a negative group of something?" One way to view it is as repeated subtraction. If the first factor being positive indicates repeated addition, then the first factor being negative indicates repeated subtraction. Therefore "negative 3 sets of negative −2" or −3(−2) means to remove 3 sets of −2 from zero. ● Students will discover that they can multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. They should then analyze and solve problems leading to the generalization of the rules for operations with integers. ● Another method for learning multiplication/division rules is to use patterns. Beginning with known facts, students can predict the answers for related facts, keeping in mind that the properties of operations apply. ● Using the language of "the opposite of"
	3.4 Divide Integers	7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then −(p/q) = (−p) /q = p/(–q). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers.		

		Also Addresses: 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. 7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		helps some students understand the multiplication of negatively signed numbers ($-4 \cdot -4 = 16$, the opposite of 4 groups of -4). Discussion about the tables should address the patterns in the products, the role of the signs in the products, and commutativity of multiplication. ● Different Algorithms: ○ Multiplication ■ Area Model ■ Partial Products ■ Lattice Algorithm ■ Traditional Multiplication ○ Division ■ Partial Quotients ■ Explicit-Trade ■ Traditional Division
	3.5 Apply Integer Operations	7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. d. Apply properties of operations as strategies to add and subtract rational numbers. 7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. c. Apply properties of operations as strategies to multiply and divide rational numbers. 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.	How can I apply properties of operations to solve problems? How can I identify which operation(s) to use when solving a real-world problem?	● It is important when performing operations that students are able to justify their steps using the properties. Although, the focus should not be on identifying the properties of operation, teachers should be using their formal names in classroom discussion, so students are able to gain familiarity with and recognize the correct terminology ● In Grade 6, students should have learned that the absolute value of a number does not take into account sign or direction; it only is a measure of distance (magnitude) from 0. Discourage students from saying that the "answer is always positive or 0" since that will lead to misconceptions when students encounter problems such as $\|4x - 2\| = 18$ in high school. Instead emphasize that

		7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		it is the "distance from 0." This is why the value of something like $\|x\| = -5$ has no solution, since distance cannot be negative.
Module 4: Operations with Rational Numbers				**3 weeks**
	Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**
	4.1 Rational Numbers	7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. Also Addresses: 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers,	What is the difference between a terminating and a repeating decimal? Why does the decimal form of a rational number terminate in 0s or eventually repeat? How can we apply the concept of rational numbers to real-life situations and make meaningful connections?	• In Grade 7 the awareness of rational and irrational numbers is initiated by observing the result of changing fractions to decimals. They can do this by making equivalent fractions with denominators using powers of ten or by using long division. • Students should be provided with families of fractions, such as, sevenths, ninths, thirds, etc. to convert to decimals using long division. The equivalent fractions can be grouped and named (terminating or repeating). Students should begin to see why these patterns occur. Knowing the formal vocabulary rational and irrational is not expected for the students. Terminating decimals fall exactly on some tick mark on a number line; however repeating decimals do not. For example 1 3 is always subdividing an interval of smaller and smaller sizes. Students can also explore patterns to determine which fractions repeat and which fractions terminate. Technology can be used to

		fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		aid students in recognizing patterns. • Students can also use their knowledge of proportions and unit rates to help them understand why they can convert a fraction to a decimal by dividing.
	4.2 Add Rational Numbers	7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. Also Addresses: 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational	(Addition and Subtraction) How can I solve real-world problems using addition and subtraction of rational numbers? What do the sums and differences of rational numbers mean in the context of a real-world problem? How can I use subtraction to find the distance between two numbers on a number line?	• See strategies for operations with integers (Module 3) for additional strategies. Please note that not all strategies will work with rational numbers, however. • Students need to become fluent in using operations and properties of operations with all rational numbers, not just with integers. • Although all properties of operations should be addressed, this cluster should especially emphasize the Additive Inverse Property, the Multiplicative Inverse Property, and the Distributive Property. • Using both contextual and numerical problems, students should explore what happens when negatives and positives are combined. Repeated opportunities over time with models will allow students to compare the results of adding and subtracting pairs of numbers, leading to the generalization of the rules. • Fractional rational numbers, decimals, and whole numbers should also be used in computations and explorations. Students should be able to give contextual examples of integer operations, write and solve equations for real-world problems, and explain how the properties of operations apply. Real-world situations could include: profit/loss, money, weight, sea level, debit/credit, football yardage, etc.

		numbers extend the rules for manipulating fractions to complex fractions.		• Strategies for modeling: ○ Integer chips ○ Number lines
	4.3 Subtract Rational Numbers	7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		
	4.4 Multiply Rational Numbers	7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers.	(Multiplication and Division) How can I use multiplication and division to solve mathematical and real-world problems? How do I interpret the products and quotients in the context of real-world problems? How can I use patterns to recognize the sign of a product when a string of numbers is multiplied? How can I divide a number line to represent decimals and fractions?	• Multiplying and dividing integers should be thought of as an extension of adding and subtracting integers. Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers. • In multiplication, the first factor indicates the number of sets, and the second factor indicates the size of the set. This can be easily modeled with situations

		7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. Also Addresses: 7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		involving a positive times a positive or a positive times a negative. A negative times a positive can be inferred using the Commutative Property of Multiplication. ● A negative times a negative can be problematic, for students want to know "How can we have a negative group of something?" One way to view it is as repeated subtraction. If the first factor being positive indicates repeated addition, then the first factor being negative indicates repeated subtraction. Therefore "negative 3 sets of negative −2" or −3(−2) means to remove 3 sets of −2 from zero. ● Another method of understanding multiplication of negative numbers is fast forwarding and rewinding students walking backwards and forwards. Students can then see that when you rewind (−) and walk backwards (−), the video shows them walking forward. An app called Reverse Vid for iPhone allows students to rewind videos. ● Students will discover that they can multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. They should then analyze and solve problems leading to the generalization of the rules for operations with integers. ● Another method for learning multiplication/division rules is to use patterns. Beginning with known facts, students can predict the answers for related facts, keeping in mind that the properties of operations apply (See Tables 1, 2, and 3 below). ● Using the language of "the opposite of"
	4.5 Divide Rational Numbers	7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then −(p/q) = (−p) /q = p/(–q). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.		

		Also Addresses: 7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers. 7.NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		helps some students understand the multiplication of negatively signed numbers ($-4 \cdot -4 = 16$, the opposite of 4 groups of -4). Discussion about the tables should address the patterns in the products, the role of the signs in the products, and commutativity of multiplication. ● Division of integers is best understood by relating division to multiplication and applying the rules. The Inverse Property of Multiplication should be used to connect division to multiplication. Since $8 \div (-2)$ is the same as $8(-\frac{1}{2})$, the rules for multiplication apply to division. In time, students will transfer the rules to division situations.
	4.6 Apply Rational Number Operations	7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. d. Apply properties of operations as strategies to add and subtract rational numbers. 7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. c. Apply properties of operations as strategies to multiply and divide rational numbers. 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating	How can I apply the properties of operations when solving problems? How can I solve real-world problems involving the four operations with rational numbers? How do I solve problems using complex fractions?	● Students should become familiar with solving problems involving complex fractions. Draw attention to the Multiplicative Identity Property and the Multiplicative Inverse Property for solving expressions with complex fractions. This concept connects nicely with cluster 7.RP.1-3.

	fractions to complex fractions. Also Addresses: 7.EE.2 In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		

Module 5: Simplify Algebraic Expressions			**3 weeks**
Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**
5.1 Simplify Algebraic Expressions	7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. 7.EE.2 In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related.	How can I make equivalent expressions? How can I use order of operations to simplify algebraic expressions? What are the properties of operations, and how can they be used as strategies to simplify and manipulate algebraic expressions? How does expanding linear expressions help in identifying equivalent forms or revealing patterns within the expression? How can you apply properties of operations to combine like terms within linear expressions and simplify them?	• It is important that students are able to justify their thinking using the properties. Although the focus should not be on identifying the properties of operation, teachers should be using their formal names in classroom discussion so students are able to gain familiarity with and recognize the correct terminology. • Provide opportunities for students to use and understand the properties of operations. These include the Commutative, Associative, Identity, and Inverse Properties of Addition and of Multiplication, the Zero Property of Multiplication, and the Distributive Property. • Subtraction should be thought of as the
5.2 Add Linear Expressions	7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	How can you use properties of operations to simplify the addition or subtraction of expressions?	

	5.3 Subtract Linear Expressions			opposite of addition, and division should be thought of as the opposite of multiplication. Note: Avoid PEMDAS as it leads to many misconceptions and errors in computation. • Writing equivalent expressions includes simple cases of factoring out a GCF. This can be illustrated using the area model that students are already familiar with. In this model the students are given the areas, and they are asked to find the lengths and widths. It might be wise to point out that, although in real-life length and width cannot be negative, our model allows for lengths and widths to be negative to illustrate the concept. • Students started combining like terms in Grade 6. Now they will extend this concept to negative numbers. It may be helpful to use Algebra tiles to illustrate this process as it will prevent future misconceptions from forming such as trying to combine 2x and $3x^2$. Students should then extend that concept to other rational numbers besides integers.
	5.4 Factor Linear Expressions		What strategies can be used to factor out common factors from linear expressions to simplify them?	
	5.5 Combine Operations with Linear Expressions		How do the order of operations help simplify expressions with multiple operations?	
Module 6: Write and Solve Equations				**4 weeks**
	6.1 Write and Solve One-Step Equations	7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q,	How do I define variables in context? How do I apply properties of operations to solve equations? How do I determine if my solution is reasonable? How does an algebraic solution compare to an arithmetic solution?	• Continue to build on students' understanding and application of writing and solving one-step equations (6th Grade) from a problem situation to problem situations that require multi-step equations and inequalities. • This is also the context for students to practice using rational numbers including integers and positive and negative fractions and decimals. It is appropriate to expect students to show their steps in their work. Students should be able to

		and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. Also Addresses: 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		move toward explaining their thinking using correct terminology. ● To assist students' assessment of the reasonableness of their answers, especially problem situations involving fractional or decimal numbers, use whole-number approximations for the computation and then compare to the actual computation. ● In connection with 7.EE.1 students should apply the properties of operations and equality found in Table 3 and Table 4 of Ohio's Learning Standards. Teachers should be using the correct terminology to justify steps when performing operations and solving equations. Although Grade 7 students should not be required to know the formal names of the properties, they should be encouraged to recognize them and use them to justify their steps when solving equations. ● Experiences in solving equations should move through Bruner's stages of concrete, pictorial, and algebraic/abstract representation. Utilize experiences with the pan balance model, hangers, tape diagrams, or Algebra tiles as a visual tool for maintaining equality (balance): • First with simple numbers; • Then with pictures symbolizing relationships; and • Finally, with rational numbers. This allows understanding to develop as the complexity of the problems increases. ● Some studies have shown that a students' fractional knowledge correlates with their ability to write equations. Therefore encourage students
	6.2 Solve Two-Step Equations: px + q = r	7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. Also Addresses: 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	How can I solve multi-step problems with positive and negative rational numbers? How do I identify the sequence of operations used to solve an equation?	
	6.3 Write and Solve Two-Step Equations: px + q = r			
	6.4 Solve Two-Step Equations: p(x + q) = r			
	6.5 Write and Solve Two-Step Equations: p(x + q) = r			

			to solve equations with fractions by using diagrams instead of just using inverse operations. This may aid in creating understanding to alleviate the misuse of fraction rules in later grades/courses. Numbers that are easily modeled should be used initially until students internalize the process.

