Mathematics

Compacted Math 7/8

2023-2024

Aligned with Ohio's Learning Standards for Mathematics (2017)

Department of Academic Services
Office of Teaching and Learning
Curriculum Division

COLUMBUS CITY SCHOOLS

Compacted Math 7/8

Curriculum Map

Year-at-a-Glance
The Year-at-a-Glance provides a high-level overview of the course by grading period, including: ● Units; ● Standards/Learning Targets; and ● Timeframes.

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Scope and Sequence
The Scope and Sequence provides a detailed overview of each grading period, including: ● Units; ● Standards/Learning Targets; ● Timeframes; ● Big Ideas and Essential Questions; and ● Strategies and Activities.

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Curriculum and Instruction Guide
The Curriculum and Instruction Guide provides direction for standards-based instruction, including: ● Unpacked Standards / Clear Learning Targets; ● Content Elaborations; ● Sample Assessments; ● Instructional Strategies; ● Instructional Resources; and ● ODE Model Curriculum with Instructional Supports.

COLUMBUS CITY SCHOOLS

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Year-at-a-Glance

Grading Period 1 — 9 weeks

1. Operations with Integers and Rational Numbers
2. Exponents and Scientific Notation
3. Real Numbers
4. Algebraic Expressions

Grading Period 2 — 11 weeks

1. Equations and Inequalities
2. Proportional Relationships
3. Linear Relationships and Slope

Grading Period 3 — 8 weeks

1. Functions
2. Geometric Figures
3. Pythagorean Theorem

Grading Period 4 — 7 weeks

1. Transformations, Congruence, and Similarity
2. Probability
3. Sampling and Statistics

Standards for Mathematical Practice

The Standards for Mathematical Practice (SMP) describe skills that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The design of each item on Ohio's state tests encourages students to use one or more Standards for Mathematical Practice.

Modeling and Reasoning are included in the eight Standards for Mathematical Practice within Ohio's Learning Standards. Each grade's blueprint identifies modeling and reasoning as an independent reporting category that will account for a minimum of 20 percent of the overall points on that grade's test.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Practice - Grade 7

Standards for Mathematical Practice - Grade 8

Modeling and Reasoning on Ohio's State Tests in Mathematics

Scope and Sequence

Students should be assessed using teacher-based resources and the ALEKS program. Students are automatically enrolled in the ALEKS course Middle School Course 2. Teachers can move students into RTI 7 if students show a need for remediation. Refer to our guide at https://tinyurl.com/CCS-ALEKS-GUIDE

Textbook information
McGraw-Hill - Reveal Math Accelerated

Grading Period 1	Module 3: Operations with Integers			2 weeks
	Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**
	3.5 Apply Integer Operations	7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. d. Apply properties of operations as strategies to add and subtract rational numbers. 7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. c. Apply properties of operations as strategies to multiply and divide rational numbers. 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.	How can I apply properties of operations to solve problems? How can I identify which operation(s) to use when solving a real-world problem?	• It is important when performing operations that students are able to justify their steps using the properties. Although, the focus should not be on identifying the properties of operation, teachers should be using their formal names in classroom discussion, so students are able to gain familiarity with and recognize the correct terminology • In Grade 6, students should have learned that the absolute value of a number does not take into account sign or direction; it only is a measure of distance (magnitude) from 0. Discourage students from saying that the “answer is always positive or 0” since that will lead to misconceptions when students encounter problems such as $\|4x - 2\| = 18$ in high school. Instead emphasize that it is the “distance from 0.” This is why the value of something like $\|x\| = -5$ has no solution, since distance cannot be negative.

		7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		
	3.6 Rational Numbers	7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then −(p/q) = (−p) /q = p/(−q). Interpret quotients of rational numbers by describing real-world contexts. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. 8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating. Also Addresses: 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.	What is the difference between a terminating and a repeating decimal? Why does the decimal form of a rational number terminate in 0s or eventually repeat? How can we apply the concept of rational numbers to real-life situations and make meaningful connections? How do I convert rational numbers from fractions to their decimal expansions? When does a rational number in simplest fractional form have a terminating decimal?	• In Grade 7 the awareness of rational and irrational numbers is initiated by observing the result of changing fractions to decimals. They can do this by making equivalent fractions with denominators using powers of ten or by using long division. • Students should be provided with families of fractions, such as, sevenths, ninths, thirds, etc. to convert to decimals using long division. The equivalent fractions can be grouped and named (terminating or repeating). Students should begin to see why these patterns occur. Knowing the formal vocabulary rational and irrational is not expected for the students. Terminating decimals fall exactly on some tick mark on a number line; however repeating decimals do not. For example 1 3 is always subdividing an interval of smaller and smaller sizes. Students can also explore patterns to determine which fractions repeat and which fractions terminate. Technology can be used to aid students in recognizing patterns. • Students can also use their knowledge of proportions and unit rates to help them understand why they can convert a fraction to a decimal by dividing. • In previous grades, students become familiar with rational numbers called decimal fractions. In Grade 7, students carry out the long division and recognize that the remainders may repeat in a

		7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		predictable pattern—a pattern creates the repetition in the decimal representation (see 7.NS.2.d). In Grade 8, they explore its occurrence. ● Ask students what will happen in long division once the remainder is 0. They can reason that the long division is complete, and the decimal representation terminates. However, if the remainder never becomes 0, then the remainder will repeat in a cyclical pattern. The important understanding is that students can see that the pattern will continue to repeat. ● Explore differences between terminating and repeating decimals.
	3.7 Add and Subtract Rational Numbers	7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, p − q = p + (−q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	Why do opposite numbers have a sum of 0? How can I model addition and subtraction? How can I use properties of operations to solve problems? How can I solve real-world problems using addition and subtraction of rational numbers? What do the sums and differences of rational numbers mean in the context of a real-world problem? How can I use subtraction to find the distance between two numbers on a number line?	● Many students struggle with the negative sign. One way to help students is to talk about values, order, and direction instead of quantities. For example, positive 5 is greater than positive 4, but −4 is greater than −5. ○ Another reason for confusion is that the negative sign can mean several things: • A sign attached to a number to form negative numbers; • A subtraction; or • An indication to take the opposite of Because of the confusion around the negative sign it may be helpful for students to understand the different meanings of the negative sign and identify which meaning is used when in a problem including the meaning shifts. ● Using both contextual and numerical problems, students should explore what happens when negatives and positives are combined. Repeated opportunities over time with models will allow students to compare the results of adding and subtracting pairs of numbers, leading to the generalization of the rules. ● Two-color counters or colored chips can be used

		7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. Also Addresses: 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.		as a physical model for adding and subtracting integers. Integer chips allow the idea of the zero pair (Additive Inverse Property) to be apparent. ● Number lines present a visual image for students to explore and record addition and subtraction results. One of the positive aspects about using a number line model is that it is not limited to integers; it also lends itself toward connections on the coordinate plane. Students can use number lines with arrows and hops. When using number lines, establishing which factor will represent the length, number, and direction of the hops will facilitate understanding. ● Students need to become fluent in using operations and properties of operations with all rational numbers, not just with integers. ● Although all properties of operations should be addressed, this cluster should especially emphasize the Additive Inverse Property, the Multiplicative Inverse Property, and the Distributive Property.
	3.8 Multiply and Divide Rational Numbers	7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. 7.NS.3 Solve real-world and mathematical	How can I use properties of operations to multiply and divide numbers? How does multiplication connect to the distributive property? How do I multiply and divide integers? How can I use multiplication and division to solve mathematical and real-world problems? How do I interpret the products and quotients in the context of real-world problems? How can I use patterns to recognize the sign of a product when a string of numbers is multiplied?	● Multiplying and dividing integers should be thought of as an extension of adding and subtracting integers. Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers. ● In multiplication, the first factor indicates the number of sets, and the second factor indicates the size of the set. This can be easily modeled with situations involving a positive times a positive or a positive times a negative. A negative times a positive can be inferred using the Commutative Property of Multiplication. ● A negative times a negative can be problematic, for students want to know "How can we have a negative group of something?" One way to view it is as repeated subtraction. If the first factor

		problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. Also Addresses: 7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.	How can I divide a number line to represent decimals and fractions?	being positive indicates repeated addition, then the first factor being negative indicates repeated subtraction. Therefore "negative 3 sets of negative −2" or −3(−2) means to remove 3 sets of −2 from zero. ● Students will discover that they can multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. They should then analyze and solve problems leading to the generalization of the rules for operations with integers. ● Another method for learning multiplication/division rules is to use patterns. Beginning with known facts, students can predict the answers for related facts, keeping in mind that the properties of operations apply. ● Using the language of "the opposite of" helps some students understand the multiplication of negatively signed numbers (−4 · −4 = 16, the opposite of 4 groups of −4). Discussion about the tables should address the patterns in the products, the role of the signs in the products, and commutativity of multiplication. ● Different Algorithms: ○ Multiplication ■ Area Model ■ Partial Products ■ Lattice Algorithm ■ Traditional Multiplication ○ Division ■ Partial Quotients ■ Explicit-Trade ■ Traditional Division ● Multiplying and dividing integers should be thought of as an extension of adding and subtracting integers. Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers.

				• In multiplication, the first factor indicates the number of sets, and the second factor indicates the size of the set. This can be easily modeled with situations involving a positive times a positive or a positive times a negative. A negative times a positive can be inferred using the Commutative Property of Multiplication. • A negative times a negative can be problematic, for students want to know "How can we have a negative group of something?" One way to view it is as repeated subtraction. If the first factor being positive indicates repeated addition, then the first factor being negative indicates repeated subtraction. Therefore "negative 3 sets of negative −2" or −3(−2) means to remove 3 sets of −2 from zero. • Another method of understanding multiplication of negative numbers is fast forwarding and rewinding students walking backwards and forwards. Students can then see that when you rewind (−) and walk backwards (−), the video shows them walking forward. An app called Reverse Vid for iPhone allows students to rewind videos. • Students will discover that they can multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. They should then analyze and solve problems leading to the generalization of the rules for operations with integers. • Another method for learning multiplication/division rules is to use patterns. Beginning with known facts, students can predict the answers for related facts, keeping in mind that the properties of operations apply (See Tables 1, 2, and 3 below). • Using the language of "the opposite of" helps some students understand the multiplication of negatively signed numbers ($-4 \cdot -4 = 16$, the opposite of 4 groups of −4). Discussion about the

			tables should address the patterns in the products, the role of the signs in the products, and commutativity of multiplication. • Division of integers is best understood by relating division to multiplication and applying the rules. The Inverse Property of Multiplication should be used to connect division to multiplication. Since 8 ÷ (−2) is the same as $8(-\frac{1}{2})$, the rules for multiplication apply to division. In time, students will transfer the rules to division situations.
3.9 Apply Rational Number Operations	7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. d. Apply properties of operations as strategies to add and subtract rational numbers. 7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. c. Apply properties of operations as strategies to multiply and divide rational numbers. 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. Also Addresses: 7.EE.2 In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and	How can I apply the properties of operations when solving problems? How can I solve real-world problems involving the four operations with rational numbers? How do I solve problems using complex fractions	• Students should become familiar with solving problems involving complex fractions. Draw attention to the Multiplicative Identity Property and the Multiplicative Inverse Property for solving expressions with complex fractions. This concept connects nicely with cluster 7.RP.1-3.

	can reveal how those quantities are related. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		

Module 4: Exponents and Scientific Notation — **3 weeks**

Lesson	Standards/Learning Targets	Big Ideas/Essential Questions	Strategies/Activities
4.1 Powers and Exponents	Foundational for 8.EE.1 Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions.	How do patterns show the properties of exponents? How do I create equivalent numerical expressions using the properties of integer exponents?	• Although students begin using whole-number exponents in Grades 5 and 6, it is in Grade 8 when students are first expected to understand, explain, and apply the properties of exponents and to extend the meaning beyond counting-number exponents. • Students should not be told these properties but rather should derive them through experience and reason. Many students who simply “memorize” the rules without understanding may confuse the rules when trying to apply them at later times. Instead students should be encouraged to discover the rules using tables, patterns, and expanded notation. As they have multiple experiences simplifying numerical expressions with exponents, these properties become natural and obvious. • Students should use multiplicative reasoning and expanded notation to gain understanding of non-positive exponents. • Another way to view the meaning of 0 and negative exponents is by applying the following principle: The properties of counting-number
4.2 Multiply and Divide Monomials	8.EE.1 Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions.		

	4.3 Powers of Monomials 4.4 Zero and Negative Exponents			exponents should continue to work for integer exponents. (See instructional strategies) ● In Grade 8 students should also have practice using negative numbers as bases. ● Have students identify the bases before solving problems as many students incorrectly only attribute the exponent to the nearest number. ● Students should also have practice using non-integer bases such as $(1.5)^3$ or $\left(\frac{3}{4}\right)^{-2}$
	4.5 Scientific Notation	8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. 8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology.	How do I identify numbers in scientific notation? How do I convert numbers from scientific notation to decimal notation? (and vice versa) How do I compare numbers in scientific notation?	● The meanings of integer exponents, especially with respect to 0 and negatives, can be further explored in a place-value chart: Thus, integer exponents support writing any decimal in expanded form like the following: $3247.568 = 3 \cdot 10^3 + 2 \cdot 10^2 + 4 \cdot 10^1 + 7 \cdot 10^0 + 5 \cdot 10^{-1} + 6 \cdot 10^{-2} + 8 \cdot 10^{-3}$. ● Expanded form and the connection to place value is important for helping students make sense of scientific notation, which allows very large and very small numbers to be written concisely, enabling easy comparison. Students can make sense of scientific notation and negative exponents by using patterns. ● To develop familiarity, go back and forth between standard notation and scientific notation for numbers. Compare numbers, where one is given in scientific notation and the other is given in standard notation. Have students place numbers written in scientific notation on a number line and order them without converting them to

			standard form. Students should come to the conclusion that when determining value, the power is more important than the coefficient. •
4.6 Compute with Scientific Notation		How are operations performed on numbers where both scientific notation and decimal notation are used? How do different technology devices show scientific notation? What are appropriately sized units when dealing with very large or very small quantities?	• Real-world problems can help students compare quantities and make sense about their relationships. Conversely scientific notation can also help students make sense of really large or small numbers by modeling situations. Students should as often as possible have a real-world situation to model when using scientific notation to help their understanding of the concept.
Module 5: Real Numbers			**2 weeks**
Lesson	**Standards/Learning Targets**	**Big Ideas/Essential Questions**	**Strategies/Activities**
5.1 Roots	8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	How do I solve for square roots and cube roots? How do I show solutions with square root and cube root symbols? How can I evaluate square roots of perfect squares and cube roots of perfect cubes?	• To help students build a conceptual understanding, connect roots to area and volume models where the area and volume are the radicand and the solution is the length of the side of the model. • Another way to explain it is that the area and volume of the square or cube represents n; and the square and cube's side length is represented by $\sqrt{n}$ and $\sqrt[3]{n}$ respectively. • Have students use geoboards, square tiles, graph paper, or unit cubes to build squares and cubes reviewing exponents in the process. Problems such as the Painted Cube Problem can then be modified to extend to square and cube roots. • Also, provide practical opportunities for students to flexibly move between forms of squared and cubed numbers. For example, If $3^2 = 9$ then $\sqrt{9} = 3$. This flexibility should be experienced symbolically and verbally with manipulatives and

			with drawings.
5.2 Real Numbers	8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating. 8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	How do I know if a real number is rational or irrational?	• It could be appropriate to use Venn diagrams/set diagrams or flowcharts to show the relationships among real, rational, irrational numbers, integers, and natural numbers. The diagram should show that all real numbers are either rational or irrational. • Students should come to the understanding that (1) every rational number has a decimal representation that either terminates or repeats and (2) every terminating or repeating decimal is a rational number. Then, they can use that information to reason that on the number line, irrational numbers must have decimal representations that neither terminate nor repeat. • In previous grades, students learned processes that can be used to locate any rational number on the number line: Divide the interval from 0 to 1 into b equal parts; then, beginning at 0, count out a of those parts. Now they can use similar strategies to locate irrational numbers on a number line. • Use an interactive number line to allow students to see how a number line can be infinitely divided. One resource is Zoomable Number Line by MathisFun. • Although students at this grade do not need to be able to prove that $\sqrt{2}$ is irrational, they minimally need to know that $\sqrt{2}$ is irrational (see 8.EE.2), which means that its decimal representation neither terminates nor repeats. Nonetheless, they should be able to approximate irrational numbers such as $\sqrt{2}$ without using the square root key on the calculator. • The $\sqrt{2}$ caused Greek Mathematicians many problems, for although they could construct it using tools, but they could not measure it. Integrating math history into the lesson may be
5.3 Estimate Irrational Numbers	8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., π^2.	How do I approximate the value of irrational numbers? How do I get more precise approximations of real numbers?	
5.4 Compare and Order Real Numbers	8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating. 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., π^2.	How do I locate the (approximate) location of an irrational number on a number line? How can I compare and order rational and irrational numbers?	