Module 7: Write and Solve Inequalities — **3 weeks**

Lesson	Standards/Learning Targets	Big Ideas/Essential Questions	Strategies/Activities
7.1 Solve One-Step Addition and Subtraction Inequalities 7.2 Write and Solve One-Step Addition and Subtraction Inequalities 7.3 Solve One-Step Multiplication and Division Inequalities with Positive Coefficients 7.4 Solve One-Step Multiplication and Division Inequalities with Negative Coefficients 7.5 Write and Solve One-Step Multiplication and Division Inequalities	7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. b. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. Also Addresses: 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	How does a negative coefficient affect the solution set of an inequality? How does a negative coefficient affect the inequality symbol? How can I interpret the solution set of an inequality in context? How do I graph the solution set of an inequality on a number line?	• In Grade 6 students wrote inequalities in the forms of x > c and c < x. in Grade 7 students use > and <. Discuss why teachers should, when graphing on a number line, a closed circle represents > and < and an open circle represents > and <. Students should also have practice solving one and two-step inequalities with rational numbers. • Present situations where the variable is both on the left and the right side of the equality. Students need to be fluent in solving inequalities where the variable is on the left and right of the inequality for later algebraic concepts using compound inequalities. Therefore, discourage students from always writing the variable on the left side of an inequality. • When graphing, teachers should avoid telling students that the inequality points the same direction on the number line as the arrow; this creates a misconception that is hard to break when students work on compound inequalities in high school. An alternative strategy is to ask students to name 3 points that make an inequality true, and

	7.6 Write and Solve Two-Step Inequalities			then draw the arrow in that direction. • Students should be able to create equations and inequalities from real-world situations where they always precisely define the variable(s). • Provide multiple opportunities for students to work with multi-step problem situations that have multiple solutions and therefore can be represented by an inequality. Students need to be aware that values can satisfy an inequality but not be appropriate for the situation, therefore limiting the solutions for that particular problem.
	Reporting Category: Geometry			**8 weeks**
	Module 8: Geometric Figures			**4 weeks**
	Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**
	8.1 Vertical and Adjacent Angles 8.2 Complementary and Supplementary Angles	7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. Also Addresses: 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. 7.EE.4 a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers.	How can I model angle relationships? How do models of angle relationships help define the relationships? How can I identify special angle pairs? How can I use equations to solve for angle pairs in real-world problems? What are the patterns among the angles of intersecting lines?	• This cluster focuses on the importance of visualization in the understanding of Geometry. Being able to visualize and then represent geometric figures on paper is essential to solving geometric problems. After much work is done on paper, Geometry software can aid in students' understanding of Geometry. • Provide students the opportunities to explore angle relationships. At first they can measure and find patterns among the angles of intersecting lines or within polygons. Then they can utilize the relationships to write and solve equations for multi-step problems. • A student often incorrectly thinks that a wide angle with short sides is smaller than a narrow angle with long sides. To confront this problem have students

		Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.		compare angles with different side lengths.
	8.3 Triangles	7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions. a. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	How can I use specifications to draw a picture or create a model of triangles and/or quadrilaterals? How can I determine if a set of side lengths and angle measures creates a unique triangle, multiple triangles, or does not create a triangle? How do I use side lengths, angel measures, and relationships between sides to determine types and properties of quadrilaterals? Is it possible to create more than one quadrilateral with the same given set of side lengths, angle measures, and relationship between sides? What is the sum of the interior angles of triangles and quadrilaterals?	• Students should regularly be exposed to shapes and figures from many perspectives and orientations, not just the prototypical example. • Many careers and everyday activities require spatial reasoning. Some research suggests that 7th grade is the optimal time for developing spatial visualization. Sketching figures can help students develop an intuitive understanding of geometry. Although drawings should become precise over time, informal free-hand sketches can help develop spatial reasoning. • Constructions facilitate understanding of geometry. Provide opportunities for students to physically construct triangles and quadrilaterals with straws, sticks, or geometry apps. This should be done prior to using rulers, compasses, and/or protractors. Have students discover and justify the side and angle conditions that will form triangles or quadrilaterals.
	8.3b Quadrilaterals	7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions. b. Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions.		
	8.4 Scale Drawings	7.G.1 Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals. a. Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale. b. Represent proportional relationships within and between similar figures. Also Addresses: 7.RP.2 Recognize and represent proportional	How can I use drawings and/or models to make sense of problems involving similar figures? How can I identify corresponding sides and angles of similar figures? What are the similarities and differences between the angle measures and side lengths in a scale drawing and its original figure? How can I use proportions to explain the relationship between side lengths in similar	• Similarity is an increase or decrease that is multiplicative in nature instead of additive; this is a new concept for students. • Although the focus of this cluster is on rectangles and triangles, it may be useful to discuss why all circles are similar. • As an introduction to scale drawings in geometry, students should be given the opportunity to explore scale factor as the number of times you multiply the

		relationships between quantities. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	figures? What is the relationship between the areas of similar figures? How does scale affect length and area? How do I compute actual lengths and areas from a scale drawing? How do I reproduce a scale drawing using a different scale? How do I draw scaled figures with proper figure labels, scale, and dimensions?	side measure of one figure to obtain the corresponding side measure of a similar figure. It is important that students first experience this concept concretely progressing to abstract contextual situations. Pattern blocks provide a convenient means of developing the foundation of scale. Choosing one of the pattern blocks as an original shape, students can then create the next-size shape using only those same-shaped blocks. After students have time to use the shapes concretely, they should also practice drawing them. • Regularly provide students with figures that are not similar to ensure that students are continually checking for similarity. • Provide opportunities for students to use scale drawings of geometric figures with a given scale. The opportunities should require them to draw and label the dimensions of the new shape. Initially, measurements should be in whole numbers, progressing to measurements expressed with rational numbers. This will challenge students to apply their understanding of fractions and decimals. • Provide word problems that require finding missing side lengths, perimeters, or areas. In addition, allow students to design their own word problems asking for missing side lengths, perimeters, and/or areas.
	8.5 Three-Dimensional Figures	7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	What are the outcomes of slicing three-dimensional figures? What two-dimensional faces result from slicing a three-dimensional figure in various	• Slicing three-dimensional figures to observe the cross sections formed helps develop three-dimensional visualization skills. Students should have the

			ways?	opportunity to physically create some of the three-dimensional figures, slice them in different ways, and describe in pictures and words what they discover. For example, use clay or playdough to form a cube, then pull string through it at different angles and record the shape(s) of the slices found. Challenges can be given: "See how many different two-dimensional cross sections you can create by slicing a cube."
Module 9: Measure Figures				**4 weeks**
	Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**
	9.1 Circumference of Circles	7.G.4 Work with circles. a. Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. b. Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems. Also Addresses: 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution,	How do I identify and define characteristics of circles? (radius, diameter, circumference (perimeter), and area) What is the relationship between the radius and diameter of a circle? How does radius and diameter relate to a circle's circumference and area? What is the difference between the radius (and/or diameter) and the length measure of the circle (circumference)? What is pi? What is the approximate value of pi? What are the formulas for the area and circumference of a circle? How do I use the formulas for area and circumference to solve real-world problems?	• Knowing that a circle is created by connecting all the points equidistant from a point (center) is essential to understanding the relationships between radius, diameter, circumference, $\pi\pi$, and area. Students can observe this by folding a paper plate several times, finding the center at the intersection, then measuring the lengths between the center and several points on the circle, the radius. Measuring the folds through the center, or diameters leads to the realization that a diameter is two times a radius. • Students are to explore relationships between area, circumference, diameter, and radius, recognizing the constant of proportionality between each of these elements without formally defining the irrational nature of π. • A visual for understanding the area of a circle can be modeled by cutting up a paper plate into 16 pieces along diameters and reshaping the pieces into
	9.2 Area of Circles			

		identifying the sequence of the operations used in each approach.		a parallelogram. Explain to students that the perimeter of the circle is the circumference. Ask students to identify where the circumference and the radius is in their new shape. Identifying the radius gives cause for conversations. They should come to the conclusion that the height is the radius and the length is ½ of the circumference. Therefore another formula for area is $A = \frac{1}{2}Cr$. Since students already found the area of circle as $A = \pi r^2$ or $A = \pi rr$, students should realize that $\frac{1}{2}C = \pi rr$. If they divide each side by r, then can simplify the formula to $\frac{1}{2}C = \pi r$. Now if they solve for C by multiplying each side by 2, they will get $C = 2\pi r$, and they may see that $2r = d$ and change the formula to get $C = d\pi$.
	9.3 Area of Composite Figures	7.G.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. Also Addresses: 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of	How do I use a picture or model to compose and decompose two- and three-dimensional figures to find area, surface area, and volume? What is the difference between linear, square, and cubic units? How do I know when to use linear, square, and cubic units? How can I solve real-world problems involving area, surface area, and volume?	• Most area problems should be given in a real-world context. • Students can find the area of regular polygons using triangles if the apothem is given. (Students do not need to use the term apothem.) • Instead of being restricted to using nets to find surface area, students may prefer to draw the different views of a structure (front, right, top). The use of formulas to find surface area should be discouraged. • Some students will prefer drawing nets and others will prefer drawing the six different views. After practice with both methods, let students use their preferred method. • Once students are comfortable finding

		the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. 7.G.4 Work with circles. a. Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. b. Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems.		the surface area of unit cubes, tell students that the cubes' lengths are rational numbers such as ¼ inch and have them calculate the surface area. • To develop their spatial reasoning skills, have students build structures with unit cubes and find the surface area of their structure. They can solve these problems by using a net or by drawing the different views of the figure (front, right, top). Also give students a net, and have them build the structure from the net.
	9.4 Volume	7.G.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. Also Addresses: 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.		• Connect the concept of volume of prisms to slicing geometric solids (7.G.3). Have students build solids using layers in the shape of the base. They can also build three-dimensional prisms and pyramids by using coffee stirrers with twist ties, modeling clay, plastic drinking straws, or rods created out of rolled newspaper. Connect triangular prisms with rectangular prisms by letting students discover that half of a rectangular prism could be a triangular prism. • Again, emphasize volume in the context of real-world problems. Use problems that connect volume to surface area. • Some students incorrectly think that the top and bottom of a prism are always the bases of a prism, not realizing that a prism can be rotated. Reinforce that the bases must be two parallel faces.

	9.5 Surface Area	7.G.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. Also Addresses: 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.		
	9.6 Volume and Surface Area of Composite Figures	7.G.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. Also Addresses: 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. 7.G.1 Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals. a. Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale. b. Represent proportional relationships within and between similar figures.		
Reporting Category: Statistics and Probability				**6 weeks**
Module 10: Probability				**3 weeks**
	Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**

	10.1 Find Likelihoods	7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event; a probability around 1/2 indicates an event that is neither unlikely nor likely; and a probability near 1 indicates a likely event.	In an experiment, how can you determine the number of possible results? How can you describe the likelihood of an event? How can I identify a question to explore using probability?	• Build the concept of expressing probability as a number between 0 and 1, inclusive. ○ Provide students with situations that have clearly defined probability of never happening as zero, always happening as 1 or equally likely to happen as to not happen as $\frac{1}{2}$. ○ Then advance to situations in which the probability is somewhere between any two of these benchmark values. ○ Use this to build the understanding that the closer the probability is to 0, the more likely it will not happen, and the closer to 1, the more likely it will happen.
	10.2 Relative Frequency of Simple Events	7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long run relative frequency, and predict the approximate relative frequency given the probability. 7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.	How can you use relative frequencies to find probabilities? How do I design a probability model? How do I use observed frequencies to answer a question using probabilities? How can I analyze results and explain possible discrepancies between observed and theoretical outcomes?	• Students can use chance experiments to collect data. They need to come to the understanding that as they increase the number of trials in their chance experiment the relative frequency (observed or experimental) over the long-run approaches the theoretical probability. Therefore if they have no way of knowing the theoretical probability (e.g., the number and kinds of tiles in a bag are hidden), they can do many, many trials to figure out an approximation of the theoretical probability. Then they could use that information to make further conclusions. Students can also use the theoretical probability to estimate the relative frequency, keeping in mind that what "should happen" does not always happen and that oftentimes the event will be at least close in value if not exact to the theoretical probability.
	10.3 Theoretical Probability of Simple Events	7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of		

		events.		
	10.4 Compare Probabilities of Simple Events	7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long run relative frequency, and predict the approximate relative frequency given the probability. 7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.		• Provide students with models of equally likely outcomes and models of not equally likely outcomes and have students determine probabilities. These outcomes are called simple events.
	10.5 Probability of Compound Events	7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language, e.g., "rolling double sixes," identify the outcomes in the sample space which compose the event.	How can you find the number of possible outcomes of one or more events? What is the difference between dependent and independent events? How can probability from a repeated chance process be used to predict the likelihood of a long-run event?	• Students should begin to expand their knowledge and understanding of finding the probabilities of simple events to finding the probabilities of compound events by creating organized lists, tables, and tree diagrams. This helps students create a visual representation of the data. From each sample space, students determine the probability (fraction, decimal, percent) of each possible outcome. • Ask guiding questions to help students create methods for creating organized lists and tree diagrams for situations with more elements such as "How many outcomes are possible?", " What does each branch of the tree diagram represent?", or "How can you use your list to find the probability of the event?"