			interesting for some students. ● Have students do explorations where precision matters. Discuss situations where precision is vital and other situations where reasonableness is more vital than precision. Although learning about significant digits formally takes place in high school, it is appropriate to talk about how intermediate rounding affects precision. The display of irrational numbers on a calculator could also be used for a discussion point. Another discussion could take place comparing results using the pi button compared to the typical approximation of 3.14. ● The concept of precision should also be tied to real-world contexts. For example, we do not buy 3.5 apples, but we may buy 3.5 lbs of ground beef, so rounding to a whole number is a precise number in terms of buying apples. Therefore, being precise should relate to the real-world context of the problems.
Module 6: Algebraic Expressions			**2 weeks**
6.1 Simplify Algebraic Expressions	7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. 7.EE.2 In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related.	How can I make equivalent expressions? How can I use order of operations to simplify algebraic expressions? What are the properties of operations, and how can they be used as strategies to simplify and manipulate algebraic expressions? How does expanding linear expressions help in identifying equivalent forms or revealing patterns within the expression? How can you apply properties of operations to combine like terms within linear expressions and simplify	● It is important that students are able to justify their thinking using the properties. Although the focus should not be on identifying the properties of operation, teachers should be using their formal names in classroom discussion so students are able to gain familiarity with and recognize the correct terminology. ● Provide opportunities for students to use and understand the properties of operations. These include the Commutative, Associative, Identity, and Inverse Properties of Addition and of Multiplication, the Zero Property of Multiplication, and the Distributive Property. ● Subtraction should be thought of as the opposite of addition, and division should be thought of as the opposite of multiplication. Note: Avoid PEMDAS as it leads to many misconceptions and

			them?	errors in computation. • Writing equivalent expressions includes simple cases of factoring out a GCF. This can be illustrated using the area model that students are already familiar with. In this model the students are given the areas, and they are asked to find the lengths and widths. It might be wise to point out that, although in real-life length and width cannot be negative, our model allows for lengths and widths to be negative to illustrate the concept. • Students started combining like terms in Grade 6. Now they will extend this concept to negative numbers. It may be helpful to use Algebra tiles to illustrate this process as it will prevent future misconceptions from forming such as trying to combine 2x and $3x^2$. Students should then extend that concept to other rational numbers besides integers.
	6.2 Add Linear Expressions	7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	How can you use properties of operations to simplify the addition or subtraction of expressions?	
	6.3 Subtract Linear Expressions			
	6.4 Factor Linear Expressions		What strategies can be used to factor out common factors from linear expressions to simplify them?	
	6.5 Combine Operations with Linear Expressions		How do the order of operations help simplify expressions with multiple operations?	
	Using Area/Surface Area/Volume Formulas (Module 12)	7.G.4 Work with circles. a. Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. b. Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems. 7.G.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. 8.G.9 Solve real-world and mathematical problems involving volumes of cones, cylinders, and spheres.	What are the components of formulas for volume/ How do volume formulas for different figures compare? How do I use dimensions to find the volume of a figure? How can I find missing measures of a figure when given the volume? What are the differences between linear, square, and cubic units? How can I use volume to model real-world situations? What is the relationship between the volume of a cube and a pyramid with the same base and height? What is the relationship between the volume of a cylinder and a cone with the same base and height?	• Most area problems should be given in a real-world context. • Students can find the area of regular polygons using triangles if the apothem is given. (Students do not need to use the term apothem.) • Instead of being restricted to using nets to find surface area, students may prefer to draw the different views of a structure (front, right, top). The use of formulas to find surface area should be discouraged. • Some students will prefer drawing nets and others will prefer drawing the six different views. After practice with both methods, let students use their preferred method. • Once students are comfortable finding the surface area of unit cubes, tell students that the cubes' lengths are rational numbers such as ¼ inch and have them calculate the surface area. • The focus in this cluster should be on relationships between solids and the real-world application of volume. Not only do students need

				to find the volume, but they should also be able to find a missing dimension given the volume. ● To develop students' spatial skills, they need practice learning how to draw three-dimensional solids such as cones, cylinders, pyramids, and spheres. ● Most students can be readily led to the understanding that the volume of a right rectangular prism can be thought of as the area of a "Base" times the height, and so because the area of the base of a cylinder is a circle whose area equals $\pi\pi rr2$ the volume of a cylinder is Vcylinder = $\pi r^2 h$ or $V = Bh$. Foam layers that have the height of 1 unit can be used to show how to build a cylinder of h height and reinforce Base × height as well. ● To explore the formula for the volume of a cone, use cylinders and cones with the same radius and height. Fill the cone with rice or water and pour it into the cylinder. Students will discover/experience that 3 full cones are needed to fill the cylinder. This non-mathematical demonstration of the formula for the volume of a cone, Vcone = $\frac{1}{3}\pi r^3 h$ or Vcone = $\frac{1}{3}Bh$, will help students make sense of the formula. ● Make sure to differentiate between the height of an object and slant height. ● To explore the formula for the sphere, use spheres, cylinders, and cones with the same radius, whereas the height of the cone and the cylinder must be the same as the radius, but the height of the sphere will be twice the radius. Discuss the relationships between the solids. Fill the sphere with rice or water and pour into the cylinder. Students will discover/experience that there is water remaining in the sphere. This water/rice will fill the cone. The students should see that the volume of a sphere = the volume of a cylinder + volume of a cone. Because r = h

			(radius = height), by using substitution, the volume of the cylinder is $\frac{3}{3}\pi r^3$, and the volume of the cone is $\frac{1}{3}\pi r^3$, so the volume of the sphere is Vsphere = $\frac{4}{3}\pi r^3$, This non-mathematical demonstration of the formula for the volume of a sphere, Vsphere = $\frac{1}{3}\pi r^3$, will help students make sense of the formula. • Students should experience many types of real-world applications using these formulas. They should be expected to explain and justify their solutions. Some examples include the following: finding the amount of space left over in a can with 3 tennis balls; finding total volume in a silo; finding how much ice cream in a cone, etc.

Grading Period 2

Module 7: Equations and Inequalities — 4 weeks

Lesson	Standards/Learning Targets	Big Ideas/Essential Questions	Strategies/Activities
7.1 Write and Solve Two-Step Equations: px + q = r 7.2 Write and Solve Two-Step Equations: p(x + q) = r	7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.	How can I solve multi-step problems with positive and negative rational numbers? How do I identify the sequence of operations used to solve an equation?	• This is the context for students to practice using rational numbers including integers and positive and negative fractions and decimals. It is appropriate to expect students to show their steps in their work. Students should be able to move toward explaining their thinking using correct terminology. • To assist students' assessment of the reasonableness of their answers, especially problem situations involving fractional or decimal numbers, use whole-number approximations for the computation and then compare to the actual computation. • Some studies have shown that a students' fractional knowledge correlates with their ability to write equations. Therefore encourage students to solve equations with fractions by using diagrams instead of just using inverse operations. This may aid in creating

			understanding to alleviate the misuse of fraction rules in later grades/courses. Numbers that are easily modeled should be used initially until students internalize the process.
7.3 Write and Solve Equations with Variables on Both Sides	8.EE.7 Solve linear equations in one variable. b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	How do I solve multi-step equations with rational coefficients? How can I determine if an equation has no solutions, one solution, or infinitely many solutions? How can I graph the solution to a linear equation on a number line?	• Properties of Operations Table 3 on page 97 of Ohio's Learning Standards in Mathematics states the Properties of Operations and Table 4 states the Properties of Equality. Teachers should be using the correct terminology to justify steps when performing operations and solving equations. • Students incorrectly think that the variable is always on the left side of the equation. Give students situations where the variable is on the right side of the equation. Emphasize using the Symmetric Property of Equality if students wish to flip the variable to the other side of the equal sign. • Equation-solving in Grade 8 should involve multi-step problems that require the use of the distributive property, collecting like terms, rational coefficients, and variables on both sides of the equation. • In Grade 7, students may have used a pan balance, number lines, or algebra tiles to solve two-step equations. Eighth grade students could review these models and build upon them. For example, algebra tiles may help prevent student errors such as incorrectly combining like terms on opposite sides of the equations. • When not using models, some students benefit from drawing a vertical line through the equals sign to separate the two sides of the equation. • Connect mathematical analysis with real-life events by using contextual situations when solving equations. Students should experience— • analyzing and representing contextual situations with equations; • identifying whether there is one solution, no
7.4 Write and Solve Multi-Step Equations			
7.5 Determine the Number of Solutions	8.EE.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).		

			solutions, or infinitely many solutions; and then • solving the equations to prove conjectures about the solutions.
7.6 Write and Solve One-Step Addition and Subtraction Inequalities 7.7 Write and Solve One-Step Multiplication and Division Inequalities 7.8 Write and Solve Two-Step Inequalities	7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. b. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.	How does a negative coefficient affect the solution set of an inequality? How does a negative coefficient affect the inequality symbol? How can I interpret the solution set of an inequality in context? How do I graph the solution set of an inequality on a number line?	● In Grade 6 students wrote inequalities in the forms of x > c and c < x. in Grade 7 students use > and <. Discuss why teachers should, when graphing on a number line, a closed circle represents > and < and an open circle represents > and <. Students should also have practice solving one and two-step inequalities with rational numbers. ● Present situations where the variable is both on the left and the right side of the equality. Students need to be fluent in solving inequalities where the variable is on the left and right of the inequality for later algebraic concepts using compound inequalities. Therefore, discourage students from always writing the variable on the left side of an inequality. ● When graphing, teachers should avoid telling students that the inequality points the same direction on the number line as the arrow; this creates a misconception that is hard to break when students work on compound inequalities in high school. An alternative strategy is to ask students to name 3 points that make an inequality true, and then draw the arrow in that direction. ● Students should be able to create equations and inequalities from real-world situations where they always precisely define the variable(s). ● Provide multiple opportunities for students to work with multi-step problem situations that have multiple solutions and therefore can be represented by an inequality. Students need to be aware that values can satisfy an inequality but not be appropriate for the situation, therefore limiting the solutions for that particular problem.

Module 1: Proportional Relationships **3 weeks**

<table>
<tr>
<td>1.1 Unit Rates Involving Ratios of Fractions</td>
<td>7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.</td>
<td>What is a unit rate, and how does it relate to ratios and fractions?
How can you compute the unit rate of a ratio of fractions?
What strategies can be used to simplify ratios of fractions before computing the unit rate?
How can unit rates be applied to real-world situations involving lengths, areas, and other quantities?</td>
<td rowspan="4">

In Grade 6 students reasoned about ratios using models such as tables, double number lines, tape diagrams, and graphs. They avoided using fraction notation for ratios and did not set up nor explicitly solve proportions. Now in Grade 7, students should be able to set up proportions using fraction notation. Note: Solving problems using cross products should be avoided.
Applications should now focus on solving unit-rate problems with more sophisticated numbers. Entries in tables and unit rates can be rational numbers including complex fractions. For scaffolding ideas and more information about ratios and rates see Model Curriculum Grade 6.RP.1-3.
Students obtain proportional reasoning when they understand that the ratio of the two quantities remains constant even though the corresponding values of the quantities may change ($y = kx$). In other words, the relationship of the first quantity compared to the amount of the second quantity is always the same regardless if the quantities increase or decrease.
It is important that students are able to differentiate between situations that are directly proportional and those that are not. Otherwise, they may haphazardly apply proportional techniques to nonproportional situations. That means they need to carefully attend to the relationships in the problem.
One way to view and reason with proportions is to use within and between relationships. Within relationships focus on making comparisons within the same units/measure-space such as 180 miles: 60 miles = 6 gallons: 2 gallons. Whereas between relationships focus on making comparisons between different units/measure-space such as 180 miles: 6 gallons = 60 miles: 2 gallons.
Have students explore graphs that are proportions and those that are not. Given

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<tr>
<td>1.2 Understand Proportional Relationships</td>
<td>7.RP.2 Recognize and represent proportional relationships between quantities.</td>
<td>How can you determine whether two quantities are in a proportional relationship? What are the key indicators or patterns to look for?
What are equivalent ratios, and how can they be used to test for a proportional relationship? How do you determine if ratios are equivalent?
What is the significance of the constant of proportionality being a unit rate? How does it relate to the relationship between the two quantities?</td>
</tr>
<tr>
<td>1.3 Tables of Proportional Relationships</td>
<td>7.RP.2 Recognize and represent proportional relationships between quantities.
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</td>
<td>What is the constant of proportionality, and how can you identify it in tables?</td>
</tr>
<tr>
<td>1.4 Graphs of Proportional Relationships</td>
<td>7.RP.2 Recognize and represent proportional relationships between quantities.
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for</td>
<td>What role does graphing on a coordinate plane play in determining whether a relationship is proportional?
What characteristics should you observe</td>
</tr>
</table>

	equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	in the graph? How does a proportional relationship appear on a graph? What does it mean for the graph to be a straight line through the origin? What is the constant of proportionality, and how can you identify it in graphs?	various graphs, they may make tables using three points on the graph and decide whether they are proportional or not. Ask students what all proportional graphs have in common. Students should come to the conclusion that a proportional graph is a straight line that goes through the origin.
1.5 Equations of Proportional Relationships	7.RP.2 Recognize and represent proportional relationships between quantities. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations.	What is the constant of proportionality, and how can you identify it in various representations such as tables, graphs, equations, diagrams, and verbal descriptions?	
1.6 Solve Problems Involving Proportional Relationships	7.RP.2 Recognize and represent proportional relationships between quantities. 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.	How can you identify and apply proportional relationships to solve multistep problems involving ratios and percentages? What strategies can be used to set up and solve multistep problems using proportional reasoning? How can you determine whether a problem requires the use of proportional relationships versus other mathematical operations or strategies?	
Module 8: Linear Relationships and Slope			**4 weeks**
8.1 Proportional Relationships and Slope	8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.	How do I graph proportional relationships? How does the constant of proportionality relate to slope?	• Students in Grade 7 represented proportional relationships as equations such as y = kx or t = pn. They also graphed proportional relationships, discovering that a graph of a proportion must go

			through the origin, and that in the point (1, r), r is the unit rate. Now in Grade 8, the unit rate of a proportion is used to introduce "the slope" of the line. ● Students need to make connections between the different representations (equations, tables, graphs) in order to come to a unified understanding that the different representations are in essence different ways of modeling the same information. ● Explicit connections need to be made between the multiplicative factor, the slope, scale factor, and an increment in a table. ● To reinforce the relationships between the x and the y, students should continually name quantities for the real-world problem they represent. They should also identify the independent and dependent variables.
8.2 Slope of a Line	Foundational for: 8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b. 8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.		
8.3 Similar Triangles and Slope	8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.	How can I use similar right triangles to find the slope of a line?	● By using coordinate grids and various sets of similar triangles, students can prove that the slopes of the corresponding sides are equal, thus making the unit rate or rate of change equal
8.4 Direct Variation	8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the	How can I use an equation to determine the relationship between two variables? How are the equations y = mx and y =	● Distance-time problems are common in mathematics. In this cluster, they serve the purpose of illustrating how the rates of two

		coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b. Also Addresses: 8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.	mx + b related?	objects can be represented, analyzed, and described in different ways: verbally, graphically, tabularly, and algebraically. Emphasize the creation of representative graphs and the meaning of various points. Then compare the same information when represented in an equation.
Grading Period 3	8.5 Slope-Intercept Form	8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.		● Use graphing utilities such as Desmos to show the lines in the form of y = mx + b as vertical translations of the equation y = mx.
	8.6 Graph Linear Equations	Foundational for: 8.EE.8 Analyze and solve pairs of simultaneous linear equations graphically.		
	Scatterplots	**8.SP.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering; outliers; positive, negative, or no association; and linear association and nonlinear association. (GAISE Model, steps 3 and 4) **8.SP.2** Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (GAISE Model, steps 3 and 4) **8.SP.3** Use the equation of a linear model to solve problems in the context of bivariate	How do I construct a scatterplot? How do I choose appropriate scales, labels and plot points based on my choices? How do I use characteristics (clusters, gaps, outliers) to describe scatterplots? How do I describe a trend (linear, curved, positive, negative, strong association, weak association, no association)?	● Scatterplots are the most common form of representations displaying bivariate data in Grade 8. Provide scatterplot of linear data and have students practice informally finding the trend line. Students could be given a scatterplot and a spaghetti noodle to determine the "best fit." Discussion should include "What does it mean for a data point to be above the line?" or "What does it mean for it to be below the line?" ● By changing the data slightly, students can have a rich discussion about the effects of the change on the graph. The study of the trend line ties directly to the algebraic study of slope and y-intercept. Students should interpret the slope and y-intercept of the trend line in the context of the data. Then students can make predictions based on the trend line. Give students a variety of data sets that intersect the y-axis at various points, so

		measurement data, interpreting the slope and intercept. (GAISE Model, steps 3 and 4)		students do not mistakenly think that all trend lines must go through the origin. ● After a trend line is fitted through the data, the slope of the line is approximated and interpreted as a rate of change, in the context of the problem. If the slope is positive, then the two variables are positively associated. Similarly if the slope is negative, then the two variables are negatively associated. Students should also be exposed to data that do not have an association. 　● Students should create and interpret scatterplots, focusing on outliers, positive, or negative association, linearity, or curvature. Assuming the data are linear, students should informally draw a trend line on the scatterplot and informally evaluate the strength of fit. They should be able to interpret visually how well the trend line fits the "cloud" of points. 　● To move students from Level A to Level B, questions should move from "Is there an association?" to "How strong is the association?" The Quadrant Count Ratio (QCR) can help students informally determine the strength between two variables. This is an important building block in building the conceptual understanding of the correlation coefficient in high school.
	Systems of Equations	**8.EE.8** Analyze and solve pairs of simultaneous linear equations graphically. a. Understand that the solution to a pair of linear equations in two variables corresponds to the point(s) of intersection of their graphs, because the point(s) of intersection satisfy both equations simultaneously. b. Use graphs to find or estimate the solution to a pair of two simultaneous linear equations in two variables. Equations should include all	How do I find or estimate which points are solutions to a pair of linear equations in two variables? How can I use graphing to solve a pair of linear equations? How do I represent the solution to a pair of linear equations in two variables? How can I determine if a pair of	● This cluster builds on the informal understanding of slope, students gained from graphing unit rates and proportional relationships in grades 6 and 7. It also builds upon the stronger, more formal understanding of slope and the relationship between two variables from 8.EE.5-6 and 8.F.4-5. ● Students will use graphing to solve pairs of simultaneous linear equations. Beginning work should involve pairs of equations with solutions that are ordered pairs of integers, making it

		three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to pairs of linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. (Limit solutions to those that can be addressed by graphing.)	linear equations has no solutions, one solution, or infinitely many solutions? How can I solve real-world problems using pairs of linear equations? What kinds of real-world problems can be solved using pairs of linear equations? How do I use two different scales when graphing pairs of linear equations in two variables?	easier to locate the point of intersection. Although students should also be able to approximate solutions that do not fall evenly onto the intersection of grid squares. • Provide opportunities for students to see and compare simultaneous linear equations in forms other than slope-intercept form (y = mx + b). Students may solve pairs of simultaneous linear equations by inspection, by graphing using slope-intercept form, or by graphing using tables of values. • Students should be able to solve simple cases by inspection. For example, x + y = 3 and x + y = 5 has no solution because x + y cannot equal both 3 and 5. • Students should have practice working with graphs that have a variety of scales including fractions and decimals. • Students have the tendency to see the intersection point of a graphed system of equations and round it to the nearest grid line cross-section. Emphasize to students that they can sometimes find intersections in the middle of the grid squares that allow for approximations to be more precise. • Students could also investigate pairs of simultaneous equations using graphing calculators or online graphing resources. They could be asked to explain verbally and in writing what, in the equation and situation, makes lines shift to different locations on the graph. • Graphing pairs of linear equations should be introduced through contextual situations relevant to eighth graders, so students can create meaning. They should explore many tasks for which they must write and graph pairs of equations with different slopes and y-intercepts. This should lead to the generalization that finding one point of intersection is the single solution to the pair of linear equations.

			• Students should relate the solution to the context of the problem, commenting on the reasonableness of their solution. • Emphasize that the solution must satisfy both equations.

Functions — **3 weeks**

Lesson	Standards/Learning Targets	Big Ideas/Essential Questions	Strategies/Activities
Identify Functions	8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. *Function notation is not required.	How do I determine if a table, graph, equation, or verbal description represents a linear or nonlinear function?	• Students should be expected to reason from a context, a graph, or a table, after first being clear which set represents the input (e.g., independent variable) and which set is the output (e.g., dependent variable). When a relationship is not a function, students should produce a counterexample: an "input value" with at least two "output values." If the relationship is a function, the students should explain how they verified that for each input there was exactly one output. • In Grade 6 students explored independent and dependent variables, and how the dependent variable changes in relation to the independent variable. In Grade 8 students need to continue identifying the independent and dependent variables in functions. Students need practice justifying the relationship between the independent and dependent variable. • In Grade 6 students explored independent and dependent variables, and how the dependent variable changes in relation to the independent variable. In Grade 8 students need to continue identifying the independent and dependent variables in functions. Students need practice justifying the relationship between the independent and dependent variable.
Function Tables		How do I identify a set of input and output values for a function?	
Construct Linear Functions	8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	How do I interpret the slope/rate of change and y-intercept/initial value of a linear function?	
Compare Functions	8.F.2 Compare properties of two functions each represented in a different way	How do properties of two functions compare when represented	• The standards explicitly call for exploring functions numerically, graphically, verbally, and

		(algebraically, graphically, numerically in tables, or by verbal descriptions).	differently?	algebraically. For fluency and flexibility in thinking, students need experiences translating among these different representations. • Students need experience translating among the different representations using different functions. For example, they should be able to determine which function has a greater slope by comparing a table and a graph. • Students need to compare functions using the same representation. For example, within a real-world context, students compare two graphs of linear functions and relate the graphs back to its meaning within the context and its quantities. Students should work with graphs that have a variety of scales including rational numbers.
	Nonlinear Functions	8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. *Function notation is not required. 8.F.3 Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. 8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	What are the characteristics of linear and nonlinear functions? How do I use characteristics of functions to determine if they are linear or nonlinear? What are examples of functions that are nonlinear?	• In Grade 8, the focus is on linear functions, and students begin to recognize a linear function from its form y = mx + b knowing that y = mx as a special case of a linear function. Students also need experiences with nonlinear functions. This includes functions given by graphs, tables, or verbal descriptions but for which there is no formula for the rule.
	Qualitative Graphs	8.F.3 Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.	How do I determine if it is reasonable to "connect the points" on a graph based on context?	• When plotting points and drawing graphs, students should develop the habit of determining, based upon the context, whether it is reasonable to "connect the dots" on the graph. In some contexts, the inputs are discrete, and connecting

			the dots is incorrect. For example, if a function is used to model the height of a stack of n paper cups, it does not make sense to have 2.3 cups.
Module 11: Geometric Figures			**3 weeks**
11.1 Vertical and Adjacent Angles 11.2 Complementary and Supplementary Angles	7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. Also Addresses: 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. 7.EE.4 a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.	How can I model angle relationships? How do models of angle relationships help define the relationships? How can I identify special angle pairs? How can I use equations to solve for angle pairs in real-world problems? What are the patterns among the angles of intersecting lines?	• This cluster focuses on the importance of visualization in the understanding of Geometry. Being able to visualize and then represent geometric figures on paper is essential to solving geometric problems. After much work is done on paper, Geometry software can aid in students' understanding of Geometry. • Provide students the opportunities to explore angle relationships. At first they can measure and find patterns among the angles of intersecting lines or within polygons. Then they can utilize the relationships to write and solve equations for multi-step problems. • A student often incorrectly thinks that a wide angle with short sides is smaller than a narrow angle with long sides. To confront this problem have students compare angles with different side lengths.
11.3 Angle Relationships and Parallel Lines	8.G.5 Use informal arguments to establish facts about ~~the angle sum and exterior angle of triangles, about~~ the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	What are the relationships between angles formed by parallel lines and a transversal?	• In Grade 7, students develop an understanding of the special relationships of angles and their measures (complementary, supplementary, adjacent, and vertical). Now in 8.G.5 the focus is on learning about the sum of the measures of the interior angles of a triangle and exterior angle of triangles by using transformations. • This might be a good time to introduce

				vocabulary of the types of angles, such as interior, exterior, alternate interior, alternate exterior, corresponding, same side interior, and same side exterior. Students are expected to recognize but not memorize this vocabulary.
	11.4 Triangles	7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions. a. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	How can I use specifications to draw a picture or create a model of triangles and/or quadrilaterals? How can I determine if a set of side lengths and angle measures creates a unique triangle, multiple triangles, or does not create a triangle? What is the sum of the interior angles of triangles?	• Students should regularly be exposed to shapes and figures from many perspectives and orientations, not just the prototypical example. • Many careers and everyday activities require spatial reasoning. Some research suggests that 7th grade is the optimal time for developing spatial visualization. Sketching figures can help students develop an intuitive understanding of geometry. Although drawings should become precise over time, informal free-hand sketches can help develop spatial reasoning. • Constructions facilitate understanding of geometry. Provide opportunities for students to physically construct triangles with straws, sticks, or geometry apps. This should be done prior to using rulers, compasses, and/or protractors. Have students discover and justify the side and angle conditions that will form triangles.
	11.5 Angle Relationships and Triangles	8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	What is the sum of the interior angles of a triangle? What is the relationship between the exterior angle of a triangle and the two remote (non-adjacent) interior angles?	• In Grade 7 students should have had some practice exploring that the sum of the angles inside a triangle equal 180 degrees. Now students use transformations to prove it. • Students can create a triangle and use rotations and transformations to line up all the angles to prove that the sum of the interior angles of a triangle equals 180 degrees. They need to be able to demonstrate and explain why the sum of the interior angles equals 180 degrees. • Students should build on this activity to explore exterior angle relationships in triangles. They can also extend this model to explorations involving other parallel lines, angles, and parallelograms formed. Students should be able to explain why

				two angles in a triangle have to be less than 180 degrees. ● Investigations should lead to the Angle-Angle criterion for similar triangles. For instance, groups of students should explore two different triangles with one, two, and three given angle measurements. Students observe and describe the relationship of the resulting triangles. As a class, conjectures lead to the generalization of the Angle-Angle criterion.
	11.6 Scale Drawings	7.G.1 Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals. a. Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale. b. Represent proportional relationships within and between similar figures. Also Addresses: 7.RP.2 Recognize and represent proportional relationships between quantities. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form	How can I use drawings and/or models to make sense of problems involving similar figures? How can I identify corresponding sides and angles of similar figures? What are the similarities and differences between the angle measures and side lengths in a scale drawing and its original figure? How can I use proportions to explain the relationship between side lengths in similar figures? What is the relationship between the areas of similar figures? How does scale affect length and area? How do I compute actual lengths and areas from a scale drawing? How do I reproduce a scale drawing using a different scale? How do I draw scaled figures with proper figure labels, scale, and dimensions?	● Similarity is an increase or decrease that is multiplicative in nature instead of additive; this is a new concept for students. ● Although the focus of this cluster is on rectangles and triangles, it may be useful to discuss why all circles are similar. ● As an introduction to scale drawings in geometry, students should be given the opportunity to explore scale factor as the number of times you multiply the side measure of one figure to obtain the corresponding side measure of a similar figure. It is important that students first experience this concept concretely progressing to abstract contextual situations. Pattern blocks provide a convenient means of developing the foundation of scale. Choosing one of the pattern blocks as an original shape, students can then create the next-size shape using only those same-shaped blocks. After students have time to use the shapes concretely, they should also practice drawing them. ● Regularly provide students with figures that are not similar to ensure that students are continually checking for similarity. ● Provide opportunities for students to use scale drawings of geometric figures with a given scale. The opportunities should require them to draw and label the dimensions of the new shape. Initially, measurements should be in whole

		(whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.		numbers, progressing to measurements expressed with rational numbers. This will challenge students to apply their understanding of fractions and decimals. • Provide word problems that require finding missing side lengths, perimeters, or areas. In addition, allow students to design their own word problems asking for missing side lengths, perimeters, and/or areas.
	11.7 Three-Dimensional Figures	7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	What are the outcomes of slicing three-dimensional figures? What two-dimensional faces result from slicing a three-dimensional figure in various ways?	• Slicing three-dimensional figures to observe the cross sections formed helps develop three-dimensional visualization skills. Students should have the opportunity to physically create some of the three-dimensional figures, slice them in different ways, and describe in pictures and words what they discover. For example, use clay or playdough to form a cube, then pull string through it at different angles and record the shape(s) of the slices found. Challenges can be given: "See how many different two-dimensional cross sections you can create by slicing a cube."
	Pythagorean Theorem			**2 weeks**
	The Pythagorean Theorem	8.G.6 Analyze and justify an informal proof of the Pythagorean Theorem and its converse. 8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	How do I identify the legs and hypotenuse of a right triangle? What is the relationship between the areas of the squares created using the side lengths of a right triangle? How can I justify the Pythagorean Theorem and its converse? How can Pythagorean Theorem determine if a triangle is a right triangle? How do I use Pythagorean Theorem to find the missing side length of a right triangle? What are the relationships of a triangle plotted on a coordinate plane? How does the distance between two	• Students should understand the Pythagorean theorem as an area relationship between the sum of the squares on the lengths of the legs and the square on the length of the hypotenuse. This can be represented as $(\text{leg a})^2 + (\text{leg b})^2 = \text{hypotenuse}^2$. The Pythagorean Theorem only applies to right triangles. Exclusively using $a^2 + b^2 = c^2$ frequently leads to student errors in identifying the parts of the triangle. Use words like leg a, leg b, and hypotenuse c. • It is important for students to see right triangles in different orientations. • Students should be given the opportunity to explore right triangles to determine the relationships between the measures of the legs

			points on a coordinate plane relate to a right triangle? How can I use Pythagorean Theorem to calculate the distance between two points on a coordinate plane? How can I find real-world lengths that cannot be measured directly using Pythagorean Theorem?	and the measure of the hypotenuse. Experiences should involve using square grid paper to draw right triangles from given measures and representing and computing the areas of the squares on each side. Students can physically cut the squares on the legs apart and rearrange them, so they fit on the square along the hypotenuse. Data should be recorded in tables, allowing for students to conjecture about the relationship among the areas. • Students can apply the Pythagorean Theorem to real-world situations involving two- and three-dimensions. Some examples of this may include designing roofs, ramp dimensions, etc. Students should sketch right triangles to model real-world situations. Challenge students to identify additional ways that the Pythagorean Theorem is or can be used in real-world situations or mathematical problems, such as finding the height of something that is difficult to physically measure, or the right triangle formed by the diagonal of a prism.
	Converse of the Pythagorean Theorem	8.G.6 Analyze and justify an informal proof of the Pythagorean Theorem and its converse.		• Previously, students have discovered that not every combination of side lengths will create a triangle. Now they need to explore situations that involve the Pythagorean Theorem to test whether or not side lengths represent right triangles. This is an opportunity to remind students that the longest side is the only possibility for the hypotenuse. Students should be able to explain why a triangle is or is not a right triangle using the converse of the Pythagorean Theorem. This might be an opportunity for students to explore Pythagorean triples.
	Distance on the Coordinate Plane	8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.		• Students in Grade 8 should extend the use of the Pythagorean Theorem to find the distance between two points. Understanding how to determine distance by using vertical and