				• Students often see skills of creating organized lists, tree diagrams, etc. as the end product. Provide students with experiences that require the use of these graphic organizers to determine the theoretical probabilities. Have them practice making the connections between the process of creating lists, tree diagrams, etc. and the interpretation of those models and tying the simulation to a real-world situation.
	10.6 Simulate Chance Events	7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations. c. Design and use a simulation to generate frequencies for compound events.		• After the basics of probability are understood, students should experience setting up a model and using simulation (by hand or with technology) to collect data and estimate probabilities for a real situation that is sufficiently complex that the theoretical probabilities are not obvious. Simulation is a procedure that will allow students to answer questions about real problems by running experiments that closely resemble the real situation. Simulation uses devices such as coins, number cubes, cups full of paper clips or legos, or cards to generate outcomes that represent real outcomes. Students may find it difficult to make the connection between device outcomes and the real outcomes of the experiment. • Use a percent grid (a 10 by 10 grid) to record data from a simulation.
Module 11: Sampling and Statistics				**3 weeks**
	Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**
	11.1 Biased and Unbiased Samples	7.SP.1 Understand that statistics can be used to gain information about a population by examining	What is the difference between a sample and a population?	• *Note: One of the changes to this cluster was the deletion of the word "random"*

		a sample of the population. a. Differentiate between a sample and a population. b. Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias. 7.SP.2 Broaden statistical reasoning by using the GAISE model: a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. For example, "How do the heights of seventh graders compare to the heights of eighth graders?" (GAISE Model, step 1) b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) d. Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4)	How can you determine whether a sample accurately represents a population? What makes a sample an accurate representation of a population? How does a sample size affect inferences made about a population? What is bias? What factors create bias?	*from the cluster statements. Students should informally learn about what a random sample is and how it is useful in statistics. Students are not required to actually use true random sampling when collecting data. Instead they should discuss which samples are the best representative of a population. However, a teacher may wish to extend to more sophisticated ideas of random sampling depending on the makeup of his or her class.* • Provide opportunities for students to use real-life situations. This shows the purpose for using sampling to make inferences about a population. • Provide students with samples from a population, including the statistical measures. Ask students guiding questions to help them make inferences from the sample. • Random sampling is a way to remove bias. Although students at this level may not be actually using true random sampling procedures when collecting data, the benefits of a random sample should be discussed. • Increasing the sample size reduces sample error, but it does not reduce bias. When students decide to select a sample from a specific group of people (friends), use the situation as an opportunity to discuss bias.
	11.2 Make Predictions	7.SP.2 Broaden statistical reasoning by using the GAISE model: a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. For example, "How do the heights of seventh graders compare to the heights of eighth graders?" (GAISE Model, step 1) b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) c. Analyze Data: Select appropriate graphical	What makes statistical questions have variability? What is the difference between a population, census, and sample? How can we collect data to answer a statistical question? How are sample surveys conducted? How do I use distributions to analyze data? How can I determine variability within a group? How do individuals compare to individuals, individuals compare to a group,	

		methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) d. Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4) Also Addresses: 7.RP.2 Recognize and represent proportional relationships between quantities. 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.	and groups compare to groups? How can I use graphical displays to summarize data in context? How can I show a distribution of data? How can I use data to draw conclusions and make generalizations? How can I use center, spread, and shape of a distribution to show differences between groups? How do I know a sample may or may not represent a population?	
	11.3 Generate Multiple Samples	7.SP.2 Broaden statistical reasoning by using the GAISE model: a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. For example, "How do the heights of seventh graders compare to the heights of eighth graders?" (GAISE Model, step 1) b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) d. Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4) Also Addresses: 7.RP.2 Recognize and represent proportional relationships between quantities.		
	11.4 Compare Two	7.SP.3 Describe and analyze distributions.	How can I explain the mean as a balance	+Add mean absolute value from Course 1

	Populations	a. Summarize quantitative data sets in relation to their context by using mean absolute deviation (MAD), interpreting mean as a balance point. b. Informally assess the degree of visual overlap of two numerical data distributions with roughly equal variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.	point? How can I explain and calculate mean absolute deviation (MAD)? How can I use MAD to summarize data? How can I summarize and describe distributions representing one population? How can I use distributions to informally compare two populations?	
	11.5 Assess Visual Overlap			

ODE Model Curriculum

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is a model curriculum that supports the standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards. The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, possible connections between topics, and some common misconceptions.

Mathematics Grade 7 Model Curriculum with Instructional Supports

Curriculum and Instruction Guide

Module 1: Proportional Relationships

Unpacked Standards / Clear Learning Targets

Learning Target	Essential Understanding	Academic Vocabulary
7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. **7.RP.2** Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems.	• A unit rate is a comparison of two quantities where the second quantity (denominator) is one. • A rate can be written as a complex fraction which can be used to find the unit rate. • A proportional relationship is a relationship between quantities. • Proportions involve vertical and horizontal multiplicative relationships. • In a table that represents a proportional relationship between y and x, $yy\ xx$ is constant. • The unit rate, which is the constant of proportionality, can be identified through models. • Proportional relationships can be written as equations using the constant of proportionality, e.g., y = kx; y = mx; t = pn. • The constant of proportionality is not always rational, e.g., $\pi\pi$. • The unit rate is the amount of change in y as x increases by one unit, e.g., in a table or graph. • Graphs that represent proportional relationships are linear and go through the origin.	Equivalent ratio Coordinate plane Proportion Proportional relationship Nonproportional relationship Ratio Unit rate Constant of Proportionality Complex fraction Constant

I Can Statements:

- I can compute unit rates associated with ratios of fractions in like or different units.
- I can compute fractional by fractional quotients.
- I can apply fractional ratios to describe rates.
- I can define the constant of proportionality as a unit rate.
- I can analyze two ratios to determine if they are proportional to one another with a variety of strategies (e.g. using tables, graphs, pictures, etc.).
- I can analyze tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships to identify the constant of proportionality.
- I can represent proportional relationships by writing equations.
- I can recognize what (0,0) represents on the graph of a proportional relationship.

Performance Level Descriptors:

Proficient:
- Compute a unit rate of two whole numbers where the unit rate is explicitly requested
- Compute a unit rate of two familiar rational numbers where the unit rate is explicitly requested
- Compute a unit rate of two rational numbers where the unit rate is not explicitly requested
- Identify proportional relationships presented in familiar contexts
- Find the whole number constant of proportionality in relationships presented in basic familiar contexts
- Represent proportional relationships in various formats
- Solve a one-step, straightforward ratio or percent problem
- Solve a one-step, straightforward real-world ratio or percent problem
- Use proportional relationships to solve routine real-world and mathematical ratio and percent problems with multiple steps

Accomplished (all of Proficient +):
- Compare unit rates in a real-world context
- Use different representations of proportional relationships to solve real-world problems
- Apply proportional relationships to routine real-world and mathematical ratio and percent problems with multiple steps.

Advanced (all of Proficient + all of Accomplished +):
- Analyze a graph of a proportional relationship in order to explain what the points (x,y) and (1,r) represent, where r is the unit rate, and use this to solve problems.
- Apply proportional relationships to **non-routine** real-world and mathematical ratio and percent problems with multiple steps.

Prior Standard(s)

6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.
6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Future Standard(s)

7.G.1 Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals.
8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the

N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
A.SSE.3c Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
c. Use the properties of exponents to transform expressions for exponential functions.

	equation y = mx + b for a line intercepting the vertical axis at b. **8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. **8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	**F.IF.8b** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions. **G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **G.C.5** Find arc lengths and areas of sectors of circles. **G.MG.2** Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 1, page 43
- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45
- Ohio's K-8 Learning Progressions, Ratio and Proportional Relationships, page 15

Instructional Strategies

In Grade 6 students reasoned about ratios using models such as tables, double number lines, tape diagrams, and graphs. They avoided using fraction notation for ratios and did not set up nor explicitly solve proportions. Now in Grade 7, students should be able to set up proportions using fraction notation. Note: Solving problems using cross products should be avoided.

Applications should now focus on solving unit-rate problems with more sophisticated numbers. Entries in tables and unit rates can be rational numbers including complex fractions. For scaffolding ideas and more information about ratios and rates see Model Curriculum Grade 6.RP.1-3.

Students obtain proportional reasoning when they understand that the ratio of the two quantities remains constant even though the corresponding values of the quantities may change (y = kx). In other words, the relationship of the first quantity compared to the amount of the second quantity is always the same regardless if the quantities increase or decrease.

It is important that students are able to differentiate between situations that are directly proportional and those that are not. Otherwise, they may haphazardly apply proportional techniques to nonproportional situations. That means they need to carefully attend to the relationships in the problem.

One way to view and reason with proportions is to use within and between relationships. Within relationships focus on making comparisons within the same units/measure-space such as 180 miles: 60 miles = 6 gallons: 2 gallons. Whereas between relationships focus on making comparisons between different units/measure-space such as 180 miles: 6 gallons = 60 miles: 2 gallons.

Have students explore graphs that are proportions and those that are not. Given various graphs, they may make tables using three points on the graph and decide whether they are proportional or not. Ask students what all proportional graphs have in common. Students should come to the conclusion that a proportional graph is a straight line that goes through the origin.

Sample Assessments and Performance Tasks

Reporting Category:
Ratios and Proportions
Standards:
7.RP. 1, 2, and 3
Approximate Portion of Test:
22% - 31%; 12 - 16 points

OST Test Specs:
Items may use all types of rational numbers.
At least one number in the ratio must be expressed as a fraction or a decimal.
Ratios can be expressed as a fraction (1/5), with a colon (1:5), or with words, e.g., per, to, each, for each, for every.

Instructional Resources

7.RP.1
Better Lesson
Shmoop
Khan Academy Videos
Dan Meyer Activities
Pizza Doubler
Yellow Starbursts
Ticket to Ride
Illustrative Mathematics
Cooking With the Whole Cup
Cider vs. Juice - Variation 1

7.RP.2
Better Lesson
Shmoop
Khan Academy Videos
Illustrative Mathematics
Sore Throat - Variation 1
Buying Coffee

7.RP.3
Dan Meyer Activity
Holes
Illustrative Mathematics
How Fast is Usain Bolt?

Adopted Resource	
Reveal: Lesson 1-1: Unit Rates Involving Ratios of Fractions Lesson 1-2: Understanding Proportional Relationships Lesson 1-3: Tables of Proportional Relationships Lesson 1-4: Graphs of Proportional Relationships Lesson 1-5: Equations of Proportional Relationships Lesson 1-6: Solve Problems Involving Proportional Relationships	**ALEKS:** Ratios, Proportions, and Measurement (ALEKS TOC): • Ratios and Unit Rates • Proportions Fractions (ALEKS TOC): • Multiplication and Division with Fractions

Return to Scope and Sequence

Module 2: Percent Problems

Unpacked Standards / Clear Learning Targets		
Learning Target **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. **7.EE.2** In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related.	**Essential Understanding** • A percent is a specific kind of ratio with a whole of 100. • All percent problems involve a part, a whole measured in some unit, and the same part and whole measured in hundredths. • Percents can be bigger than 100% and less than 1%. • Percent problems can be represented with a proportion or an equation. • Percent increase or percent decrease problems require careful attention to the referent whole by determining to what the whole (or 100% amount) a percentage refers. • Percent of change that involves an increase includes the following: tax, markups, gratuities, commissions, fees, etc. • Percent of change that involves a decrease includes the following: markdowns, discounts, etc. • Percent error is the difference between the approximate and exact value divided by the exact value.	**Academic Vocabulary** amount of error commission discount fee gratuity interest markdown markup percent error percent of change percent of decrease percent of increase principal sales tax selling price simple interest tip wholesale cost

COLUMBUS CITY SCHOOLS

Math 7

I Can Statements:

- I can recognize situations in which percentage proportional relationships apply.
- I can apply proportional reasoning to solve multistep ratio and percent problems, e.g., simple interest, tax, markups, markdowns, gratuities, commissions, fees, percent increase and decrease, percent error, etc.

Performance Level Descriptors:

Proficient:
- Solve a one-step, straightforward ratio or percent problem
- Solve a one-step, straightforward real-world ratio or percent problem
- Use proportional relationships to solve routine real-world and mathematical ratio and percent problems with multiple steps
- Recognize simple equivalent expressions

Accomplished (all of Proficient +):
- Apply proportional relationships to routine real-world and mathematical ratio and percent problems with multiple steps
- Understand that rewriting an expression can show how the quantities are related in familiar problem-solving contexts

Advanced (all of Proficient + all of Accomplished +):
- Apply proportional relationships to non-routine real-world and mathematical ratio and percent problems with multiple steps
- Understand that rewriting an expression can show how quantities are related in an unfamiliar problem-solving context

Prior Standard(s)

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Future Standard(s)

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.
A.SSE.3a,b,c Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions for exponential functions.
F.IF.8b Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
b. Use the properties of exponents to interpret expressions for exponential functions.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 1, page 43
- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45
- Ohio's K-8 Learning Progressions, Ratio and Proportional Relationships, page 15

Instructional Strategies

Because percents have been introduced as rates in Grade 6, the work with percents should continue to follow the thinking involved with rates and proportions. Solutions to problems can be found by using the same strategies for solving rates, such as looking for equivalent ratios or based upon understandings of decimals. Previously, percents have focused on those between 0 and 100, now percents above 100 and less than 1 are introduced.

Use examples when the final amount is greater than the original amount.

Proportion bars, tape diagrams, and double number lines can be useful to visualize percent increase and decrease problems. Discuss that there are two perspectives in solving percent increase and percent decrease problems. The first perspective is finding the increase or decrease and the adding/subtracting it to/from the original to get the new price. The second perspective is to combine the percents first (20% off would be 80% of the original price or a meal with a 15% tip would be 115% of the original bill), and then set up and solve the proportion or equation. Students should be able to explain why both situations work.

In percent problems, especially in percent change problems, it is vital to be able to correctly identify the whole (100% amount).

Students should solve a variety of problems involving percent including tax, interest, tip, mark-ups, mark-downs, commissions, fees, discounts etc. Since the students in Grade 6 do not formally set-up and solve proportions, they were limited to solving percent problems using ratio reasoning with models such as tables and double number line diagrams and 100 grids. Now in Grade 7 students can solve percents more formally using proportions and equations (7.EE.3-4).

Discuss how percents can be used to mislead people. They can make big numbers look small and small numbers look big. Which sounds better, a company laid off 5% of its workforce or a company laid off 2,000 people? Another example of deceptive use of percents could be choosing to use an increase instead of percent change or vice versa. For example a shirt that used to sell for $25 dollars now sells for $35, which is only a change of $10, but it is a 40% increase in price.

Absolute error is $|approximate\ value\ -\ exact\ value|$. Percent error is $\frac{|approximate\ value - exact\ value|}{|exact\ value|} \cdot 100\%$. Discuss the difference between absolute error and percent error. Explain that the percent error is useful for comparing the error to the original amount. The smaller the object is, the more precision is needed.

Sample Assessments and Performance Tasks

Reporting Categories:
Ratios and Proportions; The Number System

Standards:
7.NS.3 (R & P); 7.EE.2 (TNS)

OST Test Specs:
Items may use all types of rational numbers.
Ratios can be expressed as a fraction (1/5), with a colon (1:5), or with words, *e.g., per, to, each, for each, for every.*

Approximate Portion of Test: 22% 31%; 12 - 16 points (R & P); 28% - 37%; 15 - 19 points (TNS)	Items will require at least two steps. Percentages used can be greater than 100% or less than 1%.

Instructional Resources

Better Lesson
Shmoop
Khan Academy Videos
Illustrative Mathematics
Double Discounts

Adopted Resource

Reveal: Lesson 2-1: Percent of Change Lesson 2-2: Tax Lesson 2-3: Tips and Markups Lesson 2-4: Discounts Lesson 2-5: Interest Lesson 2-6: Commission and Fees Lesson 2-7: Percent Error	**ALEKS:** Percents (ALEKS TOC): • Percent Increase/Decrease • Understanding Percents • Interest • Percent of a Number

Return to Scope and Sequence

Module 3: Operations with Integers

Unpacked Standards / Clear Learning Targets

Learning Target	Essential Understanding	Academic Vocabulary
7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive	**Rational Numbers** • The set of integers consists of positive whole numbers, their opposites, and 0. • In a fraction the negative sign can be written in the numerator, the denominator, or out front, e.g., $\frac{-3}{4} = \frac{3}{-4} = -\frac{3}{4}$. **Addition and Subtraction** • When modeling operations with integers on a number line, the sign of the number indicates the direction and the number indicates the	Absolute Value Additive Identity Additive Inverse Property Additive Inverses Integer Opposites Zero Pair Difference

or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, p − q = p + (−q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. 7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1)(−1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then −(p/q) = (−p) /q = p/(–q). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	amount of spaces moved. • In a number line model, the subtraction sign means to change directions. • A number and its opposite are additive inverses; they have a sum of 0, i.e., $a + (-a) = 0$. • The absolute value of p − q is just the distance from p to q, regardless of direction. **Multiplication and Division** • A positive product is the result of multiplying two numbers with the same sign. • A negative product is the result of multiplying two numbers with different signs. • Division can be written using a fraction bar.	Graph Negative Integer Positive Integer Rational Sum Zero Integer

COLUMBUS CITY SCHOOLS

Math 7

I Can Statements:

- I can describe situations in which opposite quantities combine to make 0.
- I can represent and explain how a number and its opposite have a sum of 0 and are additive inverses.
- I can recognize and describe the rules when multiplying signed numbers.
- I can explain why integers can be divided except when the divisor is 0.
- I can solve real-world mathematical problems by adding, subtracting, multiplying, and dividing integers.

Performance Level Descriptors:

Proficient:
- Model addition and subtraction of simple rational numbers on the number line
- Recognize the additive inverse property
- Add, subtract, multiply, and divide integers
- Solve mathematical problems using the four operations on simple rational numbers
- Convert between familiar fractions and decimals
- Convert from fractions to decimals without technology

Accomplished (all of Proficient +):
- Solve mathematical problems using the four operations on rational numbers

Advanced (all of Proficient + all of Accomplished +):
- Interpret products and quotients of rational numbers in real-world contexts

Prior Standard(s)

4.OA.3 Solve multistep word problems posed with whole numbers and having whole number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
5.NF.5 5 Interpret multiplication as scaling (resizing).
6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.
6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.
6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge; use

Future Standard(s)

8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating.
8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology.

positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	**A.APR.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45
- Ohio's K-8 Learning Progressions, The Number System, pages 16-17

Instructional Strategies

Many students struggle with the negative sign. One way to help students is to talk about values, order, and direction instead of quantities. For example, positive 5 is greater than positive 4, but −4 is greater than −5.
Another reason for confusion is that the negative sign can mean several things:

- A sign attached to a number to form negative numbers;
- A subtraction; or
- An indication to take the opposite of

Because of the confusion around the negative sign it may be helpful for students to understand the different meanings of the negative sign and identify which meaning is used when in a problem including the meaning shifts.

Using both contextual and numerical problems, students should explore what happens when negatives and positives are combined. Repeated opportunities over time with models will allow students to compare the results of adding and subtracting pairs of numbers, leading to the generalization of the rules.

Two-color counters or colored chips can be used as a physical model for adding and subtracting integers. Integer chips allow the idea of the zero pair (Additive Inverse Property) to be apparent.

Number lines present a visual image for students to explore and record addition and subtraction results. One of the positive aspects about using a number line model is that it is not limited to integers; it also lends itself toward connections on the coordinate plane. Students can use number lines with arrows and hops. When using number lines, establishing which factor will represent the length, number, and direction of the hops will facilitate understanding.

Multiplying and dividing integers should be thought of as an extension of adding and subtracting integers. Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers.

In multiplication, the first factor indicates the number of sets, and the second factor indicates the size of the set. This can be easily modeled with situations involving a positive times a positive or a positive times a negative. A negative times a positive can be inferred using the Commutative Property of Multiplication.

A negative times a negative can be problematic, for students want to know "How can we have a negative group of something?" One way to view it is as repeated subtraction. If the first factor being positive indicates repeated addition, then the first factor being negative indicates repeated subtraction. Therefore "negative 3 sets of negative −2" or

−3(−2) means to remove 3 sets of −2 from zero.

Students will discover that they can multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. They should then analyze and solve problems leading to the generalization of the rules for operations with integers.

Another method for learning multiplication/division rules is to use patterns. Beginning with known facts, students can predict the answers for related facts, keeping in mind that the properties of operations apply.

Using the language of "the opposite of" helps some students understand the multiplication of negatively signed numbers ($-4 \cdot -4 = 16$, the opposite of 4 groups of −4). Discussion about the tables should address the patterns in the products, the role of the signs in the products, and commutativity of multiplication.

It is important when performing operations that students are able to justify their steps using the properties. Although, the focus should not be on identifying the properties of operation, teachers should be using their formal names in classroom discussion, so students are able to gain familiarity with and recognize the correct terminology

In Grade 6, students should have learned that the absolute value of a number does not take into account sign or direction; it only is a measure of distance (magnitude) from 0. Discourage students from saying that the "answer is always positive or 0" since that will lead to misconceptions when students encounter problems such as $|4x - 2| = 18$ in high school. Instead emphasize that it is the "distance from 0." This is why the value of something like $|x| = -5$ has no solution, since distance cannot be negative.

Sample Assessments and Performance Tasks

Reporting Category: The Number System **Standards**: 7.NS.1, 2, and 3; 7.EE.3 **Approximate Portion of Test**: 28% - 37%; 15 - 19 points	**OST Test Specs**: Items may use all types of rational numbers. Items must include at least 1 negative number. Students need to be able to recognize the formal names of properties. For 7.NS.2a, b, and c: items must include a negative number. Students need to be able to recognize the formal names of properties. Items involving estimation to assess reasonableness will not require the student to find the exact answer. Variables may need to be defined using appropriate units.

Instructional Resources

7.NS.1	7.NS.2	7.NS.3
Better Lesson Khan Academy Dan Meyer Activities	Better Lesson Khan Academy Videos Illustrative Mathematics:	Better Lesson Khan Academy Videos Dan Meyer

World Record Balloon Dog Gas Station Ripoff Graduation Dueling Discounts Illustrative Mathematics Bookstore Account Comparing Freezing Points Differences and Distances Differences of Integers Distances Between Houses Distances on the Number Line 2 Operations on the number line Rounding and Subtracting	Distributive Property of Multiplication Why is a Negative Times a Negative Always Positive? Temperature Change Decimal Expansions of Fractions Equivalent fractions approach to non-repeating decimals Repeating decimal as approximation Repeating or Terminating?	Graduation Illustrative Mathematics Sharing Prize Money

Adopted Resource

Reveal: Lesson 3-1: Add Integers Lesson 3-2: Subtract Integers Lesson 3-3: Multiply Integers Lesson 3-4: Divide Integers Lesson 3-5: Apply Integer Operations	**ALEKS:** Whole Numbers and Integers (ALEKS TOC): • Adding and Subtracting with Integers • Multiplying and Dividing with Integers • Plotting and Comparing Integers • Exponents and Order of Operations

Return to Scope and Sequence

Module 4: Operations with Rational Numbers

Unpacked Standards / Clear Learning Targets

Learning Target

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.
b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
c. Understand subtraction of rational numbers as adding the additive inverse, p − q = p + (−q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real world contexts.
d. Apply properties of operations as strategies to add and subtract rational numbers.
7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1)(−1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then −(p/q) = (−p) /q = p/(–q). Interpret quotients of rational numbers by describing real-world contexts.
c. Apply properties of operations as strategies to multiply and divide rational numbers.
d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or

Essential Understanding

Rational Numbers

• A rational number is any number that can be written as the quotient or fraction $\frac{P}{Q}$ of two integers, a numerator p, and non-zero denominator q.
• A rational number can be converted to a decimal using long division; the decimal form of a rational number terminates in 0s or repeats.

Addition and Subtraction

•Subtraction of rational numbers is adding the additive inverse, i.e., $p - q = p + (-q)$

Multiplication and Division

• Multiplication of rational numbers can be modeled on the number line.
• Division is the inverse of multiplication, so the same rules for rational numbers apply.
• Every quotient of integers (with a nonzero divisor) is a rational number.
• A repeating quotient has a line of the repeating numerals.