				horizontal lengths as legs of a right triangle is more important than deriving or memorizing a formula. • An extension could be having students understand how to find the midpoint as well as there is an intuitive connection to finding the distance between two points.
Module 13: Transformations, Congruence, and Similarity				**3 weeks**
Grading Period 4	13.1 Translations 13.2 Reflections 13.3 Rotations	8.G.1 Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). a. Lines are taken to lines, and line segments are taken to line segments of the same length. 8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	How can I use physical models, transparencies, or geometry software to explore transformations and verify their properties? What are the effects of transformations on two-dimensional figures using coordinates?	• Transformations should include those done both with and without coordinates. • Students should be able to appropriately label figures, angles, lines, line segments, congruent parts, and images (primes or double primes). • Students are expected to use logical thinking, expressed in words using correct terminology. They should also be using informal arguments, which are justifications based on known facts and logical reasoning. However, they are not expected to use theorems, axioms, postulates or a formal format of proof such as two-column proofs. • Students should solve mathematical and real-life problems based on understandings related to this cluster. Investigation, discussion, justification of their thinking, and application of their learning will assist them in the more formal learning of geometry standards in high school. • Initial work should be presented in such a way that students understand the concept of each type of transformation and the effects that each transformation has on an object before working within the coordinate system. • Provide opportunities for students to physically manipulate figures to discover properties of similar and congruent figures involving appropriate manipulatives, such as tracing paper, rulers, Miras, transparencies, and/or dynamic geometric software. Time should be allowed for

				students to explore the figures for each step in a series of transformations, e.g., cutting out and tracing. ● Discussion should include the description of the relationship between the preimage (original figure) and image(s) in regards to their corresponding parts (length of sides and measure of angles) and the description of the movement, (line of reflection, distance, and direction to be translated, center of rotation, angle of rotation, and the scale factor of dilation). ● Students should be able to provide a sequence of transformations required to go from a preimage to its image. Provide opportunities for students to discuss the procedure used, whether different procedures can obtain the same results, and if there is a more efficient way that can be used instead. They need to learn to describe transformations using words, numbers, drawings, and expressions. ● Although computer software is encouraged to be used in this cluster, it should not be used prematurely. Students need time to develop these geometric concepts with hands-on materials such as transparencies. ● Work in the coordinate plane follows an intuitive understanding of the transformations and should involve the mapping of various polygons by changing the coordinates using addition, subtraction, and multiplication.
	13.4 Dilations	8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	What are the relationships between coordinates of figures before and after transformations? How do I calculate scale factor?	★ See 13.1-13.3 above. ● Introduce dilation by discussing a topic such as "How do we double the size of a wiggly curve?" After much discussion, lead students toward assigning an arbitrary point, O, on the plane, and pushing every point on the squiggly line twice as far away from O. Explain to students that this is a dilation. A dilation pushes out (or pulls in) every point of the figure from its center of dilation

				proportionally by the same amount. This can be easily modeled by pushing in or pulling out an image on an overhead projector or an image drawn on a flashlight. In this case the center of dilation is O, and it can be anywhere on the plane. This can also be done by copying and pasting line segments using technology such as Microsoft Word, Powerpoint, or Smartboard. ● A dilation also has a scale factor. When doubling the size of a wiggly curve, the scale factor is 2, but a scale factor could be any number such as 1/2 or 3. Explain that in Grade 8 scale factors always have to be positive. Discuss what happens to a figure when the scale factor is less than one, compared to when the scale factor is greater than one. ● Although students should have experiences with the center of dilation being anywhere either inside or outside the figure, the expectation for Grade 8 is that they be proficient using centers of dilations at the origin and at a vertex of an image.
	13.5 Congruence and Transformations	8.G.1 Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). a. Lines are taken to lines, and line segments are taken to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. 8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Include examples both with and without coordinates.)	Which transformations preserve angle measures? Which transformations preserve side lengths?	● In Grade 6 students learned that when two shapes match exactly they have the same area. In Grade 7 they learn that two figures that "match up" or are put on top of each other are the same. In Grade 8, they learn the formal term of congruence and define it by using transformations. Students should also become familiar with the symbol for congruence (≅). ● Students should observe and discuss which properties of the polygons remained the same and which properties changed. Understandings should include generalizations about which transformations maintain size or maintain shape, as well as which transformations do not. ● A discussion can be had about the meaning of congruence. Initially one can use the informal definition of congruence being the same size and shape, but the discussion should eventually move

			toward the definition in 8.G.2 "a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations." Use the word "mapping" when discussing the overlay of two figures.
13.6 Similarity and Transformations	8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Include examples both with and without coordinates.)	Which transformations create proportional side lengths? How can I use a sequence of transformations to describe two similar figures?	• Review that in Grade 7 students learned that similar figures have sides that are proportional and angles that are congruent. Also, in 7th grade students used to say that figures are similar if they have the same shape but different size. Now they will be defining similarity in terms of transformations. Students should also become familiar with the symbol for similarity (~).
Module 9: Probability			**2 weeks**
9.1 Find Likelihoods	7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event; a probability around 1/2 indicates an event that is neither unlikely nor likely; and a probability near 1 indicates a likely event.	In an experiment, how can you determine the number of possible results? How can you describe the likelihood of an event? How can I identify a question to explore using probability?	• Build the concept of expressing probability as a number between 0 and 1, inclusive. ○ Provide students with situations that have clearly defined probability of never happening as zero, always happening as 1 or equally likely to happen as to not happen as $\frac{1}{2}$. ○ Then advance to situations in which the probability is somewhere between any two of these benchmark values. ○ Use this to build the understanding that the closer the probability is to 0, the more likely it will not happen, and the closer to 1, the more likely it will happen.
9.2 Relative Frequency of Simple Events	7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long run relative frequency, and predict the approximate relative frequency given the probability. 7.SP.7 Develop a probability model and use it	How can you use relative frequencies to find probabilities? How do I design a probability model? How do I use observed frequencies to answer a question using probabilities? How can I analyze results and explain possible discrepancies between observed and theoretical outcomes?	• Students can use chance experiments to collect data. They need to come to the understanding that as they increase the number of trials in their chance experiment the relative frequency (observed or experimental) over the long-run approaches the theoretical probability. Therefore if they have no way of knowing the theoretical probability (e.g., the number and kinds of tiles in

		to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.		a bag are hidden), they can do many, many trials to figure out an approximation of the theoretical probability. Then they could use that information to make further conclusions. Students can also use the theoretical probability to estimate the relative frequency, keeping in mind that what "should happen" does not always happen and that oftentimes the event will be at least close in value if not exact to the theoretical probability.
	9.3 Theoretical Probability	7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.		
	9.4 Compare Probabilities of Simple Events	7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long run relative frequency, and predict the approximate relative frequency given the probability. 7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.		• Provide students with models of equally likely outcomes and models of not equally likely outcomes and have students determine probabilities. These outcomes are called simple events.
	9.5 Probability of Compound Events	7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams,	How can you find the number of possible outcomes of one or more	• Students should begin to expand their knowledge and understanding of finding the probabilities of

	and simulations. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language, e.g., "rolling double sixes," identify the outcomes in the sample space which compose the event.	events? What is the difference between dependent and independent events? How can probability from a repeated chance process be used to predict the likelihood of a long-run event?	simple events to finding the probabilities of compound events by creating organized lists, tables, and tree diagrams. This helps students create a visual representation of the data. From each sample space, students determine the probability (fraction, decimal, percent) of each possible outcome. • Ask guiding questions to help students create methods for creating organized lists and tree diagrams for situations with more elements such as "How many outcomes are possible?", " What does each branch of the tree diagram represent?", or "How can you use your list to find the probability of the event?" • Students often see skills of creating organized lists, tree diagrams, etc. as the end product. Provide students with experiences that require the use of these graphic organizers to determine the theoretical probabilities. Have them practice making the connections between the process of creating lists, tree diagrams, etc. and the interpretation of those models and tying the simulation to a real-world situation.
Module 10: Sampling and Statistics			**2 weeks**
10.1 Biased and Unbiased Samples	7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population. a. Differentiate between a sample and a population. b. Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias. 7.SP.2 Broaden statistical reasoning by using the GAISE model: a. Formulate Questions: Recognize and	What is the difference between a sample and a population? How can you determine whether a sample accurately represents a population? What makes a sample an accurate representation of a population? How does a sample size affect inferences made about a population? What is bias? What factors create bias?	• *Note: One of the changes to this cluster was the deletion of the word "random" from the cluster statements. Students should informally learn about what a random sample is and how it is useful in statistics. Students are not required to actually use true random sampling when collecting data. Instead they should discuss which samples are the best representative of a population. However, a teacher may wish to extend to more sophisticated ideas of random sampling depending on the makeup of his or her class.* • Provide opportunities for students to use real-life situations. This shows the purpose for using

		formulate a statistical question as one that anticipates variability and can be answered with quantitative data. For example, "How do the heights of seventh graders compare to the heights of eighth graders?" (GAISE Model, step 1) b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) d. Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4)		sampling to make inferences about a population. • Provide students with samples from a population, including the statistical measures. Ask students guiding questions to help them make inferences from the sample. • Random sampling is a way to remove bias. Although students at this level may not be actually using true random sampling procedures when collecting data, the benefits of a random sample should be discussed. • Increasing the sample size reduces sample error, but it does not reduce bias. When students decide to select a sample from a specific group of people (friends), use the situation as an opportunity to discuss bias.
	10.2 Make Predictions	7.SP.2 Broaden statistical reasoning by using the GAISE model: a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. For example, "How do the heights of seventh graders compare to the heights of eighth graders?" (GAISE Model, step 1) b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) d. Interpret Results: Draw logical conclusions and make generalizations from the data based	What makes statistical questions have variability? What is the difference between a population, census, and sample? How can we collect data to answer a statistical question? How are sample surveys conducted? How do I use distributions to analyze data? How can I determine variability within a group? How do individuals compare to individuals, individuals compare to a group, and groups compare to groups? How can I use graphical displays to summarize data in context? How can I show a distribution of data? How can I use data to draw conclusions and make generalizations? How can I use center, spread, and shape	

		on the original question. (GAISE Model, step 4) Also Addresses: 7.RP.2 Recognize and represent proportional relationships between quantities. 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.	of a distribution to show differences between groups? How do I know a sample may or may not represent a population?	
	10.3 Generate Multiple Samples	7.SP.2 Broaden statistical reasoning by using the GAISE model: a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. For example, "How do the heights of seventh graders compare to the heights of eighth graders?" (GAISE Model, step 1) b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) d. Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4) Also Addresses: 7.RP.2 Recognize and represent proportional relationships between quantities.		

ODE Model Curriculum

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is a model curriculum that supports the standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards. The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, possible connections between topics, and some common misconceptions.

Mathematics Grade 7 Model Curriculum with Instructional Supports

Mathematics Grade 8 Model Curriculum with Instructional Supports

Curriculum and Instruction Guide

Module 3: Operations with Integers and Rational Numbers

Unpacked Standards / Clear Learning Targets		
Learning Target **Learning Target** 7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, p − q = p + (−q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. 7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1)(−1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then −(p/q) = (−p) /q = p/(–q). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide	**<u>Essential Understanding</u>** **Rational Numbers** • The set of integers consists of positive whole numbers, their opposites, and 0. • In a fraction the negative sign can be written in the numerator, the denominator, or out front, e.g., $\frac{-3}{4} = \frac{3}{-4} = -\frac{3}{4}$. • A rational number is any number that can be written as the quotient or fraction $\frac{P}{Q}$ of two integers, a numerator p, and non-zero denominator q. • A rational number can be converted to a decimal using long division; the decimal form of a rational number terminates in 0s or repeats. **Addition and Subtraction** • When modeling operations with integers on a number line, the sign of the number indicates the direction and the number indicates the amount of spaces moved. • In a number line model, the subtraction sign means to change directions. • A number and its opposite are additive inverses; they have a sum of 0, i.e., $a + (-a) = 0$. •Subtraction of rational numbers is adding the additive inverse, i.e., $p - q = p + (-q)$ • The absolute value of p − q is just the distance from p to q, regardless of direction. **Multiplication and Division** • A positive product is the result of multiplying two numbers with the same sign. • A negative product is the result of multiplying two numbers with different signs. • Division can be written using a fraction bar. • Multiplication of rational numbers can be modeled on the number line. • Division is the inverse of multiplication, so the same rules for rational	**<u>Academic Vocabulary</u>** Absolute Value Additive Identity Additive Inverse Property Additive Inverses Integer Opposites Zero Pair Difference Graph Negative Integer Positive Integer Rational Sum Zero Integer

rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	numbers apply. • Every quotient of integers (with a nonzero divisor) is a rational number. • A repeating quotient has a line of the repeating numerals	

I Can Statements:

- I can describe situations in which opposite quantities combine to make 0.
- I can represent and explain how a number and its opposite have a sum of 0 and are additive inverses.
- I can recognize and describe the rules when multiplying signed numbers.
- I can explain why integers can be divided except when the divisor is 0.
- I can solve real-world mathematical problems by adding, subtracting, multiplying, and dividing integers.
- I can identify properties of addition and subtraction when adding and subtracting rational numbers.
- I can apply properties of operations as strategies to add and subtract rational.
- I can identify subtraction of rational numbers as adding the additive inverse property to subtract rational numbers, p-q= p+ (-q).
- I can apply properties of operations as strategies to multiply and divide rational numbers.
- I can convert a rational number to a decimal using long division.
- I can add/multiply/subtract/divide rational numbers.
- I can solve real-world mathematical problems by adding, subtracting, multiplying, and dividing rational numbers, including complex fractions.

Performance Level Descriptors:

Proficient:	Accomplished (all of Proficient +):	Advanced (all of Proficient + all of Accomplished +):
• Model addition and subtraction of simple rational numbers on the number line • Recognize the additive inverse property • Add, subtract, multiply, and divide integers • Solve mathematical problems using the four operations on simple rational numbers • Convert between familiar fractions and decimals • Convert from fractions to decimals without technology	• Solve mathematical problems using the four operations on rational numbers	• Interpret products and quotients of rational numbers in real-world contexts

Prior Standard(s)	Future Standard(s)
4.OA.3 Solve multistep word problems posed with whole numbers and having whole number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators **5.NF.3** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **5.NF.5** Interpret multiplication as scaling (resizing). **6.NS.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. **6.NS.3** Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation. **6.NS.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative	**7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **8.NS.1** Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating. **8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **8.EE.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology. **A.APR.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

electric charge; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6.NS.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **6.NS.7** Understand ordering and absolute value of rational numbers.	

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45
- Ohio's K-8 Learning Progressions, The Number System, pages 16-17

Instructional Strategies

Many students struggle with the negative sign. One way to help students is to talk about values, order, and direction instead of quantities. For example, positive 5 is greater than positive 4, but −4 is greater than −5.
Another reason for confusion is that the negative sign can mean several things:

- A sign attached to a number to form negative numbers;
- A subtraction; or
- An indication to take the opposite of

Because of the confusion around the negative sign it may be helpful for students to understand the different meanings of the negative sign and identify which meaning is used when in a problem including the meaning shifts.

Using both contextual and numerical problems, students should explore what happens when negatives and positives are combined. Repeated opportunities over time with models will allow students to compare the results of adding and subtracting pairs of numbers, leading to the generalization of the rules.

Two-color counters or colored chips can be used as a physical model for adding and subtracting integers. Integer chips allow the idea of the zero pair (Additive Inverse Property) to be apparent.

Number lines present a visual image for students to explore and record addition and subtraction results. One of the positive aspects about using a number line model is that it is not limited to integers; it also lends itself toward connections on the coordinate plane. Students can use number lines with arrows and hops. When using number lines, establishing which factor will represent the length, number, and direction of the hops will facilitate understanding.

Multiplying and dividing integers should be thought of as an extension of adding and subtracting integers. Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers.

In multiplication, the first factor indicates the number of sets, and the second factor indicates the size of the set. This can be easily modeled with situations involving a positive times a positive or a positive times a negative. A negative times a positive can be inferred using the Commutative Property of Multiplication.

A negative times a negative can be problematic, for students want to know "How can we have a negative group of something?" One way to view it is as repeated subtraction. If the first factor being positive indicates repeated addition, then the first factor being negative indicates repeated subtraction. Therefore "negative 3 sets of negative −2" or −3(−2) means to remove 3 sets of −2 from zero.