Academic Vocabulary

Absolute value
Additive inverse
Difference
Graph
Integer
Negative integer
Opposites
Positive integer
Rational
Sum
Zero integer

eventually repeats. 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		

I Can Statements:

- I can identify properties of addition and subtraction when adding and subtracting rational numbers.
- I can apply properties of operations as strategies to add and subtract rational.
- I can identify subtraction of rational numbers as adding the additive inverse property to subtract rational numbers, p-q= p+ (-q).
- I can recognize and describe the rules when multiplying signed numbers.
- I can explain why integers can be divided except when the divisor is 0.
- I can apply properties of operations as strategies to multiply and divide rational numbers.
- I can convert a rational number to a decimal using long division.
- I can add/multiply/subtract/divide rational numbers.
- I can solve real-world mathematical problems by adding, subtracting, multiplying, and dividing rational numbers, including complex fractions.
-

Performance Level Descriptors:

Proficient:
- Model addition and subtraction of simple rational numbers on the number line
- Recognize the additive inverse property
- Add, subtract, multiply, and divide integers
- Solve mathematical problems using the four operations on simple rational numbers
- Convert between familiar fractions and decimals
- Convert from fractions to decimals without technology

Accomplished (all of Proficient +):
- Solve mathematical problems using the four operations on rational numbers

Advanced (all of Proficient + all of Accomplished +):
- Interpret products and quotients of rational numbers in real-world contexts

COLUMBUS CITY SCHOOLS

Math 7

Prior Standard(s)		Future Standard(s)
4.OA.3 3 Solve multistep word problems posed with whole numbers and having whole number answers using the four operations, including problems in which Remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators **5.NF.3** Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	**5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **6.NS.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction modelsG and equations to represent the problem. **6.NS.3** Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation. **6.NS.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **6.NS.7** Understand ordering and absolute value of rational numbers.	**7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **8.NS.1** Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating. **8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **A.APR.7** 7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45
- Ohio's K-8 Learning Progressions, The Number System, pages 16-17

Instructional Strategies

- In Grade 7 the awareness of rational and irrational numbers is initiated by observing the result of changing fractions to decimals. They can do this by making equivalent fractions with denominators using powers of ten or by using long division.
- Students should be provided with families of fractions, such as, sevenths, ninths, thirds, etc. to convert to decimals using long division. The equivalent fractions can be grouped and named (terminating or repeating). Students should begin to see why these patterns occur. Knowing the formal vocabulary rational and irrational is not expected for the students. Terminating decimals fall exactly on some tick mark on a number line; however repeating decimals do not. For example 1 3 is always subdividing an interval of smaller and smaller sizes. Students can also explore patterns to determine which fractions repeat and which fractions terminate. Technology can be used to aid students in recognizing patterns.
- Students can also use their knowledge of proportions and unit rates to help them understand why they can convert a fraction to a decimal by dividing.
- See strategies for operations with integers (Module 3) for additional strategies. Please note that not all strategies will work with rational numbers, however.
- Students need to become fluent in using operations and properties of operations with all rational numbers, not just with integers.
- Although all properties of operations should be addressed, this cluster should especially emphasize the Additive Inverse Property, the Multiplicative Inverse Property, and the Distributive Property.
- Students should become familiar with solving problems involving complex fractions. Draw attention to the Multiplicative Identity Property and the Multiplicative Inverse Property for solving expressions with complex fractions. This concept connects nicely with cluster 7.RP.1-3.

Sample Assessments and Performance Tasks

Reporting Category:
The Number System
Standards:
7.NS.1, 2, and 3; 7.EE.3
Approximate Portion of Test:
28% - 37%; 15 - 19 points
OST Test Specs:
Items may use all types of rational numbers.
Items must include at least 1 negative number.
Students need to be able to recognize the formal names of properties.
For 7.NS.2a, b, and c: items must include a negative number.
Students need to be able to recognize the formal names of properties.
Items involving estimation to assess reasonableness will not require the student to find the exact answer.
Variables may need to be defined using appropriate units.

COLUMBUS CITY SCHOOLS

Math 7

Instructional Resources		
7.NS.1 Better Lesson Khan Academy Dan Meyer Activities World Record Balloon Dog Gas Station Ripoff Graduation Dueling Discounts Illustrative Mathematics Bookstore Account Comparing Freezing Points Differences and Distances Differences of Integers Distances Between Houses Distances on the Number Line 2 Operations on the number line Rounding and Subtracting	7.NS.2 Better Lesson Khan Academy Videos Illustrative Mathematics: Distributive Property of Multiplication Why is a Negative Times a Negative Always Positive? Temperature Change Decimal Expansions of Fractions Equivalent fractions approach to non-repeating decimals Repeating decimal as approximation Repeating or Terminating?	7.NS.3 Better Lesson Khan Academy Videos Dan Meyer Graduation Illustrative Mathematics Sharing Prize Money

Adopted Resource	
Reveal: Lesson 4-1: Rational Numbers Lesson 4-2: Add Rational Numbers Lesson 4-3: Subtract Rational Numbers Lesson 4-4: Multiply Rational Numbers Lesson 4-5: Divide Rational Numbers Lesson 4-6: Apply Rational Number Operations	**ALEKS:** Decimals (ALEKS TOC): • Venn Diagrams and Sets of Rational Numbers

Return to Scope and Sequence

Module 5: Simplify Algebraic Expressions

Unpacked Standards / Clear Learning Targets

Learning Target

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.2 In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related.

Essential Understanding

- Equivalent expressions always have the same value even if written in different forms.
- Equivalent expressions can be generated using properties of operations (Distributive Property, Associative Properties of Multiplication, Associative Property of Addition, Commutative Property of Multiplication, Commutative Property of Addition, and Identity Property of Multiplication).
- The order of operations is used to generate equivalent algebraic expressions.
- The coefficient of a single variable is 1 even if it is not written. For example, –x = –1x and x = 1x.
- A fractional coefficient can be written in two ways, e.g., $\frac{x}{3} = \frac{1}{3}x$.
- Negative rational terms can be written in three ways, e.g., $\frac{1}{-3} = \frac{-1}{3} = -\frac{1}{3}$.
- In problems involving percentages, 100% of the variable x can be written as x = 1x.
- Factoring a GCF can be used to write an equivalent expression.
- Writing expressions in equivalent forms can serve different purposes and provide different ways of seeing a problem in context.

Academic Vocabulary

Algebraic
Coefficient
Constant
Distributive property
Exponent
Expression
Factor
Like term
Quantities
Rational
Sum
Term
Variable

I Can Statements:

- I can combine like terms with rational coefficients.
- I can factor and expand linear expressions with rational coefficients using the distributive property.
- I can apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- I can write equivalent expressions with fractions, decimals, percents, and integers.
- I can rewrite an expression in an equivalent form in order to provide insight about how quantities are related in a problem context.
- I can convert between numerical forms as appropriate.
- I can solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically.
- I can apply properties of operations to calculate with numbers in any form.
- I can assess the reasonableness of answers using mental computation and estimation strategies.

Performance Level Descriptors		
Proficient: ● Recognize simple equivalent expressions ● Apply properties of operations to factor and expand linear expressions with positive integer coefficients Apply properties of operations to factor and expand linear expressions with simple rational coefficients	Accomplished (all of Proficient +): ● Apply properties of operations to factor and expand linear expressions with rational coefficients ● Understand that rewriting an expression can show how quantities are related in familiar problem-solving contexts	Advanced (all of Proficient + all of Accomplished +): ● Apply properties of operations to factor and expand linear expression with complex rational coefficients ● Understand that rewriting an expression can show how quantities are related in an unfamiliar problem-solving context

Prior Standard(s)	Future Standard(s)	
6.EE.3 Apply the properties of operations to generate equivalent expressions. **6.EE.4** 4 Identify when two expressions are equivalent, i.e., when the two expressions name the same number regardless of which value is substituted into them.	**8.EE.7** Solve linear equations in one variable **A.APR.1** Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **A.SSE.1** Interpret expressions that represent a quantity in terms of its context. **A.SSE.2** Use the structure of an expression to identify ways to rewrite it.	**A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **N.CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

Instructional Strategies

It is important that students are able to justify their thinking using the properties. Although the focus should not be on identifying the properties of operation, teachers should be using their formal names in classroom discussion so students are able to gain familiarity with and recognize the correct terminology.

Provide opportunities for students to use and understand the properties of operations. These include the Commutative, Associative, Identity, and Inverse Properties of Addition and of Multiplication, the Zero Property of Multiplication, and the Distributive Property.

Subtraction should be thought of as the opposite of addition, and division should be thought of as the opposite of multiplication. Note: Avoid PEMDAS as it leads to many misconceptions and errors in computation.

Writing equivalent expressions includes simple cases of factoring out a GCF. This can be illustrated using the area model that students are already familiar with. In this model the students are given the areas, and they are asked to find the lengths and widths. It might be wise to point out that, although in real-life length and width cannot be negative,

our model allows for lengths and widths to be negative to illustrate the concept.

Students started combining like terms in Grade 6. Now they will extend this concept to negative numbers. It may be helpful to use Algebra tiles to illustrate this process as it will prevent future misconceptions from forming such as trying to combine 2x and $3x^2$. Students should then extend that concept to other rational numbers besides integers.

Sample Assessments and Performance Tasks

Reporting Category: The Number System **Standards**: 7.EE.1 and 2 **Approximate Portion of the Test:** 28% - 37%; 15 - 19 points	**OST Test Specs:** Items may use all types of rational numbers. Items use only linear expressions. Negative numbers and multiple operations may be used. Students need to be able to recognize the formal names of properties.

Instructional Resources

7.EE.1	7.EE.2
Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Writing Expressions	Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Ticket to Ride Guess My Number

Adopted Resource

Reveal:	**ALEKS:**
Lesson 5-1: Simplify Algebraic Expressions Lesson 5-2: Add Linear Expressions Lesson 5-3: Subtract Linear Expressions Lesson 5-4: Factor Linear Expressions Lesson 5-5: Combine Operations with Linear Expressions	Equations and Inequalities (ALEKS TOC): • Simplifying Algebraic Expressions • The Distributive Property

Return to Scope and Sequence

COLUMBUS CITY SCHOOLS

Math 7

Module 6: Write and Solve Equations

Unpacked Standards / Clear Learning Targets

Learning Target

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Essential Understanding

- Variables are used to represent a quantity.
- The order of operations is used to write and solve equations given within a context of a word problem.
- A solution is a value that makes an equation or an inequality true.
- Inverse operations may be used to solve equations and inequalities.
- Equivalent expressions always have the same value even if written in different forms.
- Equivalent expressions can be generated by using properties of operations (distributive property, associative, commutative, identity and inverse properties of multiplication and addition).
- A term includes the operational sign in front of it.

Academic Vocabulary

Constant
Distributive property
Equation
Exponent
Coefficient
Factor
Like term
Quantities
Term
Variable
Coefficients
Factor
Properties

I Can Statements:

- I can identify the sequence of operations used to solve an algebraic equation of the form $px + q = r$ and $p(x + q) = r$.
- I can fluently solve equations of the form $px + q = r$ and $p(x + q) = r$ with speed and accuracy.
- I can use variables and construct equations to represent quantities of the form $px + q = r$ and $p(x + q) = r$ from real--world and mathematical problems.

Performance Level Descriptors:

Proficient:
- Solve simple equations
- Solve two-step equations with integer coefficients
- Use variables to create and solve simple equations and inequalities that model word problems

Accomplished (all of Proficient +):
- Construct equations and inequalities with a variable to solve routine problems

Advanced (all of Proficient + all of Accomplished +):
- Construct equations and inequalities with more than one variable to solve non-routine problems
- Use variables to represent and reason with quantities in real-world and mathematical situations

Prior Standard(s)	Future Standard(s)
6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q, and x are all nonnegative rational numbers.	**8.EE.7** Solve linear equations in one variable. **8.EE.8** Analyze and solve pairs of simultaneous linear equations graphically. **A.CED.1** Create equations and inequalities in one variable and use them to solve problems.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45
- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 3, pages 46-47
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

Instructional Strategies

Continue to build on students' understanding and application of writing and solving one-step equations (6th Grade) from a problem situation to problem situations that require multi-step equations and inequalities.

This is also the context for students to practice using rational numbers including integers and positive and negative fractions and decimals. It is appropriate to expect students to show their steps in their work. Students should be able to move toward explaining their thinking using correct terminology.
To assist students' assessment of the reasonableness of their answers, especially problem situations involving fractional or decimal numbers, use whole-number approximations for the computation and then compare to the actual computation.

In connection with 7.EE.1 students should apply the properties of operations and equality found in Table 3 and Table 4 of Ohio's Learning Standards. Teachers should be using the correct terminology to justify steps when performing operations and solving equations. Although Grade 7 students should not be required to know the formal names of the properties, they should be encouraged to recognize them and use them to justify their steps when solving equations.

Experiences in solving equations should move through Bruner's stages of concrete, pictorial, and algebraic/abstract representation. Utilize experiences with the pan balance model, hangers, tape diagrams, or Algebra tiles as a visual tool for maintaining equality (balance):
- First with simple numbers;
- Then with pictures symbolizing relationships; and
- Finally, with rational numbers. This allows understanding to develop as the complexity of the problems increases.

Some studies have shown that a students' fractional knowledge correlates with their ability to write equations. Therefore encourage students to solve equations with fractions by using diagrams instead of just using inverse operations. This may aid in creating understanding to alleviate the misuse of fraction rules in later grades/courses. Numbers that are easily modeled should be used initially until students internalize the process.

Sample Assessments and Performance Tasks	
Reporting Category: The Number System **Standards**: 7.EE.4 **Approximate Portion of the Test:** 28% - 37%; 15 - 19 points	**OST Test Specs**: Items may use all types of rational numbers. Items involving estimation to assess reasonableness will not require the student to find the exact answer. Students need to be able to recognize the formal names of properties. Variables may need to be defined using appropriate units. For 7.EE.4a, equations must be of the form $px + q = r$ or $p(x + q) = r$, where p, q, and r are specific rational numbers.

Instructional Resources
Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Bookstore account Sports Equipment Set

Adopted Resource	
Reveal: Lesson 6-1: Write and Solve Equations Lesson 6-2: Solve Two-Step Equations: px + q = r Lesson 6-3: Write and Solve Two-Step Equations: px + q = r Lesson 6-4: Solve Two-Step Equations p(x+q) = r Lesson 6-5: Write and Solve Two-Step Equations: p(x+q) = r	**ALEKS:** Equations and Inequalities (ALEKS TOC): • Multi-Step Equations • Applications of Equations Whole Numbers and Integers (ALEKS TOC): • One-Step Equations

Return to Scope and Sequence

COLUMBUS CITY SCHOOLS

Math 7

Module 7: Write and Solve Inequalities

Unpacked Standards / Clear Learning Targets

Learning Target

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Essential Understanding

- Inequalities have infinitely many solutions.
- Solutions to inequalities can be represented on number line diagrams.
- Point c is not included in the graphical solution to $x > c$ or $x < c$; the number line diagram represents this with an open circle around point c.
- Point c is included in the graphical solution to $x > c$ or $x < c$; the number line diagram represents this with a closed circle at point c.
- All of the solutions to an inequality are represented with a shaded region on a number line diagram.
- The inequality $x > c$ is equivalent to $c < x$, and $x > c$ is equivalent to $c < x$.
- When multiplying or dividing both sides of an inequality by a negative number, the order of the comparison it represents is reversed.

Academic Vocabulary

Constant
Distributive property
Exponent
Coefficient
Factor
Inequality
Greater Than
Less Than
Like term
Quantities
Term
Variable
Coefficients
Factor
Properties

I can statements:

- I can graph the solution set of the inequality of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers.
- I can solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers.

Performance Level Descriptors:

Proficient:
- Identify a solution of an inequality
- Solve simple inequalities with positive integer coefficients
- Use variables to create and solve simple equations and inequalities that model word problems

Accomplished (all of Proficient +):
- Construct equations and inequalities with a variable to solve routine problems

Advanced (all of Proficient + all of Accomplished +):
- Construct equations and inequalities with more than one variable to solve non-routine problems
- Use variables to represent and reason with quantities in real-world and mathematical situations

Prior Standard(s)	Future Standard(s)
6.EE.8 Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	**A.CED.1** Create equations and inequalities in one variable and use them to solve problems. **A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45
- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 3, pages 46-47
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

Instructional Strategies

In Grade 6 students wrote inequalities in the forms of x > c and c < x. in Grade 7 students use > and <. Discuss why teachers should, when graphing on a number line, a closed circle represents > and < and an open circle represents > and <. Students should also have practice solving one and two-step inequalities with rational numbers.

Present situations where the variable is both on the left and the right side of the equality. Students need to be fluent in solving inequalities where the variable is on the left and right of the inequality for later algebraic concepts using compound inequalities. Therefore, discourage students from always writing the variable on the left side of an inequality.

When graphing, teachers should avoid telling students that the inequality points the same direction on the number line as the arrow; this creates a misconception that is hard to break when students work on compound inequalities in high school. An alternative strategy is to ask students to name 3 points that make an inequality true, and then draw the arrow in that direction.

Students should be able to create equations and inequalities from real-world situations where they always precisely define the variable(s).

Provide multiple opportunities for students to work with multi-step problem situations that have multiple solutions and therefore can be represented by an inequality. Students need to be aware that values can satisfy an inequality but not be appropriate for the situation, therefore limiting the solutions for that particular problem.

COLUMBUS CITY SCHOOLS

Math 7

Sample Assessments and Performance Tasks

Reporting Category:
The Number System
Standards:
7.EE.4
Approximate Portion of the Test:
28% - 37%; 15 - 19 points

OST Test Specs:
Items may use all types of rational numbers.
For 7.EE.4b, inequalities must be of the form $px + q > r$, $px + q \geq r$, $px + q < r$ or $px + q \leq r$, where p, q, and r are specific rational numbers.
Items may require graphing a solution to an inequality on a number line.

Instructional Resources

Better Lesson
Shmoop
Khan Academy Videos
Illustrative Mathematics
Bookstore account
Sports Equipment Set

Adopted Resource

Reveal:

Lesson 7-1: Write and Solve Inequalities
Lesson 7-2: Write and Solve One-Step Addition and Subtraction Inequalities
Lesson 7-3; Solve One-Step Multiplication and Division Inequalities with Positive Coefficients
Lesson 7-4: Solve One-Step Multiplication and Division Inequalities with Negative Coefficients
Lesson 7-5: Write and Solve One-Step Multiplication and Division Inequalities
Lesson 7-6: Write and Solve Two-Step Inequalities

ALEKS:

Equations and Inequalities (ALEKS TOC):

- One-Step Inequalities
- Writing and Graphing Inequalities
- Applications of Inequalities
- Multi-Step Inequalities

Return to Scope and Sequence

Module 8: Geometric Figures

Unpacked Standards / Clear Learning Targets

Learning Target

7.G.1 Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals.
a. Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale.
b. Represent proportional relationships within and between similar figures.

7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions.
a. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
b. Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions.

7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Essential Understanding

Special Angle Pairs

- Two angles are supplementary when their angle measures have a sum of 180 degrees.
- Two angles are complementary when their angle measures have a sum of 90 degrees.
- Vertical angles are the angles opposite each other when two lines intersect; the angles are congruent.
- Vertical angles are congruent because they are both supplementary to the same angle.
- Two angles are adjacent when they have a common side and a common vertex and do not overlap.

Drawing Geometric Figures

- The sum of all three angles in any triangle equals 180 degrees.
- The sum of all four angles in any quadrilateral equals 360 degrees.
- Three possible outcomes exist when constructing triangles with given measurements of sides and/or angles: a unique triangle, more than one triangle, or no triangle.
- Some quadrilaterals have more specific names based on relationships such as pairs of parallel sides, congruent sides, and angle relationships.

Similar Figures

- Angles are congruent if they are equal in measure. Note: 7th grade students may use the term "equal in measure" in place of congruent.
- Similar figures have corresponding angles that are congruent and corresponding side lengths that are proportional.
- Applying a scale factor greater than one results in a bigger image.
- Applying a scale factor of 1 results in a congruent image. Note: Students are not required to understand congruency of two figures until 8th grade.
- Applying a scale factor less than 1 but greater than zero results in a smaller image.

Slicing Three-Dimensional Figures

- Slicing a three-dimensional figure results in a two-dimensional shape.
- Slicing a three-dimensional figure in different ways could result in different shapes.

Academic Vocabulary

Angle
Complementary
Supplementary
Vertical angle
Side
Vertex
Adjacent
Opposite
Congruent
Common Side
Common Vertex
Geometric figure
Triangle
Quadrilateral
Unique
Similar Figures
Scale drawing
Scale Factor
Three-Dimensional
Two-Dimensional
Plane sections
Protractor
Right rectangular prisms
Right rectangular pyramid

COLUMBUS CITY SCHOOLS

Math 7

I Can Statements:

- I can describe and locate supplementary, complementary, and vertical angles in figures.
- I can describe the relationship between supplementary, complementary, and vertical angles.
- I can write equations to represent angle relationships with unknown angle measures.
- I can solve equations for unknown angles using supplementary, complementary, vertical, and adjacent angles.
- I can construct triangles from three given angle measures to determine when there is a unique triangle, more than one triangle or no triangle using appropriate tools (freehand, rulers, protractors, and technology).
- I can construct triangles from three given side measures to determine when there is a unique triangle, more than one triangle or no triangle using appropriate tools (freehand, rulers, protractors, and technology).
- I can use ratios and proportions to create scale drawing.
- I can identify corresponding sides of scaled geometric figures.
- I can compute lengths and areas from scale drawings using strategies such as proportions.
- I can solve problems involving scale drawings of geometric figures using scale factors.
- I can reproduce a scale drawing that is proportional to a given geometric figure using a different scale.
- I can determine which conditions create unique triangles, more than one triangle, or no triangle.
- I can analyze given conditions based on the three measures of angles or sides of a triangle to determine when there is a unique triangle, more than one triangle or no triangle.
- I can define slicing as the cross-section of a 3D figure.
- I can describe the two-dimensional figures that result from slicing a three dimensional figure such as a right rectangular prism or pyramid.
- I can analyze three-dimensional shapes by examining two dimensional cross sections.

Performance Level Descriptors

Proficient:

- Determine a scale from scale drawings of geometric figures and compute an actual length given a measurement in a scale drawing and the scale
- Solve problems involving scale drawings of geometric figures, including computing actual areas from a scale drawing and represent proportional relationships among similar figures
- Recognize simple geometric shapes based on given conditions
- Draw geometric shapes with given conditions

Accomplished (all of Proficient +):

- Solve real-world problems involving similar figures
- Identify the two-dimensional figures that result from non-routine slices of prisms and pyramids
- Use supplementary, complementary, vertical, and adjacent angles to solve multi-step problems with angle measurements expressed as variables in degrees.

Advanced (all of Proficient + all of Accomplished +):

- Reproduce scale drawings at a different scale to solve real-world problems

- Identify the two-dimensional figures that result from routine slices of prisms and pyramids
- Classify pairs of angles
- Determine whether a set of any three given angle or side length measurements can result in a triangle or whether a quadrilateral could be represented by given angles and/or side lengths
- Using technology or math tools, determine whether a set of any three given angle or side length measures can result in a unique triangle, more than one triangle, or no triangles at all and construct quadrilaterals with given conditions
- Use supplementary, complementary, vertical, or adjacent angles to solve problems with angles expressed as numerical measurements

Prior Standard(s)	Future Standard(s)	
4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. 6.G.1 Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems. 7.RP.2 Recognize and represent proportional relationships between quantities. 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b. 8.G.1 Verify experimentally the properties of rotations, reflections, and translations. G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. G.CO.9 Prove and apply theorems about lines and angles. Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. G.MG.3 Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.

Content Elaborations

- Ohio's K-8 Critical Area of Focus Grade 7, Number 3, pages 46-47
- Ohio's K-8 Learning Progressions, Geometry, page 21
- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 1, page 43

Instructional Strategies

This cluster focuses on the importance of visualization in the understanding of Geometry. Being able to visualize and then represent geometric figures on paper is essential to solving geometric problems. After much work is done on paper, Geometry software can aid in students' understanding of Geometry.

Provide students the opportunities to explore angle relationships. At first they can measure and find patterns among the angles of intersecting lines or within polygons. Then they can utilize the relationships to write and solve equations for multi-step problems.

A student often incorrectly thinks that a wide angle with short sides is smaller than a narrow angle with long sides. To confront this problem have students compare angles with different side lengths.

Students should regularly be exposed to shapes and figures from many perspectives and orientations, not just the prototypical example.

Many careers and everyday activities require spatial reasoning. Some research suggests that 7th grade is the optimal time for developing spatial visualization. Sketching figures can help students develop an intuitive understanding of geometry. Although drawings should become precise over time, informal free-hand sketches can help develop spatial reasoning.

Constructions facilitate understanding of geometry. Provide opportunities for students to physically construct triangles and quadrilaterals with straws, sticks, or geometry apps. This should be done prior to using rulers, compasses, and/or protractors. Have students discover and justify the side and angle conditions that will form triangles or quadrilaterals.

Similarity is an increase or decrease that is multiplicative in nature instead of additive; this is a new concept for students.

Although the focus of this cluster is on rectangles and triangles, it may be useful to discuss why all circles are similar.