Students will discover that they can multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. They should then analyze and solve problems leading to the generalization of the rules for operations with integers.

Another method for learning multiplication/division rules is to use patterns. Beginning with known facts, students can predict the answers for related facts, keeping in mind that the properties of operations apply.

Using the language of "the opposite of" helps some students understand the multiplication of negatively signed numbers ($-4 \cdot -4 = 16$, the opposite of 4 groups of −4). Discussion about the tables should address the patterns in the products, the role of the signs in the products, and commutativity of multiplication.

It is important when performing operations that students are able to justify their steps using the properties. Although, the focus should not be on identifying the properties of operation, teachers should be using their formal names in classroom discussion, so students are able to gain familiarity with and recognize the correct terminology

In Grade 6, students should have learned that the absolute value of a number does not take into account sign or direction; it only is a measure of distance (magnitude) from 0. Discourage students from saying that the "answer is always positive or 0" since that will lead to misconceptions when students encounter problems such as $|4x - 2| = 18$ in high school. Instead emphasize that it is the "distance from 0." This is why the value of something like $|x| = -5$ has no solution, since distance cannot be negative.

Sample Assessments and Performance Tasks

Reporting Category:	**OST Test Specs**:
The Number System **Standards**: 7.NS.1, 2, and 3; 7.EE.3 **Approximate Portion of Test**: 28% - 37%; 15 - 19 points	Items may use all types of rational numbers. Items must include at least 1 negative number. Students need to be able to recognize the formal names of properties. For 7.NS.2a, b, and c: items must include a negative number. Students need to be able to recognize the formal names of properties. Items involving estimation to assess reasonableness will not require the student to find the exact answer. Variables may need to be defined using appropriate units.

Instructional Resources

7.NS.1	7.NS.2	7.NS.3
Better Lesson Khan Academy Dan Meyer Activities World Record Balloon Dog Gas Station Ripoff Graduation Dueling Discounts Illustrative Mathematics Bookstore Account Comparing Freezing Points Differences and Distances Differences of Integers Distances Between Houses Distances on the Number Line 2 Operations on the number line Rounding and Subtracting	Better Lesson Khan Academy Videos Illustrative Mathematics: Distributive Property of Multiplication Why is a Negative Times a Negative Always Positive? Temperature Change Decimal Expansions of Fractions Equivalent fractions approach to non-repeating decimals Repeating decimal as approximation Repeating or Terminating?	Better Lesson Khan Academy Videos Dan Meyer Graduation Illustrative Mathematics Sharing Prize Money

Adopted Resource	
Reveal: 3.5 Apply Integer Operations 3.6 Rational Numbers 3.7 Add and Subtract Rational Numbers 3.8 Multiply and Divide Rational Numbers 3.9 Apply Rational Number Operations	**Aleks:** Whole Numbers and Integers (ALEKS TOC): ● Adding and Subtracting with Integers ● Multiplying and Dividing with Integers ● Plotting and Comparing Integers Decimals (ALEKS TOC): ● Venn Diagrams and Sets of Rational Numbers

Return to Scope and Sequence

Module 4: Exponents and Scientific Notation

Unpacked Standards / Clear Learning Targets

Learning Target

8.EE.1 Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions.

8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other.

8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology.

Essential Understanding

Exponents

Note: In 8th Grade all exponents are integers, so all notes refer to cases using integer exponents.

- $x^0 = 1$, when $x \neq 0$.
- $x^{-n} = \frac{1}{x^n}$, when $x \neq 0$.
- A coefficient is different than an exponent. For example, n^2 is different than 2n because n^2 means n × n and 2n means 2 × n, and n^3 is different than 3n because n^3 means n × n × n and 3n means 3 × n.
- A positive exponent denotes repeated multiplication of the base.
- A negative exponent denotes repeated division of the base.
- A negative exponent does not change the sign of the base.

Scientific Notation

- Scientific notation is a mathematical expression written as a decimal number greater than or equal to one but less than 10 multiplied by a power of ten, e.g. 3.1×10^4 .
- A number expressed in scientific notation that has a negative exponent is between negative one and positive one.
- A number expressed in scientific notation that has a positive exponent is greater than one or less than negative one.
- Powers of ten can be used to compare numbers written in scientific notation.

Academic Vocabulary

Base
Cube
Exponent
Power
Square
Decimal notation
Scientific notation
Standard form

I Can Statements:

- I can explain why a zero exponent produces a value of one.
- I can explain how a number raised to an exponent of 1 is the reciprocal of that number.
- I can explain the properties of integer exponents to generate equivalent numerical expressions.
- I can express numbers as a single digit times an integer power of 10.
- I can use scientific notation to estimate very large and/or very small quantities.
- I can compare quantities in scientific notation to express how much larger one is compared to the other.
- I can perform operations using numbers expressed in scientific notations and decimals.
- I can use scientific notation to express very large and very small quantities.
- I can interpret scientific notation that has been generated by technology.

- I can choose appropriate units when using scientific notation.

Performance Level Descriptors:		
Proficient: ● Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large quantities. ● Use scientific notation to represent and compare very large and very small quantities. ● Express how many times a number written as an integer power of 10 is than another number written as an integer power of 10. ● Solve routine problems that require performing operations with numbers expressed in scientific notation, including numbers written in both decimal and scientific notation and interprets scientific notation that has been generated by technology. ● Apply the properties of integer exponents to solve mathematical problems.	Accomplished (all of Proficient +): ● Solve problems involving the conversion between decimal notation and scientific notation and the comparison of numbers written in different notations.	Advanced (all of Proficient + all of Accomplished +): ● Calculate and interpret values written in scientific notation within new and unfamiliar contexts. ● Use properties of integer exponents to order or evaluate multiple numerical expressions with integer exponents.

Prior Standard(s)		Future Standard(s)
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. 5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10. 5.NBT.7 Solve real-world problems by adding, subtracting, multiplying, and dividing decimals using concrete models	6.EE.1 Write and evaluate numerical expressions involving whole number exponents. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. A.APR.1 Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction,

or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, or multiplication and division; relate the strategy to a written method and explain the reasoning used.		and multiplication; add, subtract, and multiply polynomials. F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 8, Number 4, page 55
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

Instructional Strategies

Although students begin using whole-number exponents in Grades 5 and 6, it is in Grade 8 when students are first expected to understand, explain, and apply the properties of exponents and to extend the meaning beyond counting-number exponents.

Students should not be told these properties but rather should derive them through experience and reason. Many students who simply "memorize" the rules without understanding may confuse the rules when trying to apply them at later times. Instead students should be encouraged to discover the rules using tables, patterns, and expanded notation. As they have multiple experiences simplifying numerical expressions with exponents, these properties become natural and obvious.

Students should use multiplicative reasoning and expanded notation to gain understanding of non-positive exponents.

Another way to view the meaning of 0 and negative exponents is by applying the following principle: The properties of counting-number exponents should continue to work for integer exponents.

- Therefore, the properties for exponents can also be used to help students understand $x^0 = 1$. For example, consider the following expression and simplification: $3^0 \cdot 3^5 = 3^{0+5} = 3^5$. This computation shows that when 3^0 is multiplied by 3^5, the result should be 3^5, which implies that 3^0 must be 1. Because this reasoning holds for any base other than 0, it can be reasoned that $a^0 = 1$ for any nonzero number a.
- The properties of exponents can also help students make sense of negative exponents. To make a judgment about the meaning of 3−4, the approach is similar: $3^{-4} \cdot 3^4 = 3^{-4+4} = 3^0 = 1$. This computation shows that 3^{-4} should be the reciprocal of 3^4, because their product is 1. And again, this reasoning holds for any nonzero base. Thus, we can reason that $a^{-n} = \frac{1}{a^n}$.

In Grade 8 students should also have practice using negative numbers as bases. For example, it is very difficult for students to differentiate between -3^2 and $(-3)^2$. To help students differentiate between the two forms, encourage them to write it out in expanded form: $(-3)^2 = -3 \cdot -3$ and $-3^2 = -1 \cdot 3 \cdot 3$.

Have students identify the bases before solving problems as many students incorrectly only attribute the exponent to the nearest number. For example, instead of realizing that $(4 + 2)^3 = (4 + 2) \cdot (4 + 2) \cdot (4 + 2) = 216$, they incorrectly calculate $4 + 2^3 = 12$.

Students should also have practice using non-integer bases such as $(1.5)^3$ or $\left(\frac{3}{4}\right)^{-2}$

Sample Assessments and Performance Tasks

Reporting Category: The Number System	OST Test Specs:
Standards: 8.EE.1, 3, and 4 **Approximate Portion of Test: 20% - 25%; 11 - 13 points**	Exponents will be integers. Bases will be whole number, fractions, or decimals. Variables will be used only for unknown exponents (*Ex:* $3^2 \bullet 3^x = 3^6$) All numbers will be able to be written tin the form $a \times 10^b$ where a and b are integers and $1 \leq a < 10$. Exponents may be positive or negative. Items may use all types of rational numbers. Exponents may be positive or negative.

Instructional Resources

8.EE.1	8.EE.3	8.EE.4
Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Ants versus humans Extending the Definitions of Exponents, Variation 1 Raising to the zero and negative powers	Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Ant and Elephant Orders of Magnitude Pennies to heaven	Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Ants versus humans Choosing appropriate units Giantburgers Pennies to heaven

Adopted Resource

Reveal:	Aleks:
Lesson 4-1: Powers and Exponents Lesson 4-2: Multiply and Divide Monomials Lesson 4-3: Powers of Monomials Lesson 4-4: Zero and Negative Exponents Lesson 4-5: Scientific Notation Lesson 4-6: Compute with Scientific Notation	Exponents, Polynomials, and Radicals (ALEKS TOC): • Product, Power, and Quotient Rules • Negative Exponents • Scientific Notation

	Whole Numbers and Integers (ALEKS TOC): ● Exponents and Order of Operations

Return to Scope and Sequence

Module 5: Real Numbers

Unpacked Standards / Clear Learning Targets		
Learning Target 8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating. 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., π^2. 8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	**Essential Understanding** Roots • The equation $x^2 = p$ has two solutions for x: $\sqrt{p}$ and $-\sqrt{p}$. For example, in describing the solutions to $x^2 = 36$, students can write $x = \pm\sqrt{36} = \pm 6$. • The $\sqrt{p}$ is defined to be the positive solution to the equation $x^2 = p$. For example, it is **not** correct to say that $\sqrt{36} = \pm 6$, instead it should be $\sqrt{36} = 6$. • $x^3 = p$ has only one solution for x: $\sqrt[3]{p}$ • The square root of 2 is irrational. Real Numbers • Every real number can be classified as repeating, terminating, or non-repeating, nonterminating. • Real numbers are either rational or irrational. • A rational number is any number that can be written as the quotient or fraction of two integers, $\frac{p}{q}$, where p is the numerator and q is the non-zero denominator. • Rational numbers when written as a decimal expansion are repeating or terminating. • Irrational numbers when written as a decimal expansion are non-repeating and nonterminating. • A number is classified by its simplest form, e.g., $\sqrt{25}$ is rational because 5 is rational. • Square roots may be negative, e.g., Both 6 and −6 are square roots of 36, and $\sqrt{36}$ means only the positive square root whereas $-\sqrt{36}$ means only the negative square root. • A negative sign cannot be inside a square root number at this grade level.	**Academic Vocabulary** Cube root Irrational Rational Root Square root Decimal Irrational number Rational number Repeating decimal Square root Terminating decimal

	• The roots of perfect squares and perfect cubes of whole numbers are rational. • Square roots of whole numbers that are not perfect squares are irrational. • Cube roots of whole numbers that are not perfect cubes are irrational. • Given two distinct numbers, it is possible to find both a rational and an irrational number between them.	

I Can Statements:

- I can use square root and cube root symbols as inverse operations to represent solutions to equations of the form x^2= p and x^3 =p, where p is a positive rational number.
- I can evaluate square roots of small perfect squares through $12^2 = 144$.
- I can evaluate cube roots of small perfect cubes: cube root of 1 through the cube root of 125.
- I can understand that the square root of 2 is irrational.
- I can define rational and irrational numbers.
- I can show that the decimal expansion of rational numbers repeats eventually.
- I can convert a decimal expansion, which repeats eventually into a rational number.
- I can show that every number has a decimal expansion.
- I can approximate irrational numbers as rational numbers.
- I can approximately locate and order irrational numbers on a number line.
- I can estimate the value of expression involving irrational numbers using rational approximations without a calculator.

Performance Level Descriptors:

Proficient:
- Evaluate square roots of small perfect squares
- Calculate the cube root of small perfect cubes
- Use square root and cube root symbols to represent solutions of the form x^2 = p and x^3 = p, where p is a positive rational number.
- Use the properties of natural number exponents to generate equivalent numerical expressions.
- Apply the properties of natural number exponents to solve simple mathematical problems.

Accomplished (all of Proficient +):
- Place irrational numbers on a number line in an abstract setting using variables.
- Use square root and cube root symbols to represent solutions to real-world problems resulting from equations of the form x^2 = p and x^3 = p.

Advanced (all of Proficient + all of Accomplished +):
- Explain how square roots and cube roots relate to each other and to their radicands.
- Notice and explain the patterns that exist when writing rational numbers (repeating decimals) as fractions.
- Explain how to get more precise approximations of square roots.

- Identify square roots of non-square numbers and pi as irrational numbers.
- Identify between which two whole number values a square root of a non-square number is located.
- Identify rational and irrational numbers and convert less familiar rational numbers (repeating decimals) to fraction form
- Place irrational numbers on a number line

Prior Standard(s)	Future Standard(s)	
6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. 7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.	8.G.9 N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. A.REI.4ab 4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. b. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.

Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 8, Number 4, page 55]
- [Ohio's K-8 Learning Progressions, The Number System, pages 16-17]
- [Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19]

Instructional Strategies

In previous grades, students become familiar with rational numbers called decimal fractions. In Grade 7, students carry out the long division and recognize that the remainders may repeat in a predictable pattern—a pattern creates the repetition in the decimal representation (see 7.NS.2.d). In Grade 8, they explore its occurrence.

Ask students what will happen in long division once the remainder is 0. They can reason that the long division is complete, and the decimal representation terminates.

However, if the remainder never becomes 0, then the remainder will repeat in a cyclical pattern. The important understanding is that students can see that the pattern will continue to repeat.

Explore differences between terminating and repeating decimals.

To help students build a conceptual understanding, connect roots to area and volume models where the area and volume are the radicand and the solution is the length of the side of the model.

Another way to explain it is that the area and volume of the square or cube represents n; and the square and cube's side length is represented by $\sqrt{n}$ and $\sqrt[3]{n}$ respectively. Have students use geoboards, square tiles, graph paper, or unit cubes to build squares and cubes reviewing exponents in the process. Problems such as the Painted Cube Problem can then be modified to extend to square and cube roots.

Also, provide practical opportunities for students to flexibly move between forms of squared and cubed numbers. For example, If $3^2 = 9$ then $\sqrt{9} = 3$. This flexibility should be experienced symbolically and verbally with manipulatives and with drawings.

It could be appropriate to use Venn diagrams/set diagrams or flowcharts to show the relationships among real, rational, irrational numbers, integers, and natural numbers. The diagram should show that all real numbers are either rational or irrational.

Students should come to the understanding that (1) every rational number has a decimal representation that either terminates or repeats and (2) every terminating or repeating decimal is a rational number. Then, they can use that information to reason that on the number line, irrational numbers must have decimal representations that neither terminate nor repeat.

In previous grades, students learned processes that can be used to locate any rational number on the number line: Divide the interval from 0 to 1 into b equal parts; then, beginning at 0, count out a of those parts. Now they can use similar strategies to locate irrational numbers on a number line.

Use an interactive number line to allow students to see how a number line can be infinitely divided. One resource is Zoomable Number Line by MathisFun.

Although students at this grade do not need to be able to prove that $\sqrt{2}$ is irrational, they minimally need to know that $\sqrt{2}$ is irrational (see 8.EE.2), which means that its decimal representation neither terminates nor repeats. Nonetheless, they should be able to approximate irrational numbers such as $\sqrt{2}$ without using the square root key on the calculator.

The $\sqrt{2}$ caused Greek Mathematicians many problems, for although they could construct it using tools, but they could not measure it. Integrating math history into the lesson may be interesting for some students.

Have students do explorations where precision matters. Discuss situations where precision is vital and other situations where reasonableness is more vital than precision. Although learning about significant digits formally takes place in high school, it is appropriate to talk about how intermediate rounding affects precision. The display of irrational numbers on a calculator could also be used for a discussion point. Another discussion could take place comparing results using the pi button compared to the typical approximation of 3.14.

The concept of precision should also be tied to real-world contexts. For example, we do not buy 3.5 apples, but we may buy 3.5 lbs of ground beef, so rounding to a whole number is a precise number in terms of buying apples. Therefore, being precise should relate to the real-world context of the problems.

Sample Assessments and Performance Tasks

Reporting Category: The Number System **Standards: 8.NS.1 and 2; 8.EE.2** **Approximate Portion of Test: 20% - 25%; 11 - 13 point**	**OST Test Specs:** ● Irrational numbers are limited to expressions involving π or radicals. ● Irrational expressions will only use one operation. ● Items will include square roots and cube roots. ● Radicands will be positive rational numbers. ● Items may require the identification of both solutions of the equation $x^2 = p$.

Instructional Resources

8.EE.2	8.NS.1	8.NS.2
Better Lesson Shmoop Khan Academy Videos Painted Cube or Birthday Problem • Painted Cubes from the Mathematics Centre • Painted Sides of a Cube from The University of Georgia • Painted Cube from NRICH Enriching Mathematics • The Painted Cube by CollecEDNY	Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Converting Decimal Representations of Rational Numbers to Fraction Representations Converting Repeating Decimals to Fractions Identifying Rational Numbers Repeating or Terminating?	Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Comparing Rational and Irrational Numbers Irrational Numbers on the Number Line Placing a square root on the number line

Adopted Resource

Reveal: Lesson 5-1: Roots Lesson 5-2: Real Numbers Lesson 5-3: Estimate Irrational Numbers	**Aleks:** Exponents, Polynomials, and Radicals (ALEKS TOC): ● Square Roots and Irrational Numbers ● Higher Roots and Nonlinear Equations

Lesson 5-4: Compare and Order Real Numbers	Decimals (ALEKS TOC): • Venn Diagrams and Sets of Rational Numbers

Return to Scope and Sequence

Module 6: Algebraic Expressions

Unpacked Standards / Clear Learning Targets

Learning Target	Essential Understanding	Academic Vocabulary
7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. 7.EE.2 In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related.	• Equivalent expressions always have the same value even if written in different forms. • Equivalent expressions can be generated using properties of operations (Distributive Property, Associative Properties of Multiplication, Associative Property of Addition, Commutative Property of Multiplication, Commutative Property of Addition, and Identity Property of Multiplication). • The order of operations is used to generate equivalent algebraic expressions. • The coefficient of a single variable is 1 even if it is not written. For example, –x = –1x and x = 1x. • A fractional coefficient can be written in two ways, e.g., $\frac{x}{3} = \frac{1}{3}x$. • Negative rational terms can be written in three ways, e.g., $\frac{1}{-3} = \frac{-1}{3} = -\frac{1}{3}$. • In problems involving percentages, 100% of the variable x can be written as x = 1x. • Factoring a GCF can be used to write an equivalent expression. • Writing expressions in equivalent forms can serve different purposes and provide different ways of seeing a problem in context.	Algebraic Coefficient Constant Distributive property Exponent Expression Factor Like term Quantities Rational Sum Term Variable

I Can Statements:

- I can combine like terms with rational coefficients.
- I can factor and expand linear expressions with rational coefficients using the distributive property.
- I can apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- I can write equivalent expressions with fractions, decimals, percents, and integers.
- I can rewrite an expression in an equivalent form in order to provide insight about how quantities are related in a problem context.
- I can convert between numerical forms as appropriate.

- I can solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically.
- I can apply properties of operations to calculate with numbers in any form.
- I can assess the reasonableness of answers using mental computation and estimation strategies.

Performance Level Descriptors

Proficient:	Accomplished (all of Proficient +):	Advanced (all of Proficient + all of Accomplished +):
• Recognize simple equivalent expressions • Apply properties of operations to factor and expand linear expressions with positive integer coefficients Apply properties of operations to factor and expand linear expressions with simple rational coefficients	• Apply properties of operations to factor and expand linear expressions with rational coefficients • Understand that rewriting an expression can show how quantities are related in familiar problem-solving contexts	• Apply properties of operations to factor and expand linear expression with complex rational coefficients • Understand that rewriting an expression can show how quantities are related in an unfamiliar problem-solving context

Prior Standard(s)	Future Standard(s)	
6.EE.3 Apply the properties of operations to generate equivalent expressions. **6.EE.4** 4 Identify when two expressions are equivalent, i.e., when the two expressions name the same number regardless of which value is substituted into them.	**8.EE.7** Solve linear equations in one variable **A.APR.1** Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **A.SSE.1** Interpret expressions that represent a quantity in terms of its context. **A.SSE.2** Use the structure of an expression to identify ways to rewrite it.	**A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **N.CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

Instructional Strategies

It is important that students are able to justify their thinking using the properties. Although the focus should not be on identifying the properties of operation, teachers should be using their formal names in classroom discussion so students are able to gain familiarity with and recognize the correct terminology.

Provide opportunities for students to use and understand the properties of operations. These include the Commutative, Associative, Identity, and Inverse Properties of Addition and of Multiplication, the Zero Property of Multiplication, and the Distributive Property.

Subtraction should be thought of as the opposite of addition, and division should be thought of as the opposite of multiplication. Note: Avoid PEMDAS as it leads to many misconceptions and errors in computation.

Writing equivalent expressions includes simple cases of factoring out a GCF. This can be illustrated using the area model that students are already familiar with. In this model the students are given the areas, and they are asked to find the lengths and widths. It might be wise to point out that, although in real-life length and width cannot be negative, our model allows for lengths and widths to be negative to illustrate the concept.

Students started combining like terms in Grade 6. Now they will extend this concept to negative numbers. It may be helpful to use Algebra tiles to illustrate this process as it will prevent future misconceptions from forming such as trying to combine $2x$ and $3x^2$. Students should then extend that concept to other rational numbers besides integers.

Sample Assessments and Performance Tasks

Reporting Category: The Number System **Standards**: 7.EE.1 and 2 **Approximate Portion of the Test:** 28% - 37%; 15 - 19 points	**OST Test Specs:** Items may use all types of rational numbers. Items use only linear expressions. Negative numbers and multiple operations may be used. Students need to be able to recognize the formal names of properties.

Instructional Resources

7.EE.1 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Writing Expressions	7.EE.2 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Ticket to Ride Guess My Number

Adopted Resource	
Reveal: Lesson 6-1: Simplify Algebraic Expressions Lesson 6-2: Add Linear Expressions Lesson 6-3: Subtract Linear Expressions Lesson 6-4: Factor Linear Expressions Lesson 6-5: Combine Operations with Linear Expressions	**Aleks:** Equations and Inequalities (ALEKS TOC): ● Simplifying Algebraic Expressions ● The Distributive Property

Return to Scope and Sequence

Module 7: Equations and Inequalities

Unpacked Standards / Clear Learning Targets		
Learning Target 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. 8.EE.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	**Essential Understanding** EQUATIONS • Variables are used to represent a quantity. • The order of operations is used to write and solve equations given within a context of a word problem. • A solution is a value that makes an equation or an inequality true. • Inverse operations may be used to solve equations and inequalities. • Equivalent expressions always have the same value even if written in different forms. • Equivalent expressions can be generated by using properties of operations (distributive property, associative, commutative, identity and inverse properties of multiplication and addition). • A term includes the operational sign in front of it. • Linear equations can have no solutions, one solution, or infinitely many solutions. • Linear equations are solved by using inverse operations. • Linear equations that are equivalent to $x = a$ have one solution. • Linear equations that are equivalent to $a = a$ have infinitely many solutions. • Linear equations that are equivalent to $a = b$ have no solutions. INEQUALITIES • Inequalities have infinitely many solutions. • Solutions to inequalities can be represented on number line diagrams. • Point c is not included in the graphical solution to $x > c$ or $x < c$; the	**Academic Vocabulary** Constant Distributive property Equation Exponent Coefficient Factor Inequality Greater Than Less Than Like terms Quantities Term Variable Coefficients Factor Properties Infinite solutions No solution One variable equation Inverse operation

	number line diagram represents this with an open circle around point c. • Point c is included in the graphical solution to $x \geq c$ or $x \leq c$; the number line diagram represents this with a closed circle at point c. • All of the solutions to an inequality are represented with a shaded region on a number line diagram. • The inequality $x > c$ is equivalent to $c < x$, and $x \geq c$ is equivalent to $c \leq x$. • When multiplying or dividing both sides of an inequality by a negative number, the order of the comparison it represents is reversed.	

I Can Statements:

- I can identify the sequence of operations used to solve an algebraic equation of the form px + q = r and p(x + q) = r.
- I can fluently solve equations of the form px + q = r and p(x + q) = r with speed and accuracy.
- I can use variables and construct equations to represent quantities of the form px + q = r and p(x + q) = r from real--world and mathematical problems.
- I can solve equations in one variable with variables on both sides of the equation.
- I can give examples of linear equations in one variable with one solution.
- I can give examples of linear equations in one variable with infinitely many solutions.
- I can give examples of linear equations in one variable with no solution.
- I can solve linear equations with rational number coefficients.
- I can solve equations whose solutions require expanding expressions using the distributive property and/or collecting like terms.
- I can graph the solution set of the inequality of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers.
- I can solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers.

Performance Level Descriptors:

Proficient:
7th
- Solve simple equations
- Solve two-step equations with integer coefficients
- Use variables to create and solve simple equations and inequalities that model word problems
- Identify a solution of an inequality
- Solve simple inequalities with positive integer coefficients
- Use variables to create and solve simple equations and inequalities that model word problems

Accomplished (all of Proficient +):
7th
- Construct equations and inequalities with a variable to solve routine problems

Advanced (all of Proficient + all of Accomplished +):
7th
- Construct equations and inequalities with more than one variable to solve non-routine problems
- Use variables to represent and reason with quantities in real-world and mathematical situations

8th ● Solve straightforward one or two step linear equations with integer coefficients. ● Solve straightforward multi-step linear equations with rational coefficients ● Solve routine multi-step linear equations with rational coefficients and variables on both sides and provide examples of equations that have one solution, infinitely many solutions, or no solutions	8th ● Strategically choose and use procedures to solve linear equations in one variable. ● Justify why an equation has one solution, infinitely many solutions, or no solution.

Prior Standard(s)	Future Standard(s)
6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers. **6.EE.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	**8.EE.7** Solve linear equations in one variable. **8.EE.8** Analyze and solve pairs of simultaneous linear equations graphically. **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. **A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45
- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 3, pages 46-47
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

- Ohio's K-8 Critical Areas of Focus, Grade 8, Number 1, pages 50-51
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

Instructional Strategies

Continue to build on students' understanding and application of writing and solving one-step equations (6th Grade) from a problem situation to problem situations that require multi-step equations and inequalities.

This is also the context for students to practice using rational numbers including integers and positive and negative fractions and decimals. It is appropriate to expect students to show their steps in their work. Students should be able to move toward explaining their thinking using correct terminology.
To assist students' assessment of the reasonableness of their answers, especially problem situations involving fractional or decimal numbers, use whole-number approximations for the computation and then compare to the actual computation.

In connection with 7.EE.1 students should apply the properties of operations and equality found in Table 3 and Table 4 of Ohio's Learning Standards. Teachers should be using the correct terminology to justify steps when performing operations and solving equations. Although Grade 7 students should not be required to know the formal names of the properties, they should be encouraged to recognize them and use them to justify their steps when solving equations.

Experiences in solving equations should move through Bruner's stages of concrete, pictorial, and algebraic/abstract representation. Utilize experiences with the pan balance model, hangers, tape diagrams, or Algebra tiles as a visual tool for maintaining equality (balance):
• First with simple numbers;
• Then with pictures symbolizing relationships; and
• Finally, with rational numbers. This allows understanding to develop as the complexity of the problems increases.

Some studies have shown that a students' fractional knowledge correlates with their ability to write equations. Therefore encourage students to solve equations with fractions by using diagrams instead of just using inverse operations. This may aid in creating understanding to alleviate the misuse of fraction rules in later grades/courses. Numbers that are easily modeled should be used initially until students internalize the process.

In Grade 7, students learned integer operations for the first time. They also applied the properties of operations when solving two-step equations and inequalities. Now students build on the fact that solutions maintain equality and that equations may have only one solution, many solutions, or no solutions at all.

Properties of Operations Table 3 on page 97 of Ohio's Learning Standards in Mathematics states the Properties of Operations and Table 4 states the Properties of Equality. Teachers should be using the correct terminology to justify steps when performing operations and solving equations.

Students incorrectly think that the variable is always on the left side of the equation. Give students situations where the variable is on the right side of the equation. Emphasize using the Symmetric Property of Equality if students wish to flip the variable to the other side of the equal sign.

Equation-solving in Grade 8 should involve multi-step problems that require the use of the distributive property, collecting like terms, rational coefficients, and variables on both sides of the equation.

In Grade 7, students may have used a pan balance, number lines, or algebra tiles to solve two-step equations. Eighth grade students could review these models and build upon them. For example, algebra tiles may help prevent student errors such as incorrectly combining like terms on opposite sides of the equations.

When not using models, some students benefit from drawing a vertical line through the equals sign to separate the two sides of the equation.

Connect mathematical analysis with real-life events by using contextual situations when solving equations. Students should experience—

- analyzing and representing contextual situations with equations;
- identifying whether there is one solution, no solutions, or infinitely many solutions; and then
- solving the equations to prove conjectures about the solutions.

In Grade 6 students wrote inequalities in the forms of x > c and c < x. in Grade 7 students use > and <. Discuss why teachers should, when graphing on a number line, a closed circle represents > and < and an open circle represents > and <. Students should also have practice solving one and two-step inequalities with rational numbers.

Present situations where the variable is both on the left and the right side of the equality. Students need to be fluent in solving inequalities where the variable is on the left and right of the inequality for later algebraic concepts using compound inequalities. Therefore, discourage students from always writing the variable on the left side of an inequality.

When graphing, teachers should avoid telling students that the inequality points the same direction on the number line as the arrow; this creates a misconception that is hard to break when students work on compound inequalities in high school. An alternative strategy is to ask students to name 3 points that make an inequality true, and then draw the arrow in that direction.

Students should be able to create equations and inequalities from real-world situations where they always precisely define the variable(s).

Provide multiple opportunities for students to work with multi-step problem situations that have multiple solutions and therefore can be represented by an inequality. Students need to be aware that values can satisfy an inequality but not be appropriate for the situation, therefore limiting the solutions for that particular problem.

Sample Assessments and Performance Tasks

Reporting Category: The Number System **Standards**: 7.EE.4 **Approximate Portion of the Test:** 28% - 37%; 15 - 19 points **Standards: 8.EE.7** **Approximate Portion of Test: 20% - 29%; 11 - 15 points**	**OST Test Specs**: Items may use all types of rational numbers. Items involving estimation to assess reasonableness will not require the student to find the exact answer. Students need to be able to recognize the formal names of properties. Variables may need to be defined using appropriate units. For 7.EE.4a, equations must be of the form $px + q = r$ or $p(x + q) = r$, where p, q, and r are specific rational numbers. For 7.EE.4b, inequalities must be of the form $px + q > r$, $px + q \geq r$, $px + q < r$ or $px + q \leq r$, where p, q, and r are specific rational numbers. Items may require graphing a solution to an inequality on a number line. Equations can be more complex than the forms $px + r = q$ and $p(x + r) = q$.