As an introduction to scale drawings in geometry, students should be given the opportunity to explore scale factor as the number of times you multiply the side measure of one figure to obtain the corresponding side measure of a similar figure. It is important that students first experience this concept concretely progressing to abstract contextual situations. Pattern blocks provide a convenient means of developing the foundation of scale. Choosing one of the pattern blocks as an original shape, students can then create the next-size shape using only those same-shaped blocks. After students have time to use the shapes concretely, they should also practice drawing them.

Regularly provide students with figures that are not similar to ensure that students are continually checking for similarity.

Provide opportunities for students to use scale drawings of geometric figures with a given scale. The opportunities should require them to draw and label the dimensions of

the new shape. Initially, measurements should be in whole numbers, progressing to measurements expressed with rational numbers. This will challenge students to apply their understanding of fractions and decimals.

Provide word problems that require finding missing side lengths, perimeters, or areas. In addition, allow students to design their own word problems asking for missing side lengths, perimeters, and/or areas.

Slicing three-dimensional figures to observe the cross sections formed helps develop three-dimensional visualization skills. Students should have the opportunity to physically create some of the three-dimensional figures, slice them in different ways, and describe in pictures and words what they discover. For example, use clay or playdough to form a cube, then pull string through it at different angles and record the shape(s) of the slices found. Challenges can be given: "See how many different two-dimensional cross sections you can create by slicing a cube."

Sample Assessments and Performance Tasks

Reporting Category: Geometry **Standards**: 7.G.1, 2, 3, and 5 **Approximate Portion of Test:** 20% - 25%; 11 - 13 points (with Module 9)	**OST Test Specs**: Figures are limited to triangles and special quadrilaterals Scale factors can be any positive rational number not equal to 1. Figures are limited to triangles and quadrilaterals. Items willvolus on Van Hiele Level 1 (Analysis) with some aspects of Van Hiele Level 2 (Informal Deduction/Abstraction). Items will not require knowledge of the hierarchy of quadrilaterals (e.g., all squares are rhombuses but not all rhombuses are squares) Prisms and pyramids can have bass up to six sides All slices will be parallel or perpendicular to the base of the figure Items may use all types of rational numbers Angles must be measured in degrees. Items may require the student to refer to two angles as "equal in measure". Items will not require students to understand congruency of two figures.

Instructional Resources

7.G.1	7.G.2	7.G.3	7.G.5
Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics	Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics	Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics	Better Lesson Shmoop Khan Academy Videos

Floor Plan Rescaling Washington Park	A Bug's Life	Cube Ninjas Dan Meyer Activity World Record Ballon Dog	

Adopted Resource	
Reveal: Lesson 8-1: Vertical and Adjacent Angles Lesson 8-2: Complementary and Supplementary Angles Lesson 8-3: Triangles Lesson 8-4: Scale Drawings Lesson 8-5: Three-Dimensional Figures	**ALEKS:** Lines, Angles, and Polygons (ALEKS TOC): • Angle Relationships • Classifying and Measuring Angles • Triangle Constructions and Triangle Inequalities • Classifying Triangles Ratios, Proportions, and Measurement (ALEKS TOC): • Scale Factors and Scale Drawings • Proportions Perimeter, Area, and Volume (ALEKS TOC): • Three-Dimensional Figures

Return to Scope and Sequence

Module 9: Measure Figures

Unpacked Standards / Clear Learning Targets		
Learning Target 7.G.4 Work with circles. a. Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. b. Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems. 7.G.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	**Essential Understanding** **Circles** • Points on a circle are the same distance from the center. • A circle is created by connecting all the points equidistant from the center point • The radius is the distance from any point on the circle to the center. • The diameter is a straight line that passes through a circle and its center point where both endpoints lie on the circle. • The diameter is twice the radius. • The circumference is the distance around a circle.	**Academic Vocabulary** Area Circumference Diameter Radius Circle Pi Cube Polygon Quadrilateral

	• Pi is a constant ratio of a circle's circumference divided by its diameter. • The symbol for pi is π. **Applications of Area, Surface Area, and Volume** • A right prism is a prism whose bases are parallel to one another and whose lateral faces are rectangles. • In a right prism the bases are perpendicular to the vertical sides. • Prisms can have bases other than rectangles. • The two bases of a prism are the same shape and size.	Right prism Surface area Three-dimensional Triangle Two-dimensional Volume

I Can Statements:

- I can determine the parts of a circle including radius, diameter, area, circumference, center and chord.
- I can identify π.
- I can recognize the formulas for area and circumference of a circle.
- I can determine the formulas for area and circumference of a circle, and find its area.
- I can find its circumference, given the area of a circle.
- I can justify that π can be derived from the circumference and diameter of a circle.
- I can apply the circumference or area formulas to solve mathematical and real-world problems.
- I can determine the formulas for area and volume and then procedure for finding surface area and when to use them in real-world and math problems for two and three dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.
- I can solve real-world and math problems involving area, surface area and volume of two and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Performance Level Descriptors:

Proficient:
- Identify the parts of a circle
- Calculate the circumference of a circle in mathematical problems
- Calculate the area and circumference of a circle in real-world and mathematical problems
- Calculate the area of triangles and rectangles
- Calculate the area of quadrilaterals and polygons
- Solve problems involving the area of two-dimensional objects composed of triangles, quadrilaterals, and polygons
- Calculate the volume of cubes

Accomplished (all of Proficient +):
- Given the circumference of a circle, determine its area
- Solve real-world and mathematical problems involving the surface area of three-dimensional objects composed of triangles and rectangles

Advanced (all of Proficient + all of Accomplished +):
- Solve problems using formulas for the area and circumference of a circle
- Informally describe the relationship between the two measures
- Solve complex problems involving the surface area and volume of three-dimensional figures with polygonal faces

- Calculate the volume of right rectangular prisms
- Solve routine real-world and mathematical problems involving the surface area and volume of three-dimensional objects composed of cubes and right prisms.

Prior Standard(s)	Future Standard(s)
6.G.1 Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems. 6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = $\ell \cdot w \cdot h$ and V = $B \cdot h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. 6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real world and mathematical problems.	8.G.6 Analyze and justify an informal proof of the Pythagorean Theorem and its converse. G.C.5 Find arc lengths and areas of sectors of circles. G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. G.MG.1 Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder. G.MG.3 Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.

Content Elaborations

- Ohio's K-8 Critical Area of Focus Grade 7, Number 3, pages 46-47
- Ohio's K-8 Learning Progressions, Geometry, page 21

Instructional Strategies

Knowing that a circle is created by connecting all the points equidistant from a point (center) is essential to understanding the relationships between radius, diameter, circumference, $\pi\pi$, and area. Students can observe this by folding a paper plate several times, finding the center at the intersection, then measuring the lengths between the center and several points on the circle, the radius. Measuring the folds through the center, or diameters leads to the realization that a diameter is two times a radius.

Students are to explore relationships between area, circumference, diameter, and radius, recognizing the constant of proportionality between each of these elements without formally defining the irrational nature of π.

A visual for understanding the area of a circle can be modeled by cutting up a paper plate into 16 pieces along diameters and reshaping the pieces into a parallelogram. Explain to students that the perimeter of the circle is the circumference. Ask students to identify where the circumference and the radius is in their new shape. Identifying the radius gives cause for conversations. They should come to the conclusion that the height is the radius and the length is ½ of the circumference. Therefore another formula for area is $A = \frac{1}{2}Cr$. Since students already found the area of circle as $A = \pi r^2$ or $A = \pi rr$, students should realize that $\frac{1}{2}C = \pi rr$. If they divide each side by r, then can simplify the

formula to $\frac{1}{2}C = \pi r$. Now if they solve for C by multiplying each side by 2, they will get $C = 2\pi r$, and they may see that $2r = d$and change the formula to get $C = d\pi$.

Most area problems should be given in a real-world context.

Students can find the area of regular polygons using triangles if the apothem is given. (Students do not need to use the term apothem.)

Instead of being restricted to using nets to find surface area, students may prefer to draw the different views of a structure (front, right, top). The use of formulas to find surface area should be discouraged.

Some students will prefer drawing nets and others will prefer drawing the six different views. After practice with both methods, let students use their preferred method.

Once students are comfortable finding the surface area of unit cubes, tell students that the cubes' lengths are rational numbers such as ¼ inch and have them calculate the surface area.

To develop their spatial reasoning skills, have students build structures with unit cubes and find the surface area of their structure. They can solve these problems by using a net or by drawing the different views of the figure (front, right, top). Also give students a net, and have them build the structure from the net.

Connect the concept of volume of prisms to slicing geometric solids (7.G.3). Have students build solids using layers in the shape of the base. They can also build three-dimensional prisms and pyramids by using coffee stirrers with twist ties, modeling clay, plastic drinking straws, or rods created out of rolled newspaper. Connect triangular prisms with rectangular prisms by letting students discover that half of a rectangular prism could be a triangular prism.

Again, emphasize volume in the context of real-world problems. Use problems that connect volume to surface area.

Some students incorrectly think that the top and bottom of a prism are always the bases of a prism, not realizing that a prism can be rotated. Reinforce that the bases must be two parallel faces.

Sample Assessments and Performance Tasks

Reporting Category:
Geometry
Standards:
7.G.4 and 6
Approximate Portion of Test:
20% - 25%; 11 - 13 points (with Module 8)

OST Test Specs:
Only whole circles and semicircles will be used.
Items may ask for exact answers in terms of π.
Scoring will allow for using 3.14, using the π-button on a calculator, or $\frac{22}{7}$ for the value of π
Items may use all types of rational numbers.
Two-dimensional shapes are limited to polygons
Three-dimensional shapes are limited to right prisms and pyramids.
Prisms and pyramids can have bases up to six sides.

	Items will not require the student to find the volume of a pyramid.

Instructional Resources	
7.G.4 Better Lesson Shmoop Khan Academy Videos Dan Meyer Activities Pokemon Go Cheat Broken Falcon Radar Volcano Penny Circle Coffee Traveler Popcorn Picker Dan Meyer Activities (cont.) Coin Carpet Pizza Doubler Brita Illustrative Mathematics Designs Eight Circles Other Resources Circle Tool Circles and Their Areas	7.G.6 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Stand Under the Swing Dan Meyer Activity Coin Carpet
Adopted Resource	
Reveal: Lesson 9-1: Circumference of Circles Lesson 9-2: Area of Circles Lesson 9-3: Area of Composite Figures Lesson 9-4: Volume Lesson 9-5: Surface Area Lesson 9-6: Volume and Surface Area of Composite Figures	**ALEKS:** Perimeter, Area, and Volume (ALEKS TOC): • Circumference and Area of Circles • Area of Parallelograms, Triangles, and Trapezoids • Area of Composite Figures • Volume of Prisms and Cylinders • Three-Dimensional Figures • Surface Area

Return to Scope and Sequence

Module 10: Probability

Unpacked Standards / Clear Learning Targets

Learning Target

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event; a probability around 1/2 indicates an event that is neither unlikely nor likely; and a probability near 1 indicates a likely event.

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long run relative frequency, and predict the approximate relative frequency given the probability.

7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.
b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations.
a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language, e.g., "rolling double sixes," identify the outcomes in the sample space which compose the event.
c. Design and use a simulation to generate frequencies for compound events.

Essential Understanding

- Probability is the study of the chance (likelihood) that a particular event will occur.
- The theoretical probability of an event describes how often the event will occur in an infinite number of repetitions of a chance process. It also is the long run ratio of the number of times the event occurs divided by the number of times that the chance process is repeated.
- Probability is a number between 0 and 1 that has no units. o Near 1 is most likely; near 0 is least likely; and ½ is neither likely nor unlikely.
- An outcome is a possible result of an event.
- A probability model provides a probability for each possible non-overlapping outcome for a chance process.
  - o A sample space is the collection of all possible individual outcomes.
  - o An event is an outcome or set of outcomes in an experiment; it is a subset of the sample space.
    - A simple event has one outcome.
    - A compound event has more than one outcome.
  - o The total probability of all such outcomes is 1.
- Frequency (absolute frequency) is a quantity that has no units represented by a real number greater than or equal to zero. It is the number of items occurring in a given set; it is a count. Note: Frequency has a different meaning in statistics than is used in common usage, mathematics, and physics.
- Relative (observed or experimental) frequency is the ratio of times an event occurs to the number of occasions which it might occur in the same period. e.g., if a coin is flipped 1,000 times and heads occurs 498 times, the relative frequency is 498/1000 or 0.498.

Academic Vocabulary

Event
Likelihood
Outcome
Probability
Chance Event
Long-Run Frequency
Probability Models
Relative Frequency
Chance Process
Discrepancy
Observed Frequencies
Outcomes
Uniform Probability Model
Compound Events
Frequencies
Outcome
Probability
Sample Spaces
Simple Event
Simulations
Tree Diagrams

COLUMBUS CITY SCHOOLS

Math 7

I Can Statements:

- I understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.
- I can define events as subsets of the sample space.
- I can assign a probability to an event as a ratio (number of times the event occurs to the total number of trials).
- I can discuss the likelihood of an event as a number (fractions, decimals, percents).
- I can approximate the probability of a chance event by collecting data on the chance process that produces it.
- I can observe the long-run relative frequency for a given chance event.
- I can predict the approximate relative frequency given the probability of a given chance event.
- I can develop a probability model and use it to find probabilities of events.
- I can compare probabilities from a model to observed frequencies.
- I can explain possible sources of the discrepancy in models that have a disagreement in observed frequencies.
- I can develop a uniform probability model by assigning equal probability to all outcomes
- I can use a uniform probability model to determine probabilities of events.
- I can develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.
- I can find probabilities of compound events using organized lists.
- I can find probabilities of compound events using tables.
- I can find probabilities of compound events using tree diagrams. I can find probabilities of compound events using simulations.
- I can demonstrate that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- I can represent sample spaces for compound events using organized lists.
- I can represent sample spaces for compound events using tables.
- I can represent sample spaces for compound events using tree diagrams.
- I can identify the outcomes in the sample space which compose the event.
- I can design and use a simulation to generate frequencies for compound events.

Performance Level Descriptors:

Proficient:
- Understand that probabilities are numbers between 0 and 1
- Find probabilities in straightforward situations
- Understand that a probability near 0 indicates an unlikely event, a probability near 1/2 an event that is neither unlikely or likely, and a probability near 1 indicates a likely event.

Accomplished (all of Proficient +):
- Find probabilities of compound events in a real-world context
- Use example situations to explain the differences between theoretical and experimental probabilities

Advanced (all of Proficient + all of Accomplished +):
- Explain why events are likely or unlikely and use that explanation to make predictions
- Develop a probability model and use it to find probabilities of events
- Compare theoretical probabilities (from a model) to observed frequencies (experimental); explain possible sources of the discrepancy between the two

• Compare theoretical and experimental results from a probability experiment.		measures
Prior Standard(s)	**Future Standard(s)**	
7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.	S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	S.CP.2 Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 5, page 49
- Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22 – 23

Instructional Strategies

Build the concept of expressing probability as a number between 0 and 1, inclusive.
- Provide students with situations that have clearly defined probability of never happening as zero, always happening as 1 or equally likely to happen as to not happen as
- $\frac{1}{2}$.
- Then advance to situations in which the probability is somewhere between any two of these benchmark values.
- Use this to build the understanding that the closer the probability is to 0, the more likely it will not happen, and the closer to 1, the more likely it will happen.

Students can use chance experiments to collect data. They need to come to the understanding that as they increase the number of trials in their chance experiment the relative frequency (observed or experimental) over the long-run approaches the theoretical probability. Therefore if they have no way of knowing the theoretical probability (e.g., the number and kinds of tiles in a bag are hidden), they can do many, many trials to figure out an approximation of the theoretical probability. Then they could use that information to make further conclusions. Students can also use the theoretical probability to estimate the relative frequency, keeping in mind that what "should happen" does not always happen and that oftentimes the event will be at least close in value if not exact to the theoretical probability.

Provide students with models of equally likely outcomes and models of not equally likely outcomes and have students determine probabilities. These outcomes are called simple events.

Students should begin to expand their knowledge and understanding of finding the probabilities of simple events to finding the probabilities of compound events by creating organized lists, tables, and tree diagrams. This helps students create a visual representation of the data. From each sample space, students determine the probability (fraction, decimal, percent) of each possible outcome.

Ask guiding questions to help students create methods for creating organized lists and tree diagrams for situations with more elements such as "How many outcomes are possible?", " What does each branch of the tree diagram represent?", or "How can you use your list to find the probability of the event?"

Students often see skills of creating organized lists, tree diagrams, etc. as the end product. Provide students with experiences that require the use of these graphic organizers to determine the theoretical probabilities. Have them practice making the connections between the process of creating lists, tree diagrams, etc. and the interpretation of those models and tying the simulation to a real-world situation.

After the basics of probability are understood, students should experience setting up a model and using simulation (by hand or with technology) to collect data and estimate probabilities for a real situation that is sufficiently complex that the theoretical probabilities are not obvious. Simulation is a procedure that will allow students to answer questions about real problems by running experiments that closely resemble the real situation. Simulation uses devices such as coins, number cubes, cups full of paper clips or legos, or cards to generate outcomes that represent real outcomes. Students may find it difficult to make the connection between device outcomes and the real outcomes of the experiment.

Use a percent grid (a 10 by 10 grid) to record data from a simulation.

Sample Assessments and Performance Tasks

Reporting Category: Statistics and Probability **Standards**: 7.SP.5, 6, 7, and 8 **Approximate Portion of Test:** 22% - 29%; 12 - 15 points (with Module 11)	**OST Test Specs** Items may use all types of rational numbers.

Instructional Resources

7.SP.5	7.SP.6	7.SP.7	7.SP.8
Better Lesson Shmoop Khan Academy Videos	Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Heads or Tails Rolling Dice	Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Rolling Dice How Many Buttons?	Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Waiting Times Sitting Across From Each Other

Adopted Resource

Reveal:	**ALEKS:**
Lesson 10-1: Find Likelihoods Lesson 10-2: Relative Frequency of Simple Events Lesson 10-3: Theoretical Probability of Simple Events Lesson 10-4: Compare Probabilities of Simple Events Lesson 10-5: Probability of Compound Events Lesson 10-6: Simulate Chance Events	Data Analysis and Probability (ALEKS TOC): • Frequency Tables • Probability of Simple Events • Probability of Compound Events • Simulations

Return to Scope and Sequence

Module 11: Sampling and Statistics

Unpacked Standards / Clear Learning Targets

Learning Target	**Essential Understanding**	**Academic Vocabulary**
7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population. a. Differentiate between a sample and a population. b. Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias 7.SP.2 Broaden statistical reasoning by using the GAISE model: a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. For example, "How do the heights of seventh graders compare to the heights of eighth graders?" (GAISE Model, step 1) b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) d. Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4) 7.SP.3 Describe and analyze distributions.	• Statistics is the name for the science of collecting, analyzing, and interpreting data. • A population consists of everyone in a specific group and a sample is a subset from a specific group. • Results from a sample can be generalized for a much larger population. • Sampling variability exists because the sample proportion varies from sample to sample. • Bias, a systematic favoritism in the data collection process, can occur in the way the sample is selected or in the way data are collected. • Statistics is the name for the science of collecting, analyzing, and interpreting data. • The GAISE model framework is used to analyze and interpret data and has four steps: Formulate the Question; Collect Data to Answer the Question; Analyze the Data; and Interpret Results. • Data are not just numbers; they are numbers generated with respect to a particular context and situation. • There are two types of data: categorical and numerical. • Categorical data are sorted into groups and categories. • Numerical data are measurable. • A statistical question anticipates a response that varies, from one individual to the next, and this variability is described in terms of spread and overall shape.	Bias Population Sample Convenience Sample Statistics Unbiased Data GAISE model Graphical methods Numerical measures Quantitative data Statistical reasoning Statistical question Variability Balance point Distribution Asymmetric Symmetric Mean Absolute Deviation (MAD) Numerical data Quantitative data Variability

a. Summarize quantitative data sets in relation to their context by using mean absolute deviation (MAD), interpreting mean as a balance point. b. Informally assess the degree of visual overlap of two numerical data distributions with roughly equal variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.	• A distribution shows all values of data and how often they occur. • A set of data has a distribution which can be described by its center, spread, and overall shape. • A measure of variation is a single number that describes the extent to which data vary in a distribution.	Inferences Invalid Inference Valid Inference Visual Overlap

I Can Statements:

- I understand that inferences about a population can be made by examining a sample.
- I can understand why generalizations made about a population from a sample are only valid if the sample represents that population.
- I can pose my own statistical question of interest.
- I can form questions that allow for generalizations of a population.
- I can use random selection or random assignment appropriately.
- I can compare measures of center and spread between groups using displays and values.
- I can describe potential sources of error.
- I can acknowledge that looking beyond the data is feasible by interpreting differences in shape, center, and spread.
- I can note the difference between two groups with different conditions.
- I can interpret mean as a balance point.
- I can explore, explain, and calculate the mean absolute deviation (MAD).
- I can summarize data using MAD within a context.
- I can summarize and describe distributions representing one population.
- I can informally compare distributions representing two populations using MAD, histograms, dot plots, and/or boxplots.

Performance Level Descriptors:

Proficient:
- Determine whether a sample is random
- Explain whether a sample is random
- Describe a sample of a given population
- Use the mean to compare and draw inferences about two different populations
- Use measures of center to draw comparisons about two different populations

Accomplished (all of Proficient +):
- Use measures of variability for numerical data from random samples to draw informal comparative inferences about two populations

Advanced (all of Proficient + all of Accomplished +):
- Assess the degree of visual overlap of two numerical data distributions with similar variability
- Use measures of variability for numerical data from random samples to draw informal comparative inferences about multiple populations

Prior Standard(s)	Future Standard(s)
5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (a/b) × q as a parts of a part 5.NF.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. 6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction modelsG and equations to represent the problem. 6.SP.1 Develop statistical reasoning by using the GAISE model: a. Formulate Questions b. Collect Data c. Analyze Data d. Interpret Results 6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets. S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population. S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 4, page 48
- Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22-23
- GAISE Model pages 14 – 15
    - Focus of 7th grade is Levels A – B, pages 22-59

Instructional Strategies

Note: One of the changes to this cluster was the deletion of the word "random" from the cluster statements. Students should informally learn about what a random sample is and how it is useful in statistics. Students are not required to actually use true random sampling when collecting data. Instead they should discuss which samples are the best representative of a population. However, a teacher may wish to extend to more sophisticated ideas of random sampling depending on the makeup of his or her class.

Provide opportunities for students to use real-life situations. This shows the purpose for using sampling to make inferences about a population.

Provide students with samples from a population, including the statistical measures. Ask students guiding questions to help them make inferences from the sample.

Random sampling is a way to remove bias. Although students at this level may not be actually using true random sampling procedures when collecting data, the benefits of a random sample should be discussed.

Increasing the sample size reduces sample error, but it does not reduce bias. When students decide to select a sample from a specific group of people (friends), use the situation as an opportunity to discuss bias.

Sample Assessments and Performance Tasks

Reporting Category:
Statistics and Probability
Standards:
7.SP.1, 2, and 3
Approximate Portion of Test:
22% - 29%; 12 - 15 points (with Module 10)

OST Test Specs
Items may use all types of rational numbers.
In 7.SP.1b, the focus will be on making sure that samples are representative of the population. Students will not be required to understand the difference between a non-randomized and a randomized sample.
Items will focus on Level A of the GAISE model.
In 7.SP.2, items can be aligned to one step of the GAISE model or to multiple steps.
Items will focus on progressing from Level A to Level B of the GAISE model.
Items may require knowledge of mean and median as measures of center.
Items may require knowledge of range, interquartile range, and mean absolute deviation (MAD) as measures of variation.
Items can test k knowing the order of the four steps of the model.
In 7.SP.3, visual data displays are limited to dot plots (line plots), histograms, and box plots.
Items will focus on progressing from Level A to Level B of the GAISE model.
Items may require knowledge of mean and median as a measure of center.
Items may require knowledge of range, interquartile range, and mean absolute deviation (MAD) as measures of variation.

COLUMBUS CITY SCHOOLS

Math 7

Instructional Resources		
7.SP.1 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Math Class	7.SP.2 Better Lesson Shmoop Khan Academy Videos Dan Meyer Activity Brita Illustrative Mathematics Valentine Marbles Other Resources Online Statistics Lesson Plans Sample Lesson w/ GAISE Model	7.SP.3 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics College Athletes Offensive Linemen

Adopted Resource	
Reveal: Lesson 11-1: Biased and Unbiased Samples Lesson 11-2: Make Predictions Lesson 11-3: Generate Multiple Samples Lesson 11-4: Compare Two Populations Lesson 11-5: Assess Visual Overlap	**ALEKS:** Data Analysis and Probability (ALEKS TOC): • Collecting Data • Graphs of Data • Measures of Variation • Mean, Median, and Mode

Return to Scope and Sequence