Instructional Resources

7.EE.4	8.EE.7	
Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Bookstore account Sports Equipment Set	Better Lesson Shmoop Khan Academy Videos Dan Meyer Activity Ditch Diggers	Illustrative Mathematics Coupon versus discount Sammy's Chipmunk and Squirrel Observations Solving Equations The Sign of Solutions

Adopted Resource	
Reveal: Lesson 7-1: Write and Solve Two-Step Equations: px + q = r Lesson 7-2: Write and Solve Two-Step Equations: p(x + q) = r Lesson 7-3: Write and Solve Equations with Variables on Each Side Lesson 7-4: Write and Solve Multi-Step Equations Lesson 7-5: Determine the Number of Solutions Lesson 7-6: Write and Solve One-Step Addition and Subtraction Inequalities Lesson 7-7: Write and Solve One-Step Multiplication and Division Inequalities Lesson 7-8: Write and Solve Two-Step Inequalities	**Aleks:** Equations and Inequalities (ALEKS TOC): ● Multi-Step Equations ● Applications of Equations ● Equations with Variables on Both Sides ● The Distributive Property ● Simplifying Algebraic Expressions Whole Numbers and Integers (ALEKS TOC): ● One-Step Equations

Return to Scope and Sequence

Module 1: Proportional Relationships

Unpacked Standards / Clear Learning Targets		
Learning Target **7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. **7.RP.2** Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a	**Essential Understanding** • A unit rate is a comparison of two quantities where the second quantity (denominator) is one. • A rate can be written as a complex fraction which can be used to find the unit rate. • A proportional relationship is a relationship between quantities. • Proportions involve vertical and horizontal multiplicative relationships. • In a table that represents a proportional relationship between y and x, $yy\ xx$ is constant.	**Academic Vocabulary** Equivalent ratio Coordinate plane Proportion Proportional relationship Nonproportional relationship Ratio Unit rate

coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems.	• The unit rate, which is the constant of proportionality, can be identified through models. • Proportional relationships can be written as equations using the constant of proportionality, e.g., y = kx; y = mx; t = pn. • The constant of proportionality is not always rational, e.g., $\pi\pi$. • The unit rate is the amount of change in y as x increases by one unit, e.g., in a table or graph. • Graphs that represent proportional relationships are linear and go through the origin.	Constant of Proportionality Complex fraction Constant

I Can Statements:

- I can compute unit rates associated with ratios of fractions in like or different units.
- I can compute fractional by fractional quotients.
- I can apply fractional ratios to describe rates.
- I can define the constant of proportionality as a unit rate.
- I can analyze two ratios to determine if they are proportional to one another with a variety of strategies (e.g. using tables, graphs, pictures, etc.).
- I can analyze tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships to identify the constant of proportionality.
- I can represent proportional relationships by writing equations.
- I can recognize what (0,0) represents on the graph of a proportional relationship.

Performance Level Descriptors:

Proficient:
- Compute a unit rate of two whole numbers where the unit rate is explicitly requested
- Compute a unit rate of two familiar rational numbers where the unit rate is explicitly requested
- Compute a unit rate of two rational numbers where the unit rate is not explicitly requested
- Identify proportional relationships presented in familiar contexts
- Find the whole number constant of proportionality in relationships presented in basic familiar contexts

Accomplished (all of Proficient +):
- Compare unit rates in a real-world context
- Use different representations of proportional relationships to solve real-world problems
- Apply proportional relationships to routine real-world and mathematical ratio and percent problems with multiple steps.

Advanced (all of Proficient + all of Accomplished +):
- Analyze a graph of a proportional relationship in order to explain what the points (x,y) and (1,r) represent, where r is the unit rate, and use this to solve problems.
- Apply proportional relationships to **non-routine** real-world and mathematical ratio and percent problems with multiple steps.

- Represent proportional relationships in various formats
- Solve a one-step, straightforward ratio or percent problem
- Solve a one-step, straightforward real-world ratio or percent problem
- Use proportional relationships to solve routine real-world and mathematical ratio and percent problems with multiple steps

Prior Standard(s)	Future Standard(s)	
6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	**7.G.1** Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals. **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. **8.EE.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b. **8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. **8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading	**N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **A.SSE.3c** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions. **F.IF.8b** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions. **G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **G.C.5** Find arc lengths and areas of sectors of circles. **G.MG.2** Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot.

	these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 1, page 43
- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45
- Ohio's K-8 Learning Progressions, Ratio and Proportional Relationships, page 15

Instructional Strategies

In Grade 6 students reasoned about ratios using models such as tables, double number lines, tape diagrams, and graphs. They avoided using fraction notation for ratios and did not set up nor explicitly solve proportions. Now in Grade 7, students should be able to set up proportions using fraction notation. Note: Solving problems using cross products should be avoided.

Applications should now focus on solving unit-rate problems with more sophisticated numbers. Entries in tables and unit rates can be rational numbers including complex fractions. For scaffolding ideas and more information about ratios and rates see Model Curriculum Grade 6.RP.1-3.

Students obtain proportional reasoning when they understand that the ratio of the two quantities remains constant even though the corresponding values of the quantities may change ($y = kx$). In other words, the relationship of the first quantity compared to the amount of the second quantity is always the same regardless if the quantities increase or decrease.

It is important that students are able to differentiate between situations that are directly proportional and those that are not. Otherwise, they may haphazardly apply proportional techniques to nonproportional situations. That means they need to carefully attend to the relationships in the problem.

One way to view and reason with proportions is to use within and between relationships. Within relationships focus on making comparisons within the same units/measure-space such as 180 miles: 60 miles = 6 gallons: 2 gallons. Whereas between relationships focus on making comparisons between different units/measure-space such as 180 miles: 6 gallons = 60 miles: 2 gallons.

Have students explore graphs that are proportions and those that are not. Given various graphs, they may make tables using three points on the graph and decide whether they are proportional or not. Ask students what all proportional graphs have in common. Students should come to the conclusion that a proportional graph is a straight line that goes through the origin.

Sample Assessments and Performance Tasks

Reporting Category: Ratios and Proportions **Standards:** 7.RP. 1, 2, and 3 **Approximate Portion of Test:** 22% - 31%; 12 - 16 points	**OST Test Specs:** Items may use all types of rational numbers. At least one number in the ratio must be expressed as a fraction or a decimal. Ratios can be expressed as a fraction (1/5), with a colon (1:5), or with words, e.g., per, to, each, for each, for every.

Instructional Resources		
7.RP.1 Better Lesson Shmoop Khan Academy Videos Dan Meyer Activities Pizza Doubler Yellow Starbursts Ticket to Ride Illustrative Mathematics Cooking With the Whole Cup Cider vs. Juice - Variation 1	7.RP.2 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Sore Throat - Variation 1 Buying Coffee	7.RP.3 Dan Meyer Activity Holes Illustrative Mathematics How Fast is Usain Bolt?

Adopted Resource	
Reveal: Lesson 1-1: Unite Rates Involving Ratios of Fractions Lesson 1-2: Understand Proportional Relationships Lesson 1-3: Tables of Proportional Relationships Lesson 1-4: Graphs of Proportional Relationships	**Aleks:** Ratios, Proportions, and Measurement (ALEKS TOC): ● Ratios and Unit Rates ● Proportions Fractions (ALEKS TOC): ● Multiplication and Division with Fractions

Return to Scope and Sequence

Module 8: Linear Relationships and Slope

Unpacked Standards / Clear Learning Targets

Learning Target	Essential Understanding	Academic Vocabulary
8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. **8.EE.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.	• The slope is a constant ratio between the rise and the run for any two points on a line. • A graph of a proportional relationship is a line that passes through the origin. • Only the slope, m, of the equation y = mx represents a proportional relationship. • Slope is represented by m in the equation y = mx or y = mx + b. • Corresponding angles in similar right triangles are equal. • Corresponding sides of similar triangles are proportional. • A line in the form y = mx and intersects the origin. • A line in the form y = mx + b intersects the y-axis at (0, b) with b being the y-intercept. *Note: A linear function has neither a slope nor a y-intercept. But the graph of a linear function has both.* • A relationship between two variables can be represented as a graph, table, equation, or verbal description.	Constant of proportionality Constant of variation Direct variation Initial value Linear relationship Rate of change Proportional relationship Slope Y-intercept Origin Corresponding angles Corresponding sides Similar triangles Intersect

I Can Statements:

- I can determine the rate of change by the definition of slope.
- I can graph proportional relationships from data or equations.
- I can compare/contrast slope and rate of change.
- I can compare two different proportional relationships represented in different ways.
- I can interpret the unit rate of proportional relationships as the slope of a graph.

Performance Level Descriptors:

Proficient:
- Graph proportional relationships, interpreting the unit rate as the slope
- Graph proportional relationships, interpreting the unit rate as the slope and compare two different proportional relationships using the same representation

Accomplished (all of Proficient +):
- Apply understanding of slope to solve routine problems graphically and algebraically

Advanced (all of Proficient + all of Accomplished +):
- Apply understanding of slope to solve non-routine problems graphically and algebraically

- Graph proportional relationships, interpreting the unit rate as the slope and compare two different proportional relationships using different representations
- Determine the slope of a line given a graph

Prior Standard(s)	Future Standard(s)
7.RP.2 Recognize and represent proportional relationships between quantities.	**8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 8, Number 1, page 50-51
- Ohio's K-8 Critical Areas of Focus, Grade 8, Number 3, pages 53-54
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

Instructional Strategies

Students in Grade 7 represented proportional relationships as equations such as $y = kx$ or $t = pn$. They also graphed proportional relationships, discovering that a graph of a proportion must go through the origin, and that in the point (1, r), r is the unit rate. Now in Grade 8, the unit rate of a proportion is used to introduce "the slope" of the line.

Students need to make connections between the different representations (equations, tables, graphs) in order to come to a unified understanding that the different representations are in essence different ways of modeling the same information.

Explicit connections need to be made between the multiplicative factor, the slope, scale factor, and an increment in a table.

To reinforce the relationships between the x and the y, students should continually name quantities for the real-world problem they represent. They should also identify the independent and dependent variables.

By using coordinate grids and various sets of similar triangles, students can prove that the slopes of the corresponding sides are equal, thus making the unit rate or rate of change equal.

Use graphing utilities such as Desmos to show the lines in the form of $y = mx + b$ as vertical translations of the equation $y = mx$.

Sample Assessments and Performance Tasks

Reporting Category: Equations and Expressions	OST Test Specs:
Standards: 8.EE.5 and 6 **Approximate Portion of Test: 20% - 29%; 11 - 15 points**	Items may use all types of rational numbers. Items pertain only to direct proportional relationships. All triangles will be right triangles and in a coordinate grid.

Instructional Resources

8.EE.5	8.EE.6
Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Coffee by the Pound Comparing Speeds in Graphs and Equations Peaches and Plums Sore Throats, Variation 2 Stuffing Envelopes Who Has the Best Job?	Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Slopes Between Points on a Line

Adopted Resource

Reveal:	ALEKS:
	Graphs, Functions, and Sequences (ALEKS TOC): • Proportional Relationships • Slope • Direct and Inverse Variation • Equations of Lines • Tables and Graphs of Lines • Ordered Pairs

	Ratios, Proportions, and Measurement (ALEKS TOC): ● Proportions ● Ratios and Unit Rates ● Similar Figures

Return to Scope and Sequence

Functions

Unpacked Standards / Clear Learning Targets		
Learning Target **8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. *Function notation is not required. **8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **8.F.3** Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	**Essential Understanding** • A function is a rule that assigns each input exactly one output. • The graph of a function is a set of ordered pairs consisting of an input and a corresponding output. • Functions can be represented as an equation, graph, table, and verbal description. • Properties of graphs of linear functions include slope/rate of change, y-intercept/initial value, x-intercept, where the slope is increasing, constant, or decreasing. • A vertical line has an undefined slope, where y is not a function of x. • A graph of a linear function is a non-vertical straight line. • A nonlinear function is a function whose graph is not a straight line. • A table represents a linear function when constant differences between input values produce constant differences between output values. • Linear functions have a constant rate of change. • Some functions are not continuous	**Academic Vocabulary** Domain Input Output Range Initial value Linear function Rate of change Slope Y-intercept

I Can Statements:

- I can define a function.
- I can determine if an equation represents a function.
- I can apply a function rule for any input that produces exactly one output.
- I can generate a set of ordered pairs from a function and graph the function.

- I can recognize the equation y=mx+b is the equation of a function whose graph is a straight line where m is the slope and b is the y-intercept
- I can provide examples of nonlinear functions using multiple representations (tables, graphs, and equations).
- I can compare the characteristics of linear and nonlinear functions using various representations.
- I can determine the rate of change (slope) and initial value (y-intercept) from two (x,y) values, a verbal description, values in a table, or graph.
- I can construct a function to model a linear relationship between two quantities.
- I can relate the rate of change and initial value to real world quantities in a linear function in terms of the situation modeled and in terms of its graph or a table of values.

Performance Level Descriptors:

Proficient:
- Identify whether a relation is a function from a graph or a mapping
- Given tables of ordered pairs, determine if the relation is a function
- Complete a table to show a relation that is or is not a function
- Compare properties (i.e. slope, y-intercept, values) of two functions in a graph
- Compare properties (i.e. slope, y-intercept, values) of two functions represented in the same way (algebraically, graphically, or verbal descriptions)
- Compare properties (i.e. slope, y-intercept, values) of two functions each represented in a different way (algebraically, graphically, numerically in tables, or verbal descriptions)
- Given a straight forward qualitative description of a functional relationship between two quantities, sketch a graph
- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values
- Construct a function to model a linear relationship between two quantities

Accomplished (all of Proficient +):
- Justify whether two functions represented in different ways are equivalent or not by comparing their properties

Advanced (all of Proficient + all of Accomplished +):
- Explain why a function is linear or nonlinear
- Interpret qualitative features of a function in a context
- Strategically and efficiently choose different ways to represent functions in solving a variety of problems

Prior Standard(s)	Future Standard(s)		
7.RP.2 Recognize and represent proportional relationships between quantities. **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. **8.EE.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.	**A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **F.BF.1** Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from context. **F.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. **F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).	**F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions. **F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	**F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically **F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context. **S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **G.CO.2** 2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 8, Number 2, page 52
- Ohio's K-8 Learning Progressions, Functions, page 20

Instructional Strategies

Students should be expected to reason from a context, a graph, or a table, after first being clear which set represents the input (e.g., independent variable) and which set is the output (e.g., dependent variable). When a relationship is not a function, students should produce a counterexample: an "input value" with at least two "output values." If the relationship is a function, the students should explain how they verified that for each input there was exactly one output.

In Grade 6 students explored independent and dependent variables, and how the dependent variable changes in relation to the independent variable. In Grade 8 students need to continue identifying the independent and dependent variables in functions. Students need practice justifying the relationship between the independent and dependent variable.

In Grade 6 students explored independent and dependent variables, and how the dependent variable changes in relation to the independent variable. In Grade 8 students need to continue identifying the independent and dependent variables in functions. Students need practice justifying the relationship between the independent and dependent variable.

The standards explicitly call for exploring functions numerically, graphically, verbally, and algebraically. For fluency and flexibility in thinking, students need experiences translating among these different representations.

Students need experience translating among the different representations using different functions. For example, they should be able to determine which function has a greater slope by comparing a table and a graph.

Students need to compare functions using the same representation. For example, within a real-world context, students compare two graphs of linear functions and relate the graphs back to its meaning within the context and its quantities. Students should work with graphs that have a variety of scales including rational numbers.

In Grade 8, the focus is on linear functions, and students begin to recognize a linear function from its form $y = mx + b$ knowing that $y = mx$ as a special case of a linear function. Students also need experiences with nonlinear functions. This includes functions given by graphs, tables, or verbal descriptions but for which there is no formula for the rule.

When plotting points and drawing graphs, students should develop the habit of determining, based upon the context, whether it is reasonable to "connect the dots" on the graph. In some contexts, the inputs are discrete, and connecting the dots is incorrect. For example, if a function is used to model the height of a stack of n paper cups, it does not make sense to have 2.3 cups.

Sample Assessments and Performance Tasks

Reporting Category: Equations and Expressions **Standards: 8.EE.5 and 6** **Approximate Portion of Test: 20% - 29%; 11 - 15 points**	**OST Test Specs:** Items may use all types of rational numbers. Items pertain only to direct proportional relationships. All triangles will be right triangles and in a coordinate grid.

Instructional Resources

8.F.1	8.F.2	8.F.3
Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Foxes and Rabbits Function Rules Introducing Functions Pennies to heaven The Customers US Garbage, Version 1	Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Battery Charging	Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Introduction to Linear Functions

8.F.4	**8.F.5**
Better Lesson Shmoop Khan Academy Videos Dan Meyer Activity 25 Billion Apps Illustrative Mathematics Baseball Cards Chicken and Steak, Variation 1 Chicken and Steak, Variation 2 Delivering the Mail, Assessment Variation Distance across the channel Downhill High School Graduation Video Streaming	Better Lesson Shmoop Khan Academy Videos Dan Meyer Activity Joulies Illustrative Mathematics Bike Race Distance Riding by the Library Tides

Adopted Resource	
Reveal: Located in Math 8 - Course 3 Lesson 5-1: Identify Functions	**ALEKS:** Graphs, Functions, and Sequences (ALEKS TOC): • Introduction to Functions

Lesson 5-2: Function Tables Lesson 5-3: Construct Linear Functions Lesson 5-4: Compare Functions Lesson 5-5: Nonlinear Functions Lesson 5-6: Qualitative Graphs	● Tables and Graphs of Lines ● Graphs of Functions ● Applications

Return to Scope and Sequence

Module 11: Geometric Figures

Unpacked Standards / Clear Learning Targets		
Learning Target 7.G.1 Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals. a. Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale. b. Represent proportional relationships within and between similar figures. 7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions. a. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. 7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. 7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. 8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	**Essential Understanding** **Special Angle Pairs** • Two angles are supplementary when their angle measures have a sum of 180 degrees. • Two angles are complementary when their angle measures have a sum of 90 degrees. • Vertical angles are the angles opposite each other when two lines intersect; the angles are congruent. • Vertical angles are congruent because they are both supplementary to the same angle. • Two angles are adjacent when they have a common side and a common vertex and do not overlap. **Angle Relationships** • The sum of the measure of the interior angles of a triangle is 180 degrees. • Any exterior angle of a triangle is congruent to the sum of the measures of the two remote interior angles of the triangle. **Drawing Geometric Figures** • The sum of all three angles in any triangle equals 180 degrees. • The sum of all four angles in any quadrilateral equals 360 degrees. • Three possible outcomes exist when constructing triangles with given measurements of sides and/or angles: a unique triangle, more than one triangle, or no triangle. • Some quadrilaterals have more specific names based on relationships such as pairs of parallel sides, congruent sides, and angle relationships. **Similar Figures** • Angles are congruent if they are equal in measure. Note: 7th grade	**Academic Vocabulary** Angle Complementary Supplementary Vertical angle Angle Sum Side Vertex Adjacent Opposite Congruent Common Side Common Vertex Geometric figure Triangle Quadrilateral Unique Similar Figures Scale drawing Scale Factor Three-Dimensional Two-Dimensional Plane sections Protractor Right rectangular prisms Right rectangular pyramid

	students may use the term "equal in measure" in place of congruent. • Similar figures have corresponding angles that are congruent and corresponding side lengths that are proportional. • Applying a scale factor greater than one results in a bigger image. • Applying a scale factor of 1 results in a congruent image. Note: Students are not required to understand congruency of two figures until 8th grade. • Applying a scale factor less than 1 but greater than zero results in a smaller image. **Slicing Three-Dimensional Figures** • Slicing a three-dimensional figure results in a two-dimensional shape. • Slicing a three-dimensional figure in different ways could result in different shapes.	

I Can Statements:

- I can describe and locate supplementary, complementary, and vertical angles in figures.
- I can describe the relationship between supplementary, complementary, and vertical angles.
- I can write equations to represent angle relationships with unknown angle measures.
- I can solve equations for unknown angles using supplementary, complementary, vertical, and adjacent angles.
- I can construct triangles from three given angle measures to determine when there is a unique triangle, more than one triangle or no triangle using appropriate tools (freehand, rulers, protractors, and technology).
- I can construct triangles from three given side measures to determine when there is a unique triangle, more than one triangle or no triangle using appropriate tools (freehand, rulers, protractors, and technology).
- I can use ratios and proportions to create scale drawing.
- I can identify corresponding sides of scaled geometric figures.
- I can compute lengths and areas from scale drawings using strategies such as proportions.
- I can solve problems involving scale drawings of geometric figures using scale factors.
- I can reproduce a scale drawing that is proportional to a given geometric figure using a different scale.
- I can determine which conditions create unique triangles, more than one triangle, or no triangle.
- I can analyze given conditions based on the three measures of angles or sides of a triangle to determine when there is a unique triangle, more than one triangle or no triangle.
- I can define slicing as the cross-section of a 3D figure.
- I can describe the two-dimensional figures that result from slicing a three dimensional figure such as a right rectangular prism or pyramid.
- I can analyze three-dimensional shapes by examining two dimensional cross sections.

Performance Level Descriptors

Proficient:	Accomplished (all of Proficient +):	Advanced (all of Proficient + all of Accomplished +):
• Determine a scale from scale drawings of geometric figures and compute an actual length given a measurement in a scale drawing and the scale • Solve problems involving scale drawings of geometric figures, including computing actual areas from a scale drawing and represent proportional relationships among similar figures • Recognize simple geometric shapes based on given conditions • Draw geometric shapes with given conditions • Identify the two-dimensional figures that result from routine slices of prisms and pyramids • Classify pairs of angles • Determine whether a set of any three given angle or side length measurements can result in a triangle or whether a quadrilateral could be represented by given angles and/or side lengths • Using technology or math tools, determine whether a set of any three given angle or side length measures can result in a unique triangle, more than one triangle, or no triangles at all and construct quadrilaterals with given conditions • Use supplementary, complementary, vertical, or adjacent angles to solve problems with angles expressed as numerical measurements • Determine missing angle measures in triangles with exterior angles and/or angles formed by parallel lines cut by a transversal	• Solve real-world problems involving similar figures • Identify the two-dimensional figures that result from non-routine slices of prisms and pyramids • Use supplementary, complementary, vertical, and adjacent angles to solve multi-step problems with angle measurements expressed as variables in degrees. • Give an informal argument that a triangle can only have one 90° angle	• Reproduce scale drawings at a different scale to solve real-world problems • Solve a variety of real-world and mathematical problems involving the angles in triangles and those formed by when parallel lines are cut by a transversal, and give informal arguments

Prior Standard(s)	Future Standard(s)	
4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in	8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the	G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. G.SRT.4 Prove and apply theorems about triangles.

real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. 6.G.1 Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems. 7.RP.2 Recognize and represent proportional relationships between quantities. **6.G.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	equation $y = mx + b$ for a line intercepting the vertical axis at b. 8.G.1 Verify experimentally the properties of rotations, reflections, and translations. G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. G.CO.9 Prove and apply theorems about lines and angles. G.CO.10 Prove and apply theorems about triangles. G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).	G.GPE.4 Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. G.MG.3 Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.

Content Elaborations

- Ohio's K-8 Critical Area of Focus Grade 7, Number 3, pages 46-47
- Ohio's K-8 Learning Progressions, Geometry, page 21
- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 1, page 43

- Ohio's K-8 Critical Areas of Focus, Grade 8, Number 3, pages 53-54
- Ohio's K-8 Learning Progressions, 6-8 Geometry, page 21

Instructional Strategies

This cluster focuses on the importance of visualization in the understanding of Geometry. Being able to visualize and then represent geometric figures on paper is essential to solving geometric problems. After much work is done on paper, Geometry software can aid in students' understanding of Geometry.

Provide students the opportunities to explore angle relationships. At first they can measure and find patterns among the angles of intersecting lines or within polygons. Then they can utilize the relationships to write and solve equations for multi-step problems.

A student often incorrectly thinks that a wide angle with short sides is smaller than a narrow angle with long sides. To confront this problem have students compare angles with different side lengths.

Students should regularly be exposed to shapes and figures from many perspectives and orientations, not just the prototypical example.

Many careers and everyday activities require spatial reasoning. Some research suggests that 7th grade is the optimal time for developing spatial visualization. Sketching figures can help students develop an intuitive understanding of geometry. Although drawings should become precise over time, informal free-hand sketches can help develop spatial reasoning.

Constructions facilitate understanding of geometry. Provide opportunities for students to physically construct triangles and quadrilaterals with straws, sticks, or geometry apps. This should be done prior to using rulers, compasses, and/or protractors. Have students discover and justify the side and angle conditions that will form triangles or quadrilaterals.

Similarity is an increase or decrease that is multiplicative in nature instead of additive; this is a new concept for students.

Although the focus of this cluster is on rectangles and triangles, it may be useful to discuss why all circles are similar.

As an introduction to scale drawings in geometry, students should be given the opportunity to explore scale factor as the number of times you multiply the side measure of one figure to obtain the corresponding side measure of a similar figure. It is important that students first experience this concept concretely progressing to abstract contextual situations. Pattern blocks provide a convenient means of developing the foundation of scale. Choosing one of the pattern blocks as an original shape, students can then create the next-size shape using only those same-shaped blocks. After students have time to use the shapes concretely, they should also practice drawing them.

Regularly provide students with figures that are not similar to ensure that students are continually checking for similarity.

Provide opportunities for students to use scale drawings of geometric figures with a given scale. The opportunities should require them to draw and label the dimensions of the new shape. Initially, measurements should be in whole numbers, progressing to measurements expressed with rational numbers. This will challenge students to apply their understanding of fractions and decimals.

Provide word problems that require finding missing side lengths, perimeters, or areas. In addition, allow students to design their own word problems asking for missing side lengths, perimeters, and/or areas.

Slicing three-dimensional figures to observe the cross sections formed helps develop three-dimensional visualization skills. Students should have the opportunity to physically create some of the three-dimensional figures, slice them in different ways, and describe in pictures and words what they discover. For example, use clay or playdough to form a cube, then pull string through it at different angles and record the shape(s) of the slices found. Challenges can be given: “See how many different two-dimensional cross sections you can create by slicing a cube."

In Grade 7, students develop an understanding of the special relationships of angles and their measures (complementary, supplementary, adjacent, and vertical). Now in 8.G.5 the focus is on learning about the sum of the measures of the interior angles of a triangle and exterior angle of triangles by using transformations.

This might be a good time to introduce vocabulary of the types of angles, such as interior, exterior, alternate interior, alternate exterior, corresponding, same side interior, and same side exterior. Students are expected to recognize but not memorize this vocabulary.

In Grade 7 students should have had some practice exploring that the sum of the angles inside a triangle equal 180 degrees. Now students use transformations to prove it.

Students can create a triangle and use rotations and transformations to line up all the angles to prove that the sum of the interior angles of a triangle equals 180 degrees. They need to be able to demonstrate and explain why the sum of the interior angles equals 180 degrees.

Students should build on this activity to explore exterior angle relationships in triangles. They can also extend this model to explorations involving other parallel lines, angles, and parallelograms formed. Students should be able to explain why two angles in a triangle have to be less than 180 degrees.

Investigations should lead to the Angle-Angle criterion for similar triangles. For instance, groups of students should explore two different triangles with one, two, and three given angle measurements. Students observe and describe the relationship of the resulting triangles. As a class, conjectures lead to the generalization of the Angle-Angle criterion.

Sample Assessments and Performance Tasks

Reporting Category:
Geometry
Standards:
7.G.1, 2, 3, and 5
Approximate Portion of Test:
20% - 25%; 11 - 13 points (with Module 9)

Standards: 8.G.5

Approximate Portion of Test: 28% - 37%; 15 - 19 points
OST Test Specs:
Figures are limited to triangles and special quadrilaterals
Scale factors can be any positive rational number not equal to 1.
Figures are limited to triangles and quadrilaterals.

Items willvolus on Van Hiele Level 1 (Analysis) with some aspects of Van Hiele Level 2 (Informal Deduction/Abstraction). Items will not require knowledge of the hierarchy of quadrilaterals (e.g., all squares are rhombuses but not all rhombuses are squares) Prisms and pyramids can have bass up to six sides All slices will be parallel or perpendicular to the base of the figure Items may use all types of rational numbers Angles must be measured in degrees. Items may require the student to refer to two angles as "equal in measure". Items will not require students to understand congruency of two figures. ● Facts are limited to angle sum, exterior angles of triangles, angles created when parallel lines are cut by a transversal and the angle-angle criterion for similarity of triangles. ● For the converse of the Pythagorean Theorem, only perfect squares are used. ● Items that apply the Pythagorean Theorem are aligned to 8.G.7 or 8.G.8. ● If the triangle is part of a three-dimensional figure, a graphic of the three-dimensional figure will be included. ● Points must either be at the intersection of two grid lines or their coordinates must be given.

Instructional Resources		
7.G.1 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Floor Plan Rescaling Washington Park	7.G.2 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics A Bug's Life	
7.G.3 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics Cube Ninjas Dan Meyer Activity World Record Ballon Dog	8.G.5 Better Lesson Shmoop Khan Academy Videos Illustrative Mathematics A Triangle's Interior Angles Congruence of Alternate Interior Angles via Rotations Find the Angle	

7.G.5 Better Lesson Shmoop Khan Academy Videos	Find the Missing Angle Rigid motions and congruent angles Similar Triangles I Similar Triangles II Street Intersections Tile Patterns II: hexagons Tile Patterns I: octagons and squares
Adopted Resource	
Reveal:	**ALEKS:** Lines, Angles, and Polygons (ALEKS TOC): • Angle Relationships • Classifying and Measuring Angles • Triangle Constructions and Triangle Inequalities • Classifying Triangles Ratios, Proportions, and Measurement (ALEKS TOC): • Scale Factors and Scale Drawings • Proportions Perimeter, Area, and Volume (ALEKS TOC): • Three-Dimensional Figures

Return to Scope and Sequence

Pythagorean Theorem

Unpacked Standards / Clear Learning Targets

Compacted Math 7/8

Learning Target -	Essential Understanding Extended Understanding •	Academic Vocabulary

Ultimate Learning Target Type:	Broad Learning Target: – The student can Underpinning Knowledge Learning Targets: – The student – Underpinning Skills Learning Targets: – The student – Underpinning Reasoning Learning Targets: – The student can

Prior Standard(s)	Future Standard(s)

Content Elaborations

Compacted Math 7/8

Instructional Strategies

Sample Assessments and Performance Tasks

OST Test Specs:

Instructional Resources

Adopted Resource

Reveal:	Aleks:

Return to Scope and Sequence

Module 13: Transformations, Congruence, and Similarity

Unpacked Standards / Clear Learning Targets		
Learning Target -	**Essential Understanding** **Extended Understanding** •	**Academic Vocabulary**

Ultimate Learning Target Type:	**Broad Learning Target:** – The student can **Underpinning Knowledge Learning Targets:** – The student – **Underpinning Skills Learning Targets:** – The student – **Underpinning Reasoning Learning Targets:** – The student can

Prior Standard(s)	Future Standard(s)

Content Elaborations

Instructional Strategies

Sample Assessments and Performance Tasks

OST Test Specs:

Instructional Resources	
Adopted Resource	
Reveal:	**Aleks:**

Return to Scope and Sequence

Module 9: Probability

Unpacked Standards / Clear Learning Targets		
Learning Target -	**Essential Understanding** **Extended Understanding** •	**Academic Vocabulary**

<table>
<tr><td>Ultimate Learning Target Type:</td><td colspan="2"><u>Broad Learning Target:</u>
– The student can

<u>Underpinning Knowledge Learning Targets:</u>
– The student
–
<u>Underpinning Skills Learning Targets:</u>
– The student
–

<u>Underpinning Reasoning Learning Targets:</u>
– The student can</td></tr>
<tr><td>Prior Standard(s)</td><td colspan="2">Future Standard(s)</td></tr>
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Content Elaborations
Instructional Strategies

Sample Assessments and Performance Tasks
OST Test Specs:

Instructional Resources
Adopted Resource

Reveal:	Aleks:

Return to Scope and Sequence

Module 10: Sampling and Statistics

Unpacked Standards / Clear Learning Targets		
Learning Target -	**Essential Understanding** **Extended Understanding** •	**Academic Vocabulary**

Ultimate Learning Target Type:	**Broad Learning Target:** – The student can **Underpinning Knowledge Learning Targets:** – The student – **Underpinning Skills Learning Targets:** – The student – **Underpinning Reasoning Learning Targets:** – The student can

Prior Standard(s)	Future Standard(s)

Content Elaborations		
Instructional Strategies		
Sample Assessments and Performance Tasks		
OST Test Specs:		

Instructional Resources	
Adopted Resource	
Reveal:	**Aleks:**

Return to Scope and Sequence